WRITTEN PRELIMINARY Ph.D EXAMINATION

Department of Applied Economics
Jan./Feb. - 2010
Trade, Development and Growth

For students electing
Macro (8701) & Micro (8703) option

Instructions

• Identify yourself by your code letter, not your name, on each question
• Start each question’s answer at the top of a new page
• You are requested to answer a total of FOUR questions
• Answer ONE question from Set One
• Answer THREE questions from Set Two
• You have four hours to complete this examination
I. Separability in economic models of agricultural households.

For this problem you will check whether separability holds when the assumptions to the model are changed. To start, here is the basic model for a two person household, as given in Apec 8703:

\[
U(c_1, c_2, l_1, l_2)
\]

\[
p(c_1 + c_2) + wL^h + rA^h = pF(L, A) + w(L_1^m + L_2^m) + rA^m
\]

\[
L = L_1^f + L_2^f + L^h
\]

\[
A = A^f + A^h
\]

\[
E^A = A^f + A^m, \quad E^L_i = L_1^f + L_2^f + l_i \text{ for } i = 1, 2
\]

\[
c_i, l_i, L_1^f, L_1^m, L^h, A^f, A^m, A^h \geq 0 \text{ for } i = 1, 2
\]

where \( p = \) price of consumption good
\( w = \) market wage rate
\( r = \) market rate for renting land
\( L = \) labor
\( A = \) land
\( F(L, A) = \) prod. function for consumption good
\( E = \) family endowment (exogenously given)

The household wants to maximize utility, as given in equation (1), subject to the constraints in equations (2), (3), (4), (5) and (6).

1. Suppose that you are looking at households in North Korea. Hiring agricultural labor has been outlawed because it contradicts the government ideology, and working for wages is also outlawed. In other words, there is no labor market at all, but there still is a well functioning land market. Rewrite the above model for the case where there is no labor market. Note that you should find that the model simplifies, and that some notation can be removed.
2. Does separability hold when there is no labor market? One way to determine this is to examine whether the household’s profit maximization decision is made without reference to the utility function. Assume that the production technology is constant returns to scale (CRS); this implies that you can write the production function as $F(L, A) = LF(1, A/L) = Lf(A/L)$, where $f(A/L)$ is output per unit of labor. Let “cash profits”, denoted by $\Pi_{\text{cash}}$, be defined as: $\Pi_{\text{cash}} = pF(L, A) - rA$. Show that the first order condition for maximizing cash profits with respect to $A$ implies that $f(A/L)$ equals $r/p$, where $f(A/L)$ is $\partial f(A/L)/\partial (A/L)$. [Hint: First work out the relationship between $\partial F(A, L)/\partial A$ and $f(A/L)$.]

3. Use your result in 2., along with the assumption that $F(A, L)$ is convex with respect to both $A$ and $L$, to show that the optimal value of $A/L$ for maximizing profit is uniquely determined by $r/p$.

4. Does your answer to 3. imply that, assuming constant returns to scale, the absence of a labor market still leads to separability as long as there is a land market? If not, is there some property that holds which is similar to separability? Give the intuition for your answer. Please be brief!

II. Trade distortion and growth

During the recent financial crises which coincided with a sharp rise in primary commodity prices, India, Russia and several other countries instituted taxes and export quotas on their agricultural exports in order to lessen impacts of higher food prices on consumers. This question asks you to explain how interventions to restrict exports are likely to affect a lower income economy that is a net exporter of agricultural goods and a net importer of industrial goods.

Consider a three sector, small, competitive and open "Ramsey - endogenous savings" economy that produces three final goods (industry, agriculture and services), employing labor and capital to produce the industrial and service sector good, and labor, capital ($k$) and land ($h$) to produce the agriculture good. Households hold homothetic preferences and exchange the services of these resources for wages, rents to capital and land, which they allocate to consumption of the three goods and to savings. The economy is
in transition growth with capital stock in period $t = 0$, less than its long-run equilibrium level. For purposes here, let the rate of technological change and the rate of population growth equal zero, let the industrial sector be the most capital intensive sector and let the service sector be the most labor intensive of the three sectors. To capture the effect of the policy to restrict agricultural exports, let the export tax equivalent of a quota on agricultural exports be $\tau > 0$, so that the price faced by agriculture is $p_a = p^w_a (1 - \tau)$ where $p^w_a$ is the world price of the agricultural good. To limit the scope of this question:

1. Characterize the intra-temporal equilibrium conditions for this economy (you need not state the primitives, just their dual counter parts like cost and profit functions)

2. From this characterization, derive/state -in general form- the reduced form equations for wages $w$, the capital rental rate $r^k$, and the three supply functions $y_m$ (industry), $y_s$ (services) and $y_a$ (agriculture).

3. Using the fact that, for an interior solution, the price of home goods can also be expressed as a function of the prices of the two traded goods, $p_m, p_a$ and factor endowments, that is

$$p_s = P^s (p_m, p_a, k, h)$$

and given (1.), (2.), explain why for $\tau > 0$ compared to $\tau = 0$,

(a) the transition of wages is lower $\dot{w} (t) / w (t) |_{\tau > 0} \leq \dot{w} (t) / w (t) |_{\tau = 0}$

(b) the transition of capital payments is higher $\dot{r}^k (t) / r^k (t) |_{\tau > 0} \geq \dot{r}^k (t) / r^k (t) |_{\tau = 0}$

(c) the transition of the price of home goods is lower $\dot{p}_s (t) / p_s (t) |_{\tau > 0} \leq \dot{p}_s (t) / p_s (t) |_{\tau = 0}$

(d) the rate of capital deepening is higher $\dot{k} (t) / k (t) |_{\tau > 0} \geq \dot{k} (t) / k (t) |_{\tau = 0}$

4. Explain the likely effects of $\tau > 0$ relative to $\tau = 0$, on the

(a) Rental rate of agricultural land

(b) Agricultural supply, and
III. Modeling non-agricultural household businesses.

This question asks you to modify the standard agricultural household model to non-agricultural businesses. It focuses on trying to understand why the return to capital in those businesses is often much higher than the prevailing interest rate (rental price for capital). The model is the following. The household wants to maximize expected utility, where utility depends only on consumption, and the choice variables are capital used in production ($K$), amount of household assets used in production $A_K$, and amount of borrowed capital ($B$). For simplicity, the household’s labor supply is assumed to be fixed and all of that labor works on the household business. More formally, we have:

$$\text{Maximize : } E[U(c)]$$

conditional on:

$$c = \varepsilon F(K) - rB + r(E^A - A_K)$$

$$K = A_K + B; \text{ (capital is from household stock or from borrowing)}$$

$$B \leq \bar{B}; \text{ (credit constraint)}$$

Note that $\varepsilon$ is a random variable with $E[\varepsilon] = 1$, and $E^A$ is the household’s endowment of assets (any assets not used in production can be rented out at the interest rate $r$), and household labor is not shown in the $F(\cdot)$ function because it does not vary.

1. Assume that the credit constraint is binding, so that $B = \bar{B}$. Write down the Lagrangian for this maximization problem, using $\lambda$ to denote the Lagrangian multiplier on the capital constraint. Don’t forget that you are maximizing expected utility.
2. Consider the special case where insurance markets work perfectly, which you can interpret as assuring that $\varepsilon$ always equals 1. However, still assumed that credit constraints are binding. For your answer to 1., find the first order condition with respect to capital and use it to write an expression for the marginal product of capital. Compared to the case of no credit constraints (and still assuming perfect insurance), is the marginal productivity of capital under a binding credit constraint higher or lower? [Hint 1: For this no uncertainty case, you can take out the expectations sign before doing any derivations.] [Hint 2: It is useful to use the capital constraint to substitute $B$ out of the utility function before differentiating.]

3. Next, assume that the credit constraint is not binding, but that $\varepsilon$ is a random variable (this is the uncertainty case). Set up the Lagrangean, derive the first order condition with respect to the amount of borrowed (not with respect to capital), and write an expression for the marginal product of capital. Compared to the case of no uncertainty (and no credit constraints), is the marginal productivity of capital for the uncertainty case higher or lower? [Hint 1: Assume the household wants to borrow (so that it does not rent out its capital), in which case $E_A = A_k$ and then $K = B + E_A$. Substitute out $K$, so no need for any Langrangean multiplier, and differentiate with respect to $B$.][Hint 2: You need to develop a covariance term after deriving the first order condition.][Hint 3: for this model the derivative of an expectation is the expectation of the derivative of the term inside the expectations brackets]

4. Suppose you have data from a sample of households that operate small businesses. The person who collected the data asked each household to answer some questions that can be used to measure households’ level of risk aversion. How would you use the data to determine whether the cause of marginal returns to capital being higher than the interest rate is due to riskiness in production?

IV. Debt and financial crises

Consider the case of a country that accumulated external debt because of fiscal deficits caused by spending that had no direct effects on productivity
1. Depict and explain the short-run equilibrium of such a country for which the external debt suggests the country is living beyond its means. (You may use graphics for part of your answer to this question). Discuss the effects on the

   (a) The domestic terms of trade,
   (b) The production of traded and home goods, and
   (c) On factor payments.

2. It is often the case (with Argentina and Turkey as prime examples) that some foreign and domestic investors providing increments of new capital investment to the country become aware of the instability of the short-run equilibrium. They respond by withholding new investments, and even engaging in capital flight. That is, they withdraw some of their "capital" from the country and invest in other "safe havens" such as U.S. treasury bills. Discuss how "capital flight" can affect the short-run equilibrium (1.a), (1.b) and (1.c) above.

3. Now, given your answer to 2., suppose foreign creditors are no longer willing to sustain the country's trade imbalance. Depict and explain the short-run equilibrium that could obtain at the depth of the crises. Discuss the effects on the

   (a) The domestic terms of trade,
   (b) The production of traded and home goods, and
   (c) On factor payments, and .
   (d) General household welfare

V. Measuring productivity & welfare effects of R&D

Measuring Productivity:

1. In a one output, two input world, use a carefully labeled diagram and associated explanation to show the effects of relative input price changes on measured multi-factor productivity when using a Laspeyres versus a Paasche indexing procedure to aggregate inputs, absent technical change.
2. With technical change, and using a Laspeyers index to aggregate inputs, do measured changes in multifactor productivity under- or overestimate the extent of technical change? Use a second, carefully labeled diagram and associated explanation to answer this question, again in a one output, two input world.

Measuring the Welfare Effects of R&D:

3. Using a clearly labeled figure of a multi-market model and associated explanation, evaluate the statement:

“Research spillovers to the rest of the world increase the national welfare of an innovating, exporting country compared with a situation of no spillovers, true or false.”

VI. Growth theory

This question draws upon your knowledge of growth theory.

Given the following primitives:

\[
\int_0^\infty \frac{e^{1-\theta} - 1}{1 - \theta} e^{(n-\rho)t} dt : \text{present value of utility}
\]

the flow budget constraint

\[
\dot{k} = w + k \left( r^k - n - \delta \right) - c
\]  

(7)

the technology

\[
Y = A \left( K^{\sigma} \left( e^{xt} L \right)^{1-\alpha} \right)^{1-\sigma} X^\sigma, \text{ where } X = \mu Y.
\]

Note \( \sigma \) = share in total cost of intermediate \( X \)

which, when rearranged and expressed in intensive form by dividing by \( e^{xt} L \) (units per effective work), yields

\[
\hat{y} = A^{\frac{1}{1-\sigma}} \mu^{\frac{\sigma}{1-\sigma}} \hat{k}^\alpha
\]  

(8)

1. State the present value Hamiltonian and
(a) Derive the Euler condition for this economy, and
(b) Briefly discusses its meaning for the case where $r^k > \rho + \delta$

To save time and avoid simple errors, take as given that firms in this economy maximize profits

$$\max_k \hat{\pi} = A^{\frac{r^k}{r^w}} \mu^{\frac{r^w}{r^k}} \hat{k}^\alpha (1 - \mu) - \hat{w} - r^k \hat{k}$$  \hspace{1cm} (9)

such that, $\hat{\pi} = \hat{y}^* (1 - \mu) - \hat{w}^* - r^k \hat{k}^* = 0$ for each $t$.

2. Use the Euler condition derived in 1. above (which you will need to express in "hat" form), and the budget constraint (7) (which will also need to be expressed in "hat" form), and the maximizing behavior of firms in (9) to derive the two differential equations for this model.

3. Solve for the steady state, more specifically,

(a) Use one of the equations derived in 2. above to solve for the steady state level of $\hat{k}^{ss}$.

(b) Log-differentiate the equation obtained in 3.a with respect to the scale parameter $A$, (i.e., to obtain $\hat{A}/A$) to show the change in $\hat{k}^{ss}/\hat{k}^{ss}$, and discuss the role/importance of the cost-share parameter $\sigma$.

4. Substitute the equation obtained in 3.a into the production function (8)

(a) Explain the meaning of the resulting equation

(b) Log-differentiate the equation obtained in 4.a with respect to the scale parameter $A$, (i.e., to obtain $\hat{A}/A$) to show the change in $\hat{y}^{ss}/\hat{y}^{ss}$, and discuss the role/importance of the cost-share parameter $\sigma$. 