WRITTEN PRELIMINARY Ph.D. EXAMINATION

Department of Applied Economics

Summer - 2006

Natural Resource and Environmental Economics

Instructions:

• Identify yourself by your code letter, not your name, on each question.

• Start each question's answer at the top of a new page.

• Answer two of the first three questions
  (choose two among questions 1-3: Environmental Econ).

• Answer two of the second three questions
  (choose two among questions 4-6: Natural Resource Econ).
ENVIRONMENTAL ECONOMICS: Choose 2 among problems 1, 2, 3

1. A firm produces output \( y \), which results in an ambient level \( x \) of a given pollutant. This is a nonpoint pollution problem, in that the source of \( x \) cannot be traced to a particular firm. Ambient levels can be reduced by abatement activity, denoted \( a \). The firm's (overall) cost function, depending on output and abatement activity, is \( C(y,a) \). The ambient pollution level is given by \( x(a,e) \), where \( e \) is a random variable. Let \( \bar{x} \) denote the target level of ambient pollution. Stochastic ambient levels are described by the c.d.f. \( F(\bar{x},a) \), describing the probability that \( x < \bar{x} \). An environmental regulator considers imposing an incentive scheme to induce the firm to abate optimally. The chosen scheme is given by

\[
T(x) = \begin{cases} 
  t(x - \bar{x}) + k & \text{if } x > \bar{x} \\
  t(x - \bar{x}) & \text{otherwise} 
\end{cases}
\]

where \( k \) is a lump-sum tax imposed if \( x \) exceeds the standard and \( t \) is a tax/subsidy imposed proportional to the excess. The benefits to abatement \( a \), compared to no abatement, are given by \( B(x(0,e) - x(a,e)) \).

a. Suppose it is the short run and there is only one firm. The regulator wishes to induce the optimal levels of production and abatement. The risk-neutral firm wishes to maximize expected profits. With \( p \) as output price, write the objective functions for the regulator and the firm. Derive the set of (two) FONCs arising from each problem.

b. Let \( y^* \) and \( a^* \) denote the socially optimal levels of \( y \) and \( a \). Derive an incentive scheme that would induce the firm (in the short run) to choose \( y^* \) and \( a^* \). Is the scheme uniquely determined?

c. Now suppose it is the long run and there are \( N \) identical firms. Inverse demand facing the industry is given by \( p(Ny) \). Derive three conditions that now characterize an efficient outcome, yielding \( a^* \), \( y^* \), and \( N^* \).

d. Suppose the benefits function \( B \) is linear. Show that \( k = 0 \), \( t = B' \), and \( \bar{x} = E[x(0,e)] \) ensures that aggregate output and abatement will be optimal.
2. There are two regions each with a set of firms emitting a pollutant. In region 1, the abatement cost curve for firm \( i \) is \( C_i(A_i) \), where \( A_i \) is the amount of abatement by firm \( i, i = 1, 2, \ldots I \). Emissions by firm \( i \) are given by

\[
E_i = \begin{cases} 
E_i - A_i & \text{for } E_i \geq A_i \\
0 & \text{otherwise}
\end{cases}
\]

In region 2, the abatement cost curve for firm \( j \) is \( C_j(A_j) \), where \( A_j \) is the amount of abatement by firm \( j, j = 1, 2, \ldots J \). Emissions by firm \( j \) are given by

\[
E_j = \begin{cases} 
E_j - A_j & \text{for } E_j \geq A_j \\
0 & \text{otherwise}
\end{cases}
\]

Assume that each abatement cost function is an increasing convex function. Emissions cause pollution damages. Because of prevailing winds, pollution from region 1 also affects region 2. Pollution damages in the two regions are:

\[
D_1 \left( \sum_{i=1}^{I} \alpha E_i \right) + D_2 \left( \sum_{i=1}^{I} \beta E_i + \sum_{j=1}^{J} \gamma E_j \right)
\]

Assume that each pollution damage function is an increasing convex function.

a. Characterize optimal pollution abatement.

b. A “cap-and-trade” regulatory system is imposed in which the government allocates a total of \( X \) permits (each permit allows a firm to emit one unit of emissions) and firms may trade permits with any other firm (including firms in the other region). Characterize the equilibrium outcome assuming that all firms are price takers in the permit market. Under what circumstances, if any, does this regulator system result in optimal pollution abatement?

c. Design a type of “cap-and-trade” regulatory system that results in optimal pollution abatement for any parameter values.
3. Two firms in a given airshed emit a pollutant in the amounts $e_1 = 2000$ and $e_2 = 1000$ respectively. Their abatement cost functions are given by $C_1(a_1) = 0.01a_1^2$ and $C_2(a_2) = 0.02a_2^2$ respectively. The environmental regulator has decided that emissions should be reduced by 1000, from 3000 to 2000.

a. Determine the Pigouvian tax required to achieve this reduction.

b. Suppose the regulator decides to employ a permit-trading scheme, distributing 1000 permits to each firm. Determine the equilibrium permit price and emissions quantities for each firm, assuming the permit market is perfectly competitive.

c. Again assuming the permit market is competitive, now imagine that citizens experience aggregate marginal benefits of abatement of $MB(q) = 50-a/50$. Citizens can organize costlessly, are willing to pay MB for permits, and are allowed to buy permits from firms and retire them. Determine the equilibrium permit price and emissions quantities for each firm. How many permits will citizens buy and retire? Is this outcome socially optimal?

d. Finally, assuming that the polluting industry is organized as in part c. above, and that $MB(q) = 50-a/50$, consider the case in which individual citizens behave independently. (That is, organization is prohibitively costly. For example, there may be an infinity of citizens.) How many permits will citizens buy and retire? Again determine the equilibrium price and emissions quantities for each firm.
4. Global climate change is caused in part by the accumulation of carbon dioxide (CO\(_2\)) in the atmosphere. CO\(_2\) is a stock pollutant: once emitted, it remains in the atmosphere for a long time.

Suppose that the benefits of emitting \(x\) tons of CO\(_2\) are equal to \(bx(t)-hx(t)^2\), where \(x(t)\) is emissions at time \(t\), and \(b\) and \(h\) are constants. Also, suppose that the damage caused by higher temperatures can be expressed as a linear function of the stock: \(cs(t)\), where \(s(t)\) is the stock and \(c\) is a constant. Thus, the net present value of the CO\(_2\) emissions path is given by:

\[
\int_{0}^{\infty} (bx - hx^2 - cs)e^{-rt} \, dt
\]

where \(r\) is the interest rate. The evolution of \(s\) is governed by the differential equation:

\[
\frac{ds}{dt} = x - \delta s,
\]

where \(\delta\) is the rate at which excess CO\(_2\) is naturally removed from the atmosphere.

a. Write down the Hamiltonian and necessary conditions for this infinite horizon problem.

b. Derive the isoclines for \(x\) and \(s\) (where \(\dot{x} = 0\) and where \(\dot{s} = 0\)). Depict these isoclines on a phase diagram with \(x\) on the vertical axis. Include directional arrows for each quadrant and sketch in some sample trajectories, including the approach to the equilibrium. What type of equilibrium exists in this system?

c. Suppose it is suddenly discovered that the damages from global climate change are much larger than previously thought. Draw an appropriate phase diagram and show how the path of CO\(_2\) emissions would change.

d. What would the optimal paths of \(x\) and \(s\) be if CO\(_2\) were never cleared from the atmosphere (that is, if \(\delta=0\))? Discuss.
Consider a mining industry that faces a linear inverse demand curve given by 
\[ p = a - bq \] where \( p \) is price, \( q \) is the amount of ore demanded and \( a \) and \( b \) are parameters. Furthermore, assume that \( a > b > 0 \). The discount rate is denoted by \( \delta \). The equation of motion for the stock of exhaustible resource, \( R \), is given by 
\[ \dot{R} = -q. \] Extraction costs are quadratic and are given by \( C = \frac{c q^2}{2} \) where \( c \) is a parameter greater than 0. The terminal time, \( T \), is not given. The only restriction on the terminal state, \( R(T) \), is that it is nonnegative.

a. Write the current value Hamiltonian of the industry optimization problem. Write the necessary conditions, including the transversality conditions.

b. Use these conditions to write the optimal policy function \( q(t) \) as a function of the unknown time of resource exhaustion, \( T \).

c. Write the mathematical condition that enables you to determine \( T \), the time of resource exhaustion.

d. It was found that the mining of the ore creates two types of environmental damages: first, the opening of the mine has created a one time eruption of mercury into the local groundwater and second, the mining process creates debris proportional to the amount of ore extracted. Assume that these two externalities are internalized within the mining firm. Write the optimization problem of a manager of this mine. Discuss how these environmental problems would affect the extraction path. (no need to derive F.O.C.)

Consider a renewable groundwater aquifer with a stochastic recharge. The dynamics of the water stock, \( x \), are given by: 
\[ dx = [f(x) - q(t)] dt + \sigma(x) dz \]
where \( q(t) \) is water extracted at time \( t \), \( f(x) \) is strictly concave, \( \sigma(x) \) is increasing function of the stock with \( \sigma(0)=0 \), and \( dz \) is white noise. A social planner maximizes an expected value of infinite stream of discounted profits from water consumption, 
\[ V = \mathbb{E} \int_t^{\infty} \Pi(\tau) e^{(\delta - r)\tau} d\tau \] by choosing \( q(t) \).

a. Write the dynamic optimization problem of the social planner and the Bellman equation of dynamic programming. What is the condition for water extraction to be positive (in terms of the value function, \( v \))? 

b. Derive the “Golden rule” of “steady state” water extraction.

c. Why is the “golden rule” that you derived not useful to determine day to day extraction as in the standard fisheries model?

d. It turns out that the profit from water consumption is a quadratic function of \( q \), and that \( \sigma \) is a constant that does not depend on stock size. Can you suggest a deterministic analysis that will provide the same optimal extraction path?