WRITTEN PRELIMINARY Ph.D. EXAMINATION

Department of Applied Economics

January 28, 2016

Consumer Behavior and Household Economics

Instructions

• Identify yourself by your code letter, not your name, on each question

• Start each question’s answer at the top of a new page

• There are SIX questions in this exam

• You are to answer a total of FOUR questions

• You MUST answer Question 1

• You MUST answer EITHER Question 2 OR Question 3, but NOT BOTH

• You must CHOOSE 2 of the 3 questions from Questions 4, 5 and 6

• You have four hours to complete the examination
QUESTION 1: Consumer Demand Theory

Consider a consumer with the following cost function:

\[
\log[c(u, p)] = \sum_{k=1}^{N} \alpha_k \log(p_k) + u \prod_{k=1}^{N} p_k^{\beta_k}.
\]

where there are N goods, u is utility, p_k is the price of good k (k goes from 1 to N), and there are N \(\alpha\) parameters and N \(\beta\) parameters.

a) Derive this consumer’s Hicksian demand for good i, where i is any value from 1 to N.

b) Derive this consumer’s indirect utility function.

c) Derive this consumers Marshallian (Walrasian) demand for good i.

d) Consider the adding up constraint (which is also known as Walras’ law). What conditions should the \(\alpha\) and \(\beta\) parameters satisfy to ensure that the adding up constraint holds for all possible prices and levels of utility?

e) Consider a particular good i. What values of the parameters determine whether good i is a luxury or a necessity? Note that it may not be necessary to consider all parameters of the system when examining a particular good.
Attention: Of Questions 2 and 3, you MUST answer ONE of them, but answer ONLY one of them. Do NOT answer BOTH of them.

QUESTION 2. Present-Biased Utility.

1. You have to write your second year paper. You can write it on Thursday (tired), Friday (tired), Saturday (in shape), or Sunday (in shape). You have a present-biased utility function.

The rewards for the four periods are $v = (15.5, 16, 22, 22)$ and the costs for the four periods are $c = (2, 5, 9, 17)$.

The cost is immediate but the reward is delayed. For simplicity, let $\beta = 1/2$ and $\delta = 1$.

a) If you are a time-consistent agent, when would you write the paper?

b) If you are a sophisticate, when would you write the paper?

c) If you are a naïf, when would you write the paper?

Show how you derive your results, and describe the intuition of your results.
**QUESTION 3: Prospect Theory**

Consider a Prospect Theory expected-value maximizer with a value function defined over gains and losses relative to a reference point 0, defined as no gains or losses.

He has some money invested, and each day the value of his investments goes up by $3000 with probability \( \frac{1}{4} \) or down by $1000 with probability \( \frac{3}{4} \), and the probability of “up” or “down” on the second day is independent of what happened on the first day.

Suppose first that he has the choice of checking his portfolio’s performance either at the end of each day, or only at the end of the second day. However, even if he chooses to check at the end of each day, he still cannot change his portfolio after the first day. His expected value is additive across days, so that if he checks at the end of each day, his total expected value equals his expected value from the first day plus his expected value from the second day. But if he checks only at the end of the second day, his total expected value is just his expected value from the sum of both days’ outcomes. (That is, he experiences his gains or losses whenever he checks, whether it is at the end of each day or only at the end of both days.)

Which will he prefer, to check his portfolio’s performance at the end of each day or to check only at the end of the second day? Explain, both algebraically and intuitively.
Attention: Of Questions 4, 5 and 6, you MUST answer TWO of them, but answer only TWO of them. Do NOT answer all THREE of them.

**QUESTION 4: Almost Ideal Demand System and Welfare**

Consider the Almost Ideal Demand System derived in Deaton and Muellbauer (1980). The log cost function for the model is of the form:

\[
\log C(u, p) = a(p) + b(p)u
\]

where,

\[
a(p) = \alpha_0 + \sum_i \alpha_i \ln p_i + 0.5 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j
\]

\[
b(p) = \beta_0 \prod_j p_j^{\beta_j}
\]

and where \(u\) is utility and \(p\) is price.

a) What restrictions on the parameters of the AIDS model are implied by the theory? Please be specific.

b) Explain in detail how you will obtain estimates of demand curves for \(N\) nondurable goods (including food) using this model. Your discussion should include (but not limited to) which equation(s) you would estimate, what data you would use, which estimator you would use, how you would impose/test restrictions, if any.

c) The government is considering extending the value added tax to food by 15%. They ask you to evaluate the welfare consequences of this change. The cost of a price increase can be measured by compensating variation (CV) as:

\[
-CV(p^0, p^1, y) = c(p^1, u^0) - c(p^0, u^0),
\]

where \(y\) is income, and superscripts 0 and 1 denote the initial value and the value after the change, respectively. Explain in detail how you would obtain the measure of CV using the estimates of the AIDS model.

d) Now suppose that due to time or data constraints you cannot estimate a demand system to measure CV. One of your colleagues suggests that you can calculate first- and second-order approximations to CV, by using a Taylor series approximation of \(c(p^1, u^0)\) around \(p^1 = p^0\). Derive these approximations as suggested by your colleague and briefly discuss their advantages/limitations compared to estimating a demand system to calculate CV.

e) Explain why first-order approximation always overestimates the impact of a price increase on welfare compared to the second-order approximation.

---

\({}^1\) An n-order Taylor series approximation of \(f(x)\) around \(x=a\) is given as:

\[
f(x) \approx f(a) + f'(a)(x - a) + \frac{f''(a)}{2} (x - a)^2 + \cdots + \frac{f^n(a)}{n!} (x - a)^n
\]
QUESTION 5: Estimating the Almost Ideal Demand System with Micro Data

Suppose that you seek to estimate own- and cross-price elasticities of demand for competing brands in a differentiated products market. To this end, you collected price and quantity data for a panel of $J$ brands across $N$ cities for $T$ periods. You advisor asked you estimate the following AIDS model to obtain the elasticities:

$$s_{int} = \alpha_{in} + \beta_i \ln \left( \frac{y_{nt}}{p_{nt}} \right) + \sum_{j=1}^{J} \gamma_{ij} \ln p_{jnt} + \epsilon_{int},$$

where $s_{int}$ is the $i^{th}$ brand revenue share in city $n$ in period $t$, $p_{int}$ is price, $y_{nt}$ is overall expenditure, $P_{nt}$ is a price index, $\alpha, \beta, \gamma$ are model parameters to be estimated, and $\epsilon$ is the error term.

a) Carefully explain the potential dimensionality problem with this estimation strategy. Why should you be concerned about the problem?

b) Please carefully explain how you will modify the econometric model to deal with the dimensionality problem. In your discussion:

i. If you need any additional assumptions about the underlying utility function and/or the econometric model for your strategy to work, please explain.

ii. Discuss the conditions under which your assumptions are reasonable, or might be tenuous?

iii. Please clearly describe any additional variables and/or regression equations that you may introduce.

c) Carefully explain the potential endogeneity problem with the estimation strategy, and its consequences.

d) Propose a set of instruments to deal with the endogeneity problem. Please be as specific as possible. What is the underlying identifying assumption for your instruments?
QUESTION 6: A Random Utility Model of Demand

Suppose you were asked to estimate brand level demand in a differentiated product market. You observe aggregate market shares, price and product characteristics of \( J \) brands in each quarter-market combination \( t \). Assume that consumers have preferences over characteristics and each consumer purchases one of the \( J \) brands or an outside option. Denote the utility of individual \( i \) from product \( j \) in market \( t \) as:

\[
 u_{ijt} = x_j \beta - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt},
\]

where \( p \) is price, \( x \) and \( \xi \) are the observed and unobserved characteristics of product \( j \), \( \beta \) and \( \alpha \) are parameters to be estimated. Assume that \( \epsilon_{ijt} \sim i.i.d. \) Type I Extreme Value, so that the model is logit. In this model, the predicted market share of product \( j \) is equivalent to the logit probability, and is given as:

\[
 s_{jt} = \frac{\exp(\delta_{jt})}{\sum_{k=0}^{J} \exp(\delta_{kt})}
\]

where \( \delta_{jt} = x_j \beta - \alpha p_{jt} + \xi_{jt} \). Suppose \( E(\xi p) \neq 0 \), but you have a vector of instruments, \( Z \), such that \( E(\xi Z) = 0 \). The utility from outside good is equal to zero, \( u_{0t} = 0 \).

a) Propose a set instruments to be used in \( Z \)? Please be as specific as possible. What is the underlying identifying assumption for your instruments?

b) Explain in detail how you would estimate the logit model to obtain consistent estimates of \( (\hat{\alpha}, \hat{\beta}) \).

c) Please discuss limitations of the logit model as implied by the following own- and cross-price elasticities of demand:

\[
 \eta_{jk} = \frac{\partial s_j}{\partial p_k s_j} = \begin{cases} 
 \alpha p_j (1 - s_j) & \text{if } j = k \\
 -\alpha p_k s_k & \text{if } j \neq k
\end{cases}
\]

One of your colleagues suggested that you should estimate the following model to overcome the limitations of the logit model in estimating the price elasticities:

\[
 u_{ijt} = x_j \beta_i - \alpha_i p_{jt} + \xi_{jt} + \epsilon_{ijt},
\]

where,

\[
 \begin{pmatrix} 
 \alpha_i \\
 \beta_i
\end{pmatrix} = \begin{pmatrix} 
 \alpha \\
 \beta
\end{pmatrix} + \begin{pmatrix} 
 \Pi \alpha \\
 \Pi \beta
\end{pmatrix} D_i + \begin{pmatrix} 
 \Sigma \alpha \\
 \Sigma \beta
\end{pmatrix} v_i,
\]

where \( D \) and \( v \) are the observed and unobserved individual characteristics, \( \Pi \) and \( \Sigma \) are parameters to be estimated. The distribution of \( v \) is assumed to be normal.

d) Do you agree with your colleague that this model overcomes the limitations of the logit? Please explain.

e) Outline a method to estimate the above model. Please be as specific as possible. How is it related to the estimation algorithm of the logit in 2?