WRITTEN PRELIMINARY Ph.D. EXAMINATION

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MANAGERIAL AND PRODUCTION ECONOMICS

Instructions:

- Identify yourself by code letter, not your name, on each question.
- Start each question at the top of a new page.
- Answer the first question.
- Answer three of the remaining five questions.
- You have four hours to complete this examination.
1. A risk averse employee’s effort, $X$, is the only variable input in a production process that produces a single output, $Y$. The firm’s production function is:

$$Y = e^{b_i} X^{b_i} + u, \quad 0 < b_i \leq 1,$$

where $u$ is a normally distributed random variable with a mean equal to zero and a variance given by the expression:

$$V(u) = e^{c_i} X^{c_i} \sigma^2, \quad c_i \leq 0 \text{ or } 1 \leq c_i.$$

Therefore,

$$E(Y) = e^{b_i} X^{b_i} \quad \text{and} \quad V(Y) = e^{c_i} X^{c_i} \sigma^2.$$

In addition to a fixed salary, $\alpha$, the employee receives a percentage, $\beta$, of the value of output, $Y$. For simplicity, we will assume output sells for a constant price of $S$ per unit. Also assume that the fixed salary is large enough to meet the employee’s reservation utility. The risk preferences of the employee are described by an additively separable multiattribute utility function with two arguments: cash income ($\alpha + \beta Y$) and effort ($X$). He has constant absolute risk aversion, $\lambda$, for income, and the effort component of his utility function is described by a simple quadratic form. Therefore, his expected utility function is given by the expression:

$$EU(\alpha + \beta Y, X) = E(\alpha + \beta Y) - \frac{\lambda}{2} V(\alpha + \beta Y) - \gamma X^2$$

$$= E(\alpha + \beta Y) - \frac{\lambda}{2} \beta^2 V(Y) - \gamma X^2,$$

where $\alpha$, $\beta$, $\lambda$, and $\gamma$ are positive parameters.

a. There are two possible ranges of values for the parameter $c$. For each range, show whether effort is risk increasing or risk reducing.

b. Formulate a maximization problem to determine the optimal level of effort for this employee. Then derive the first order necessary condition for a maximum and check the second order condition that is sufficient to ensure that this condition does identify a maximum. You do not need to rearrange the first order condition to get an explicit expression for $X$.

c. Derive an expression for $\frac{\partial X}{\partial \beta}$ and determine its sign or the conditions when it is positive or negative, giving particular attention to the relationship between the value of $c$ and the sign of $\frac{\partial X}{\partial \beta}$. What do your results imply about the effectiveness of revenue sharing as a means for encouraging more effort from employees?
2. Employees in a company’s sales force have opportunities to participate in training programs designed to help them develop skills that will make them more effective. But participation in these programs comes at a cost, since it takes time away from building a larger customer base. At time $t$, an employee’s sales skill level is represented by a human capital index, $HC(t)$, with possible values on the unit interval $[0,1]$. The dynamics of $HC(t)$ is described by the following expression:

$$HC(t+1) = HC(t) + \alpha \cdot TR(t) \cdot HC(t) \cdot [1 - HC(t)],$$

where $TR(t)$ is the percent of time devoted to training ($0 \leq TR(t) \leq 1$) and $\alpha$ is a known parameter. An employee’s customer base, $CB(t)$, is affected by the percent of time devoted to sales activities, $(1 - TR(t))$, and by the level of human capital, $HC(t)$. The dynamics of $CB(t)$ is described by the following expression:

$$CB(t+1) = [\delta \cdot CB(t) + \beta \cdot HC(t) \cdot (1 - TR(t))^{\gamma} \cdot CB(t)] \cdot u(t),$$

where $\delta$ is a customer base attrition parameter such that $0 < \delta < 1$, $\beta$ is a positive parameter, $\gamma$ is a parameter such that $0 < \gamma < 1$, and $u(t)$ is a strictly positive random variable with a mean of one and a constant variance.

Employees receive small incentive payments for time spent in training programs and a commission on sales. Annual compensation, $C(t)$, is given by the following expression:

$$C(t) = a \cdot TR(t) + \phi \cdot (b \cdot CB(t)^{c_1} \cdot HC(t)^{c_2} \cdot (1 - TR(t))^{c_3}),$$

where $a$ is a positive payment for participation in training ($\$/\%time), $\phi$ is a sales commission rate, and $b \cdot CB(t)^{c_1} \cdot HC(t)^{c_2} \cdot (1 - TR(t))^{c_3}$ is a production function for sales (measured in dollars), with positive parameters $b$, $c_1$, $c_2$, and $c_3$ and $0 < c_i < 1 \ldots c_i$.

a. Formulate a dynamic programming model that can identify the training policy that will maximize an employee’s expected present value of compensation over a 30 year career. You need not consider the employee’s source or level of income after she retires, and you can assume that $TR(30)=0$, regardless of the values for $HC(30)$ and $CB(30)$. Let the parameter $\rho$ represent an annual discount factor in your model. Be sure to explicitly identify the state and control variables. Your formulation should also include: the objective function, the state equation(s) (as given above), any other relevant constraints on states or controls, a verbal definition of the optimal value function (which is denoted as $V(HC(t), CB(t), t)$), the recurrence relation, and any relevant boundary conditions.

b. How would you expect an employee’s allocation of time to sales training to change over her career if her initial human capital index low? Explain the reasoning for your answer.

c. Briefly describe how you might use this model to help an employer determine levels of the training participation payment, $a$, and commission rate, $\phi$, that will maximize the present value of sales net of compensation over a representative employee’s career.
3. Consider a farmer who uses input $x$ to produce output $y$. The relationship between the input and output is stochastic such that $y = f(x,e)$ where $e$ is a random variable with density $g(e)$. The price the farmer receives per unit of output is $p$, while the price paid per unit of input is $r$. Other costs of production are $c$, which can be normalized to 0 for convenience. Profit is then

$$\pi = ry - rx.$$ 

The farmers utility of profit is $U(\pi)$ where $U'(\pi) > 0$ and $U''(\pi) \leq 0$.

a. Set up the utility maximization problem for the farmer. Derive and interpret the first order conditions.

b. Is it ever optimal for a risk neutral ($U''(\pi) = 0$) and risk averse ($U''(\pi) < 0$) farmer to use the same amount of the input? If so, under what conditions?

c. Is it ever optimal for a risk neutral farmer to use less of the input than a risk averse farmer? If so, under what conditions?

d. Discuss the intuition of your results in terms of how the random variable affects the marginal productivity of the input.
4. A firm uses a single input, $x$, to produce the two outputs, $y_1$ and $y_2$. The production technology mapping the input $x$ into outputs is represented by the Generalized Leontief output distance function:

$$D_0(x,y_1,y_2) = \frac{\beta_1 y_1 + \beta_2 y_2 + \beta_{12} (y_1 y_2)^{0.5}}{\alpha x} + \gamma_1 x y_1 + \gamma_2 x y_2,$$

where $\alpha, \beta_1, \beta_2, \beta_{12}, \gamma_1,$ and $\gamma_2$ are parameters.

a. Show that $D_0$ is homogeneous of degree +1 in outputs.

b. Show that $D_0$ is homogeneous of degree zero in its parameters.

c. Let $p_1$ and $p_2$ represent the (strictly positive) price of outputs $y_1$ and $y_2$, respectively.

   i. Define the revenue function, and derive the revenue function associated with $D_0$.

   ii. What is the firm's supply function for $y_1$?

   iii. Prove the revenue function is nonnegative; non-decreasing in prices; and convex in prices.
5. Assume a firm produces output according to the Cobb-Douglas production function \( Y = L^a K^\beta \) where \( Y \) is output, \( L \) is labor, \( K \) is capital and \( a \) and \( \beta \) are known parameters. Furthermore assume \( a > 0, \beta > 0 \), and \( a + \beta < 1 \). Suppose the firm can sell all it wants at a price of \( P \) and can buy all labor at a price \( w \) and capital at a price \( r \).

a. Derive the firm’s optimal labor demand and supply function assuming labor is variable, but capital is fixed at \( K_0 \).

b. Derive a competitive firm’s optimal labor demand and supply function assuming labor and capital are variable.

c. Is the elasticity of supply with respect to \( P \) greater when capital is fixed at \( K_0 \) or variable? Explain.

d. Is the elasticity of the labor demand with respect to \( w \) greater when capital is fixed at \( K_0 \) or variable? Explain.
6. You have been given farm level data from farm/household units in four villages in Andra Pradesh, India. The survey includes data for N households in each village. For each farm, the major inputs were fertilizer, fuel, labor, land, seed, and irrigation water. You have price and level data for each of the input categories. The major outputs were rice, chickpeas, spinach, and sugar cane. You have price and level data for each of the outputs. Assume the input and output prices were the same across villages.

a. You have been asked to evaluate the relative efficiency of production of each village. Give a brief description of how you would conduct such a study. Your discussion should include at least the following points: (i) the function(s) you would estimate or calibrate, (ii) the specific functional form(s) you would use, (iii) the restrictions you would place on the parameters of the functional form, (iv) a description of the parametric or nonparametric approach to your analysis, e.g., what first-order conditions would you be estimating or calibrating, etc.?

b. If you did not have irrigation water prices, describe how you would calculate the shadow value of irrigation water.