Ph.D. Preliminary Examination
MICROECONOMIC THEORY
Applied Economics Graduate Program
August 2018

The time limit for this exam is four hours. The exam has four sections. Each section includes two questions. Answer one question from each section. Before turning in your exam, number each page of your answers in sequential order, and identify the question (for example, III.2 for section III and question 2). Indicate, by circling below, the questions you completed. If you answer more than one question in a section and do not circle a question corresponding to that section, the first of the two that appears in your solution paper will be graded.

STUDENT ID LETTER: _______ (Fill in your code letter)

Answer one question from each section. Indicate the one you answered by circling:

Section I: Question 1 Question 2
Section II: Question 1 Question 2
Section III: Question 1 Question 2
Section IV: Question 1 Question 2

*** TURN IN THIS SHEET WITH YOUR ANSWER PAGES ***
Part I

Answer at most one question from Part I.
Question I.1

Consider a consumer who maximizes utility over two time periods. The focus of this question is on food consumption, so in period 1 utility is a function of only two factors, the utility from consuming food, denoted by \( v_1(f) \), and the “disutility” from the cost of food, \( pf \), where \( p \) is the price and \( f \) is the amount of food bought. Thus utility in period 1 is:

\[
\text{utility in period 1} = v_1(f) - pf, \quad \text{where } v_1'() > 0.
\]

In period 2, there is no consumption of food, but the consumer suffers disutility from being overweight from the food consumption in period 1. That is, let \( w \) be weight in period 2, and let weight be a linear function of food consumption: \( w = kf \), where \( k > 0 \). Then utility in period 2 is:

\[
\text{utility in period 2} = v_2(w), \quad \text{where } v_2'() < 0.
\]

(a) Assume that, from a life-cycle perspective, the consumer discounts utility in period 2 by a factor \( \delta \), where \( 0 < \delta \leq 1 \). Denote the consumer’s life-cycle utility by \( U \). What is the consumer’s life-cycle utility, accounting for discounting, over the two time periods? Express it as a function of \( f \) only, so that \( w \) is substituted out.

(b) Show the first-order condition for the consumer’s maximization of life-cycle utility. You can assume an interior solution.

(c) Use your answer to part b) to show what happens to food consumption, and thus what happens to the consumer’s weight in period 2, when the price of food increases. This is done by using total differentiation (or the implicit function theorem). You can assume that both \( v_1 \) and \( v_2 \) are concave, so that \( v_1''() < 0 \) and \( v_2''() < 0 \). Explain the intuition for this result in 2–3 sentences.

(d) Next, use your answer to part (b) to show what happens to the consumer’s weight when he or she becomes more patient. That is, show what happens when \( \delta \) increases. Explain the intuition for this result in 2–3 sentences.

(e) Up to now we have assumed that increased weight in period 2 reduces utility, but suppose that the consumer is malnourished, so that he or she actually has higher utility in period 2 if his or her weight increases. That is, \( v_2'(w) > 0 \). Do your answers to parts (c) and (d) change in this situation? You can continue to assume that \( v_2''() < 0 \), that is that there is decreasing marginal utility of weight as weight increases. There is no need to give any intuition for your answer.
Question I.2

Consider a consumer with the following Bernoulli utility function, where \( x \) is an amount of money:

\[
  u(x) = \frac{x^{1-\sigma} - 1}{1-\sigma}.
\]

(a) For this Bernoulli utility function, what is the coefficient of absolute risk aversion? What is the coefficient of relative risk aversion?

(b) What values of \( \sigma \) imply that this consumer is risk averse? Explain your answer by referring to one or more definitions of risk aversion.

(c) Next, consider a more general expression for \( u(x) \), which has two parameters, \( \gamma \) and \( \sigma \):

\[
  u(x) = \frac{1}{\gamma} \left[ 1 - e^{-\gamma(x^{1-\sigma} - 1)/(1-\sigma)} \right],
\]

where \( e \) is the base of the natural logarithms. For this Bernoulli utility function, what is the coefficient of absolute risk aversion? What is the coefficient of relative risk aversion?

(d) For what values of \( \gamma \) and \( \sigma \) is this consumer risk averse? Consider three different cases: both \( \gamma \) and \( \sigma \) are positive, both are negative, and one is negative while the other is positive.

(e) It can be shown that the Bernoulli utility function in part (a) is a special case of the more general Bernoulli utility function in part (c). Based on your answers for (a) and (c), what restrictions on \( \gamma \) and/or \( \sigma \) reduce the general Bernoulli utility function in part (c) to the “special case” in part (a)? Do not derive this by looking at the two utility functions, which is hard to do, but instead look at your answers to (a) and (c).
Part II

Answer at most one question from Part II.
Question II.1

Consider the production possibility set (PPS):

$$\text{PPS} = \left\{ (q, -z) \in \mathbb{R}_+^2 \times \mathbb{R}_-^2 : z_1^\alpha x_2^\beta \geq q_a^2 + q_b^2 \right\},$$

where $z_1$ and $z_2$ are inputs; $q_a$ and $q_b$ are outputs; and $\alpha > 0$ and $\beta > 0$ are constant parameters. This PPS is nonempty, satisfies free disposal of inputs and outputs, is strictly convex, and is closed.

(a) Under what additional condition(s) on $\alpha$ and $\beta$ (if any) will this technology exhibit non-increasing returns to scale? Justify your response.

The cost function for this PPS assuming $\alpha = 1/4$ and $\beta = 3/4$ is

$$c(r, q) = \theta r_1^{1/4} r_2^{3/4} (q_a^2 + q_b^2),$$

where $r_1 > 0$ and $r_2 > 0$ are input prices, and $\theta = 7/4$.

(b) Suppose output prices are $p_a > 0$ and $p_b > 0$.

(i) Find the profit-maximizing supplies assuming an interior solution.

(ii) Use these supplies to find the unconditional input demands. **Hint: To save time, you need not simplify your answer. You will also save time by using duality results instead of deriving the profit function.**

(c) Now also suppose $q_a$ and $q_b$ represent output and $p_a > 0$ and $p_b > 0$ are output prices in two different states of an uncertain world.

(i) Derive the revenue cost function, where $R_a \geq 0$ and $R_b \geq 0$ are the revenues in states $a$ and $b$. **Hint: Think before you write because this is simpler than it might first seem.**

(ii) Use this revenue cost function to derive the certainty equivalent revenue.
Question II.2

Consider a producer’s revenue cost function for a world with two inputs denoted by \( n = 1, 2 \) and two outputs denoted by \( m = 1, 2 \) in two states denoted by \( s = a, b \):

\[
c(r, p, R) = r_1^{\frac{1}{2}} r_2^{\frac{1}{2}} \left( \frac{R_a^2}{(p_a^1)^{\frac{1}{2}} (p_a^2)^{\frac{1}{2}}} + \frac{R_b^2}{(p_b^1)^{\frac{1}{2}} (p_b^2)^{\frac{1}{2}}} \right),
\]

where \( r \in \mathbb{R}_+^2 \) is a vector of input prices, \( p \in \mathbb{R}_+^4 \) is a vector of state-contingent output prices, and \( R \in \mathbb{R}_+^2 \) is a vector of state-contingent revenues. The PPS used to derive this revenue cost function is nonempty, strictly convex, closed, and satisfies strong free disposal in outputs and inputs.

(a) Derive the producer’s conditional input demand for input 1 and conditional supply for output 1 in state \( a \). \textit{Hint: These should be conditional on some or all of the input prices, and state contingent output prices and revenues.}

(b) Suppose the producer’s preferences for profit in each state are described by the risk-averse utility function

\[
W(\pi) = \alpha \ln(\pi_a) + \beta \ln(\pi_b),
\]

where \( \pi \in \mathbb{R}_+^2 \), which is derived from a preference relation that is rational, continuous, and strictly monotonic.

(i) What are the producer’s subjective beliefs about the probability of state \( a \) and state \( b \)?

(ii) Use these probability beliefs to derive the producer’s absolute risk premium.

(c) Now assume the producer is risk neutral and believes that the probability of states \( a \) and \( b \) are \( \phi_a > 0 \) and \( \phi_b > 0 \) such that \( \phi_a + \phi_b = 1 \).

i) Derive this producer’s expected profit maximizing revenues assuming an interior solution exists.

ii) Given these expected profit-maximizing revenues, what are the unconditional demand for input 1 and output 1 in state \( a \)? \textit{Hint: To save time, you need not simplify.}
Part III

Answer at most one question from Part III.
Question III.1

Consider the extensive form of the two-player static game of complete information in the figure below.

(a) What is each player’s pure strategy set? Write down the normal form for this game.

(b) Assuming $x = 75$, find all the pure and mixed strategy Nash equilibria.

(c) Assuming $x = 150$ and the game is played twice with the outcome of the first game revealed to the players before the second game is played, find all the pure strategy subgame perfect Nash equilibria.

(d) Assume $x = 150$ and that the game is repeated infinitely with the players seeing the outcome of each game before the next game is played. Also assume players discount future payoffs by $1 \geq \delta \geq 0$ per period. Find the minimum $\delta$ needed for a subgame perfect equilibrium to exist where Player A’s and B’s strategies in period $t$ are

$$
s_t^A = \begin{cases} 
C_p & \text{for } t = 0 \\
C_p & \text{for } t > 0, \text{ and if } s_{t'}^A = C_p \text{ and } s_{t'}^B = c_p \text{ for all } t' < t \\
D_p & \text{otherwise}
\end{cases}
$$

and

$$
s_t^B = \begin{cases} 
c_p & \text{for } t = 0 \\
c_p & \text{for } t > 0, \text{ and if } s_{t'}^A = C_p \text{ and } s_{t'}^B = c_p \text{ for all } t' < t \\
D_p & \text{otherwise}
\end{cases}
$$

\[\text{Figure 1: (Player A’s Payoff, Player B’s Payoff)}\]
Question III.2

Consider the extensive form of the two-player static game of incomplete information in the figure below.

(a) What is each player’s strategy set (don’t forget Nature)?

(b) Write down a normal form for this game and identify the unique pure-strategy Bayesian Nash equilibrium.

(c) Suppose this static game of incomplete information became a dynamic game of incomplete information because Player A chooses to reveal its choice of L or R to Player B before Player B makes its choice (e.g., Player B’s current information set \{v_1, v_2, v_3, v_4\} is split into two different information sets, \{v_1, v_3\} and \{v_2, v_4\}).

(i) How does this change the players’ strategy sets?

(ii) Find the unique pure-strategy perfect Bayesian equilibrium for this new game.

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Figure 2: (Player A’s Payoff, Player B’s Payoff)
Part IV

Answer at most one question from Part IV.
Question IV.1

Consider an exchange economy with two goods, indexed by superscripts $i = 1, 2$, and two households, indexed by subscripts, $j = 1, 2$. Endowments are

$$\omega_1 = (1, 0) \quad \text{and} \quad \omega_2 = (0, 1).$$

Prices are in the simplex, so that $p^1 + p^2 = 1$. We allow zero prices. In each of the following specifications of utility functions, first derive the offer curves for the two consumers. Then, if the economy has one or more competitive equilibria, compute them all. If none exist, explain why not and also explain which part of the conditions required for the existence theorem I (strict convexity) fail to hold. Be specific about which assumption is violated. (Note that an economy can have an equilibrium even when the theorem does not apply.)

(a) Utilities are $U_1(x^{1\,}_1, x^{2\,}_1) = \min[x^{1\,}_1, 2x^{2\,}_1]$ and $U_2(x^{1\,}_2, x^{2\,}_2) = \min[2x^{1\,}_2, x^{2\,}_2]$. For this part only sketch an Edgeworth box containing the endowment, at least two indifference curves for each consumer, and the equilibrium or equilibria, both the allocation and the associated price line for each.

(b) Utilities are $U_1(x^{1\,}_1, x^{2\,}_1) = \max[x^{1\,}_1, x^{2\,}_1]$ and $U_2(x^{1\,}_2, x^{2\,}_2) = \max[2x^{1\,}_2, x^{2\,}_2]$.

(c) Utilities are $U_1(x^{1\,}_1, x^{2\,}_1) = (x^{1\,}_1)^2 + (x^{2\,}_1)^2$ and $U_2(x^{1\,}_2, x^{2\,}_2) = x^{1\,}_2 x^{2\,}_2$. 

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Question IV.2

Recall Sen’s theorem about the conflict between the Pareto principle and Minimal Liberalism.

(a) Define minimal liberalism and the Pareto principle, in both cases being sure to be clear about your notation.

(b) Now recall Sen’s theorem:

**Theorem.** There is no social decision function that can simultaneously satisfy conditions (U), (P), and (L*).

Sen considers three possibilities in turn, with two voters who are decisive over some pair in each:

(i) The two pairs of alternatives \((x, y)\) over which 1 is decisive, and \((z, w)\) over which 2 is decisive, are the same pair.

(ii) The two pairs share an outcome, say \(z = x\).

(iii) There are two pairs of completely distinct alternatives: \((x, y)\) over which 1 is decisive; and \((z, w)\) over which 2 is decisive.

Provide the proof of the theorem, including the argument for each of the three parts.

(c) Describe a realistic situation in which the condition of minimal liberalism is a reasonable condition to impose upon a collective decision process.