WRITTEN PRELIMINARY Ph.D EXAMINATION

Department of Applied Economics
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Trade, Development and Growth

For students electing
Macro (8701) & Micro (8703) option

Instructions

• Identify yourself by your code letter, not your name, on each question
• Start each question’s answer at the top of a new page
• You are requested to answer a total of FOUR questions
• Answer ONE question from Set One
• Answer THREE questions from Set Two
• You have four hours to complete this examination
SET ONE:

Required Question; Answer ONE Question (I or II but not both)

I. Measurement of inequality and poverty

This is a set of questions regarding the measurement of inequality and poverty

1. Explain what a “Pigou-Dalton transfer” is, and what it implies for “good” indices of inequality.

2. Consider the first Theil inequality index:

\[
T_1(y) = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{\mu_y} \ln \left( \frac{y_i}{\mu_y} \right)
\]

where \( y_i \) is the income of person \( i \) and \( \mu_y \) is mean income. Show that it satisfies the population independence axiom and the mean income independence axiom. [Hint: For the population independence axiom just consider the case where each person in the original population is replaced by \( k \) identical people in the “new” population].

3. Next, show that a Pigou-Dalton transfer increases inequality for the first Theil inequality measure, as given above). [Hint: Just look at the terms for the two people whose incomes are changed by the transfer, and consider a very small (“marginal”) transfer.]

4. Next consider poverty indices. Recall that the FGT poverty index takes the form:

\[
P_\alpha = \left( \frac{1}{n} \right) \sum_{i=1}^{n} \left[ \frac{(z - y_i)}{z} \right]^{\alpha} \times I(z \geq y_i)
\]

where \( I(z \geq y_i) = 1 \) if \( z \geq y_i \) and \( I(z \geq y_i) = 0 \) if \( z < y_i \), and \( z \) is the poverty line. This becomes the “headcount” index when \( \alpha = 0 \). What potential problem does the “headcount” poverty index have if it is used as a guide for providing income transfers to the poor? Suppose \( \alpha = 1 \), in which case the FGT index becomes the “poverty gap” index. What potential problem does the “poverty gap” poverty index have if it is used as a guide for providing income transfers to the poor?
5. Finally, this question investigates how poverty indices may (or may not) be related to Social Welfare Functions. A typical social welfare function takes the following form:

\[ SW = \sum_{i=1}^{n} \lambda_i u(y_i) \]

where \( \lambda_i \)'s are fixed weights, utility is a function of income (which for this problem you can assume is equal to consumption), and \( u'(y_i) \geq 0 \) and \( u''(y_i) \leq 0 \). Consider the FGT poverty index with \( \alpha = 2 \):

\[ P_2 = (1/n) \sum_{i=1}^{n} (z - y_i)/z]^2 \times I(z \geq y_i) \]

for which \( I(z \geq y_i) = 1 \) if \( z \geq y_i \) and \( I(z \geq y_i) = 0 \) if \( z < y_i \), and \( z \) is the poverty line. Is it possible to express \( P_2 \) as a social welfare function of the type given above? If no, explain why not. If yes, show the expression (explain what is \( \lambda_i \) and what is \( u(y_i) \)). [Hints: 1. This is not a big complicated math problem; instead you need to think conceptually. 2. You can ignore \((1/n)\) in the expression for \( P_2 \).]

II. Poverty and economic growth

Suppose we characterize poverty by a condition where, in time period \( t = 0 \), consumption per worker \( c(0) \) is just slightly greater than that needed to sustain life, i.e., \( c(0) \approx c^e \) where \( c^e \) is the minimum level of consumption need to sustain life. For simplicity, let the economy be closed. This implies that this economy is just productive enough in period \( t = 0 \) to produce \( c(0) \) plus saving (which could be zero) so that the representative person is living at the margin of survival. Let the economy-wide production function (GDP) be

\[ Y(t) = AK(t)^\alpha L(t)^{1-\alpha}, \quad 0 < \alpha < 1 \]

utility of the representative household is given by

\[ U = \int_0^\infty \frac{(c(t) - c^e)^{1-\theta} - 1}{1-\theta} e^{(\theta-1)t} dt \]

and the flow budget constraint is (we omit the \( t \) notation)

\[ \dot{k} = w + k(r - n - \delta) - c \]
where $k$ is the capital stock per worker, $w$ is the wage rate, $r^k$ is the rate firms pay households for the use of capital, $n$ is the rate of growth in the work force, $\delta$ is depreciation and $c$ is the level of consumption per worker.

1. Derive the Euler condition for this economy, and discuss how:

(a) the intertemporal elasticity of substitution $1/\theta$ affects the growth rate of consumption and implicitly the returns to capital $k$
(b) subsistence consumption needs, $c^o$, affects the growth rate of consumption and implicitly the returns to capital $k$

2. Solve this model for the steady state rate of return to $k$, i.e., solve for $r^{k,ss}$

(a) How does the level of subsistence consumption needs, $c^o$ affect $r^{k,ss}$?
(b) What does your answer to 2.a imply about the transition to long-run equilibrium when initial conditions are $c(0) \gtrless c^o$, compared to the case where $c(0) \gg c^o$ (i.e., $c(0)$ is much greater than $c^o$)

3. Discuss the implications of this model for foreign aid that is designed to

(a) Increase the rate of growth so that the economy can approach the steady state in a shorter period of time
(b) How might foreign assistance, even in this simple framework, increase the "level" of the steady state?
SET TWO:

Answer THREE of the following four questions (III to VI)

III. Growth theory

Consider a small and open economy that is endowed in period $t = 0$, with $L (0)$ number of workers, $K (0)$ units of capital stock and $H$ amount of land. The country faces world prices $p_m$ for manufactured goods, and $p_a$ for agricultural goods. Technology and household preferences are of the typical neoclassical form assumed in growth theory. The economy is in transition to a long-run equilibrium growth path, i.e., the country’s initial capital stock is less than long-run levels, $K (0) < K^*$. Let the rate of Harrod neutral technological change augmenting labor services to be $x$, the labor force grows at rate $n$, and the Harrod rate of growth in land productivity is $\eta = x + n$. The country’s gross domestic product can be expressed in effective labor terms as

$$ \hat{\text{gdp}} (t) = p_m \hat{y}_m (t) + p_a \hat{y}_a (t) + p_s (t) \hat{y}_s (t) = \hat{w} (t) + r^k (t) \hat{k} (t) + \hat{\pi} (t) H, \quad t = 0, 1, \ldots $$

where $\hat{y}_j (t)$ is the $j$-th sectors output per effective worker, $p_s (t)$ is the price of the good produced and sold on the domestic market only, $\hat{w} (t)$ is the wage rate per effective worker, $r^k (t)$ is the rate of return to capital paid by firms, $\hat{k} (t)$ is capital stock per effective worker, and $\hat{\pi} (t)$ is the rent to land per effective worker required for the land rental market to clear at each $t$. Given assumptions regarding technology and preferences, the economy’s supply and factor rental functions are well defined. Using the supply functions for $\hat{y}_j (t), j = a, m, s$, or the factor payment functions for $\hat{w} (t), r^k (t)$, and $\hat{\pi} (t)$, the equilibrium-transition path for $t = 0, 1, \ldots$, we can obtain the economy’s GDP function for each $t$. Denote this result in un-normalized form as:

$$ \text{GDP} (t) = \mathbf{G} \left( p_m, p_s (t), p_a, L (0) e^{x+n} t, K (t), e^{(x+n) t} H \right) = \hat{\text{gdp}} (t) \left[ L (0) e^{(x+n) t} \right] $$

or to avoid confusion, we can state this equation as

$$ \text{GDP} (t) = \mathbf{G} \left( p_m, p_s (t), p_a, A (t) L (t), K (t), B (t) H \right) = \hat{\text{gdp}} (t) \left[ A (t) L (t) \right] $$

where

$$ A (t) \equiv e^{x t}, \quad L (t) = L (0) e^{n t}, \quad B (t) \equiv e^{\eta t} $$

We assume the equation is continuous and differentiable.
1. What are the homogeneity properties of the GDP function:

   (a) In prices \( p_m, p_n(t), p_a \)?
   
   (b) In endowments, \( A(t) L(t), K(t), B(t) H \)?

2. What do these properties imply about the properties of:

   (a) Supply functions?
   
   (b) The factor rental rate functions?

3. In transition growth, what is the rate of growth in \( K(t) \) relative to the rate of growth in labor services \( A(t) L(t) \)?

4. Suppose the production of the manufactured good \( Y_m(t) \) is the most capital intensive sector in the economy. Discuss and/or show how the evolution of \( \{ p_n(t), A(t) L(t), K(t), B(t) H \} \) are likely to affect the evolution of \( Y_m(t) \) over time.

5. What is the long-run rate of growth of:

   (a) \( GDP(t) \), and
   
   (b) \( K(t) \)?

IV. Land redistribution in developing countries.

This problem examines what happens when the government redistributes land to households to achieve both equity and efficiency goals. Assume that there are \( n \) households, and that the total land that can be distributed is \( L_{TOT} \). Assume as well that the government has the following social welfare function:

\[
SW = \Sigma_i^n w_i U(c_i)
\]

where \( w_i \) is the “weight” of household \( i \) in the social welfare function, \( U(c_i) \) is the utility of household \( i \), which depends on the consumption of that household, \( c_i \).
1. Suppose that utility has the functional form $U(c_i) = \ln(c_i)$. Assume also that the household uses its land ($L_i$) and other characteristics ($x_i$, which is a vector) to generate its consumption according to the following function: $\ln(c_i) = \alpha + \beta \ln(L_i) + \gamma x_i$. Derive the first order conditions for the government’s maximization problem with respect to the allocation of land to each household, subject to the constraint that

$$\sum_{i=1}^{n} L_i = L_{TOT}$$

2. Suppose that the government assigns weights ($w_i$) according to each household’s characteristics ($x_i$). In addition, there is a random element $v_i$ that is uncorrelated with $x_i$. Thus we have $\ln(w_i) = \delta'x_i + v_i$. Use your answer to part 1) to show that the “optimal” (in the sense of maximizing social welfare) allocation of land to household $i$ is the sum of a constant term, a linear function of $x$, and a random error term. [Hint: express the optimal $L_i$ in terms of $\ln(L_i)$.] Given your answer, is it possible to use data on farm households to estimate the welfare weights the government uses to allocate land to households (assuming that land is allocated for this purpose)? If not, why not? If so, what data do you need?

3. Using the expression for $\ln(c_i)$ given in part 1), and your answer to 2), express $\ln(c_i)$ as a function of a constant term, a linear function of $x$, and an error term.

4. Now, consider the “efficient” allocation of land, which is defined as the allocation of land that maximizes total consumption over the population. Show the first order conditions for this maximization problem. [Hint: for any variable $x$, if $\ln(x) = f(x)$ then $x = e^{f(x)}$.]

5. Using your answer to 4), express the “efficient” allocation of land as a function of household characteristics ($x_i$). [Do NOT include $c_i$ or $\ln(c_i)$ in your answer.]

6. Look at your answers to part 4) and part 5). Suppose that someone asserts that the government really wasn’t maximizing social welfare, but instead it was always just maximizing total consumption. If that is the case, what is the relationship between your coefficients on the $x_i$ variables in your answers to parts 2) and 5)? How can you use
your answer to part 3) to determine whether this person’s assertion is correct? [Hint: This is not a math question; it is a (rather tricky) conceptual question.]

V. Trade distortion and growth

World agriculture is highly distorted. Advanced countries subsidize agriculture causing these countries to export more agriculture goods then they otherwise would, thus depressing world agricultural prices. Moreover, the studies directed by Krueger, Schiff and Valdes in the 1980s, and a recently completed IBRD supported study directed by Anderson updating this work through 2007, find that many low income countries protect the import competing sector of their economies which is an implicit tax on agriculture in addition to the already downward distortion in world prices.

Make assumptions about the relative factor intensity of labor, capital and land in the production of goods in this economy and, assume it is a net importer of industrial goods and a net exporter of agricultural goods. For purposes here, let technological change $x$ and the growth in the labor force $n$ equal zero ($x = n = 0$). The first part of the question asks you to explain the transition path of the economy with distortions in place. The second part asks you to discuss its transition path if all distortions were removed.

1. With the trade protection in place, explain the "economics" giving rise to the transition path of the following endogenous variables (recall that the zero profit conditions will look something like; $C^m(w, r) = (1 + \tau)$, $C^s(w, r) = p_s$ where $\tau$ is the tariff rate (say 0.2) so that the domestic price for manufactures is higher than the world price, $w$ is the wage rate, $r$ is the capital rental rate, $p_s$ is the price of home goods, and $C^j(w, r)$ is the unit cost function for sector $j = m$(manufacturing), $s$(home good)).

   (a) $\dot{w}/w$, $\dot{r}/r$ (factor payments to labor, capital, respectively)
   (b) $\dot{p}_s/p_{s\cdot\cdot}$, $\dot{\pi}/\pi$ (the price of home goods, and land rental payments)
   (c) $\dot{y}_j/y_j$, $j = m$(manufacturing), $a$(agriculture), $s$(home good).
   (d) $\dot{g}\dot{d}/gdp$
   (e) What is the rate of growth of these values (given assumptions here) in the steady-state?
2. Now, suppose the country is in long-run equilibrium (i.e., the steady state) AND trade protection is removed, i.e., \( \tau = 0 \). Explain the economics of the country’s re-adjustment back to long-run equilibrium.

(a) First, compare and explain briefly the levels of selected variables when the steady state is reached under (2) compared to the case of (1). (Comment, typically we would choose some point in transition, but for purpose here we use the steady state equilibrium to make the question more clear). The variables are

i. Prices

\[
(w_{ss})^{protection} : (w_{ss})^{no~protection} ; (r_{ss})^{protection} : (r_{ss})^{no~protection} ;
\]

\[
(p_{s}^{pretection} : (p_{s}^{no~protection})
\]

ii. Quantities

\[
(y_{m}^{ss})^{protection} : (y_{m}^{ss})^{no~protection} ; (y_{s}^{ss})^{protection} : (y_{s}^{ss})^{no~protection} ;
\]

\[
(y_{a}^{ss})^{protection} : (y_{a}^{ss})^{no~protection}
\]

(b) What is the rate of growth in the undistorted economy of \( \frac{\Delta d}{d} \) in the steady-state?

3. Summarize in one paragraph the overall effect from the removal of distortions on this economy.

VI. Economic implications of R&D

Using standard welfare surplus approaches (and clearly and carefully labeled graphs) answer, illustrate and explain the following:

1. When supply and demand elasticities are of equal but opposite signs, producers and consumers share equally in the benefits from R&D. True or false, explain.

2. With a parallel research induced shift in supply, producers are always better off. True or false, explain.
3. In a large country-in-trade setting, absent international transfers of technology, consumers and producers in importing countries that do not innovate are both made worse off by research-induced supply shifts elsewhere in the world. True or false, explain.

4. From a national perspective, a large-in-trade, innovating country that exports is better off overall if its research results spillover to other countries. True or false, explain.