The Theory of Optimal Hedging Horizons and its Application to Dairy Risk Management in the United States

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11-09-2016

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Introduction

What makes a risk management program effective? Hundreds of scientific journal articles have been written on the topic, and nearly all of them conflate this question with "how much of the cash position should be hedged?" They, thus, seek to find what is known formally as the 'optimal hedge ratio' (e.g. Lien and Tse 2002, Chan et al. 2003, Chavas and Klemme 2003, 2004, Garcia and Leuthold 2004). Although there are numerous variations on this theme, all of these articles essentially examine the basis risk. Specifically, previous research shows that the lower is the correlation between unanticipated shocks to cash prices and changes in the futures prices, the lower should be the optimal hedge ratio, which results in less effective hedging with respect to a particular futures contract. Conversely, when cash price shocks strongly correspond to changes in futures prices, basis risk is considered to be small, optimal hedge ratio should be higher, resulting in a more effective hedging than that in the former scenario.

However, even if cash price shocks could be perfectly offset by gains in the hedging account, another fundamental question remains ignored by nearly entire scholarly literature on hedging – *when* should one hedge? In other words, how many months in advance of the actual marketing should a hedge be initiated? A few papers that considered hedging horizons (Geppert 1995, Malliaris and Urrutia 1991) focused on the second-order issues such as estimating the magnitude of the basis risk at different hedging horizons.

In the area of dairy risk management, previous literature presents us with conflicting findings. Maynard, Wolf and Gearhardt (2005) find that hedging may reduce milk price variance by 50-

60% in most regions. In contrast, Neyard, Tauer and Gloy (2013) claim that risk management activities do no result in significant change in variance of net farm income, compared to the reference scenario of selling milk and procuring inputs on a monthly cash basis. Bozic et al. (2012) have shown that shocks to milk prices dissipate slowly, approximately over a period of 9 months and Bozic et al. (2016) demonstrate that hedging distant horizons may substantially reduce threshold semivariance of dairy income over feed costs margins. In this article, we argue that the apparently discordant results from past research can be reconciled by evaluating the risk management effectiveness using different hedging horizons. We find that that even a disciplined risk management program that faces no basis risk may fall terribly short of offering any meaningful protection, if hedges are placed just a few months ahead of the period for which protection is desired. On the other hand, hedging with more deferred contracts can substantially reduce dairy price and margin risk. We refer to this issue as the *optimal hedging horizon* problem.

In the second section, we develop the mathematical model of the hedging problem when spot prices or margins are mean reverting. We use simulation methods to estimate the optimal hedging horizon in dairy risk management. In the third section we introduce a novel statistical test to test if dairy margins are indeed mean-reverting. Unlike most previous efforts at evaluating time series properties of spot prices that rely on unit root and stationarity tests applied to observed spot prices, we follow Bessembinder et al. (1995) and infer properties of the spot price process from the behavior of the term structure of synthetic futures income over feed cost margins. In the fourth section, we employ a novel method for calibrating the speed of the mean reversion of dairy margins. Conclusions summarize our findings and offer implications for design of private and public dairy risk management policies and instruments.

Optimal hedging under mean-reverting spot prices and margins

Economic theory of competitive markets stipulates that economic profits must converge to zero in the long-run, as free entry and exit of agents ensure that neither below- nor above-average profit margins can be sustained in the long run. Because there can be no extra economic profits in the long run, it must follow that profit margins must have a mean-reverting stochastic process such as described by Uhlenback and Orstein (1930). If input costs are stationary, then milk prices will be stationary as well. In this section we assume milk prices are mean-reverting and analyze the implications for dairy risk management.

Let the stochastic process for spot prices be given by:

$$y_{t+1} = -\alpha\left(y_t - \mu\right) + y_t + \varepsilon_t \tag{1.1}$$

Where $\alpha \in (0,1)$ and ε_t are zero-mean, independent and identically distributed shocks with standard deviation σ_ε. By consecutive substitution, it follows that the spot price in period $t+k$ can be written as

$$y_{t+k} = (1-\alpha)^k y_t + \left[1-(1-\alpha)^k\right]\mu + \sum_{i=1}^{k}(1-\alpha)^{i-1}\varepsilon_{t+k-i} \tag{1.2}$$

Denote futures price for a contract expiring in period T as observed at time t with $F_{t,T}$. Assume the contract settles against the spot price y_{t+k}, i.e. $T = t+k$ and $F_{T,T} = y_T = y_{t+k}$. Since shocks are unpredictable, $E_t\left(\varepsilon_{t+i}\right) = 0, \forall i > 0$. With the assumption that futures markets are efficient and unbiased, it follows that:

$$F_{t,T} = E_t\left(y_{t+k}\right) \tag{1.3}$$

From the unpredictability of shocks, and equations (1.2) and (1.3):

$$F_{t,T} = E_t\left(y_{t+k}\right) = (1-\alpha)^k y_t + \left[1-(1-\alpha)^k\right]\mu \tag{1.4}$$

Since $\alpha \in (0,1)$, futures prices for deferred contracts will approach the unconditional mean price μ. In the limit,

$$E_t\left[\lim_{k\to\infty} y_{t+k}\right] = \mu \tag{1.5}$$

Define the minimum needed futures price as

$$y^* = \mu - \delta, \delta > 0 \tag{1.6}$$

We formulate the hedger's objective as choosing the minimum hedging horizon k such that whenever it is time to initiate the next hedging round, the futures price is higher than the minimum needed futures price y^* in γ percent of cases:

$$\Pr\left[F_{t,t+k} \le y^*\right] \le \gamma, \forall t \tag{1.7}$$

For example, setting $\gamma = 0.05$ would insure that hedging results in acceptable hedged price level in 95% of hedging attempts.

Assume that the unconditional distribution of the spot price at time t is given by the normal distribution $N\left(\mu,\sigma\right)$. From (1.4), the unconditional variance of futures prices for contracts expiring in k periods will be

$$Var\left(F_{t,t+k}\right) = \left(1-\alpha\right)^{2k}\sigma^2 \tag{1.8}$$

The probability that the hedge will result in unacceptably low hedged price is equal to the cumulative distribution function, evaluated at y^* :

$$\Pr\left[F_{t,t+k} \le y^*\right] = CDF\left(y^*;\mu,\sigma,\alpha,k\right) \tag{1.9}$$

Since $\alpha \in \left(0,1\right)$, increase in the hedging horizon k reduces the unconditional variance of k-horizon futures prices:

$$k_2 > k_1 \Rightarrow \left(1-\alpha\right)^{2k_2}\sigma^2 < \left(1-\alpha\right)^{2k_1}\sigma^2 \tag{1.10}$$

From (1.9) and (1.10), increase in the hedging horizon reduces the probability of unacceptable hedging outcomes:

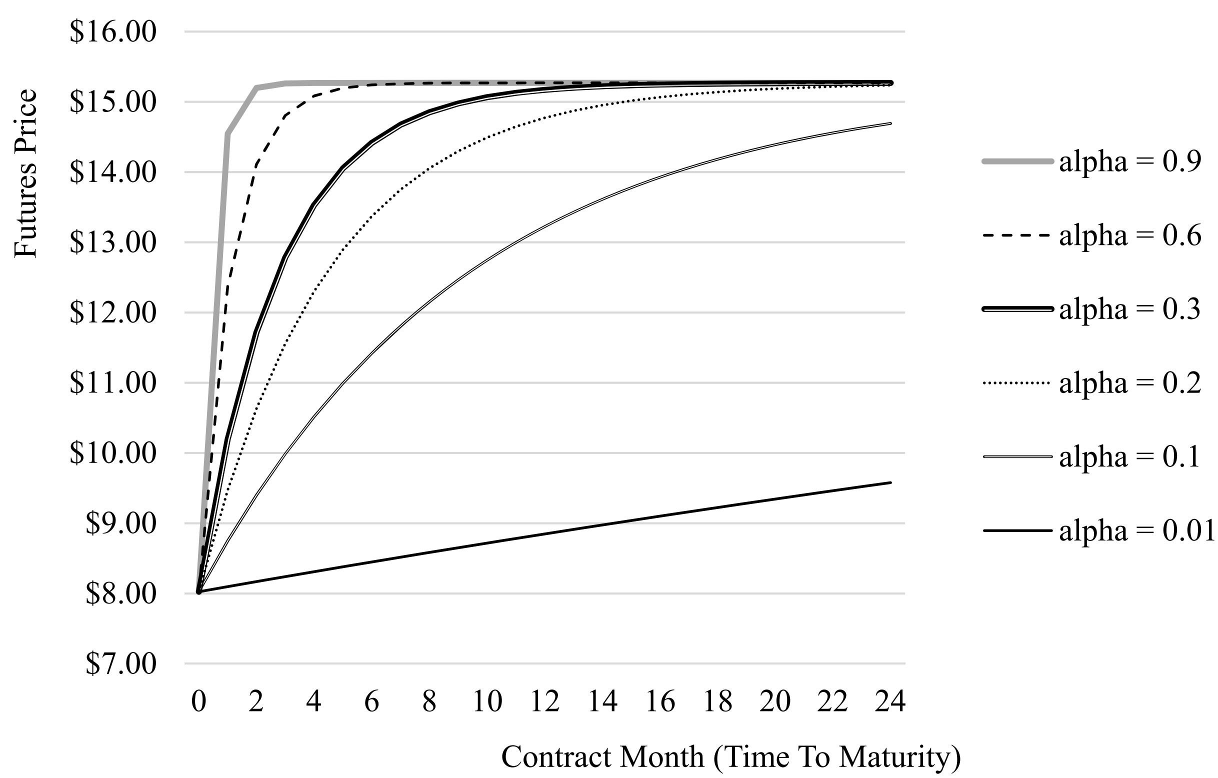
$$\frac{\partial CDF\left(y^*;\mu,\sigma,\alpha,k\right)}{\partial k} < 0 \tag{1.11}$$

The optimal hedging horizon problem can now be expressed formally as:

$$\begin{aligned} &\min k: \\ &s.t.\ CDF\left(y^*;\mu,\sigma,\alpha,k\right) \le \gamma \end{aligned} \tag{1.12}$$

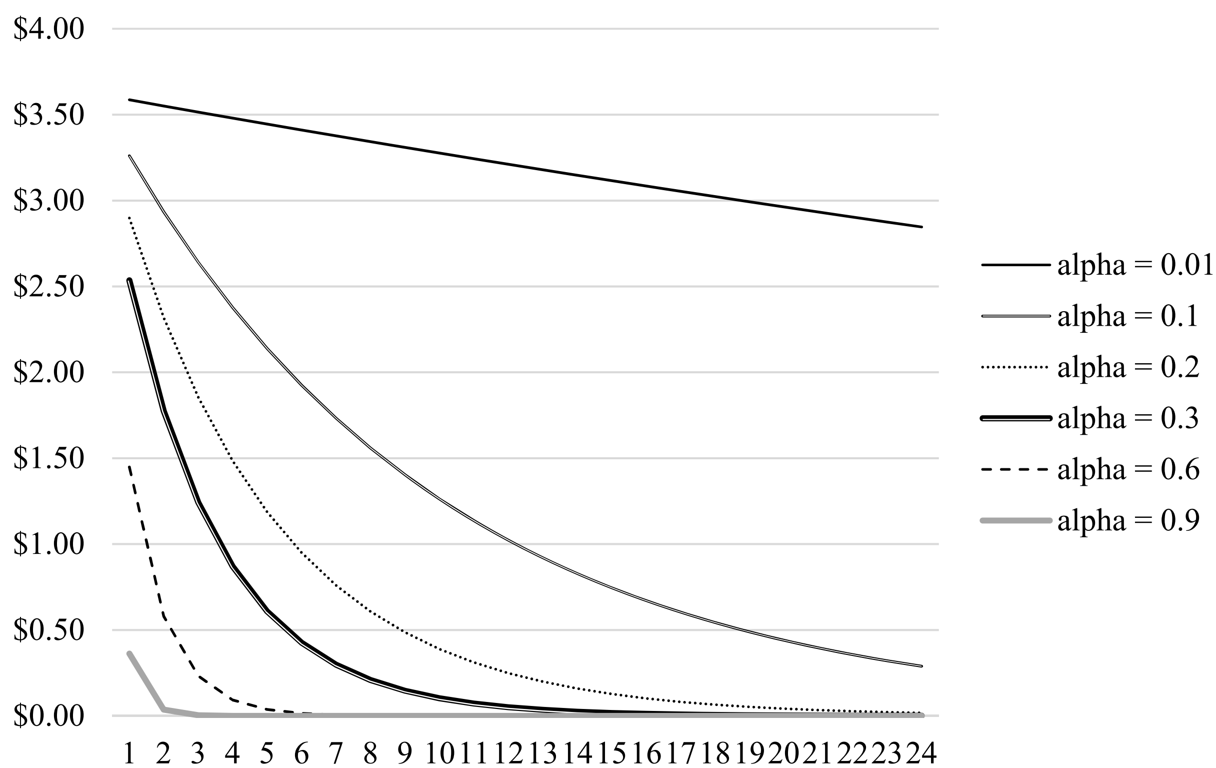
Let us apply this model to Class III milk prices. The mean Class III milk price over 2000-2016 period was \$15.27/cwt, and the standard deviation was \$3.62. The lowest Class III milk price observed over this period was \$9.11 in March of 2003. In Figure 1, we examine the shape of the term structure of futures prices, under the above described model assumptions, if the spot price were to be \$8.00/cwt, i.e. at an extremely low level. We simulate the term structures under six different levels of α coefficient, ranging from 0.01 (very slow mean reversion) to 0.9 (extremely fast mean reversion).

Figure 1. Simulated term structures of Class III milk futures prices under various assumptions on the speed of mean reversion of Class III milk prices.



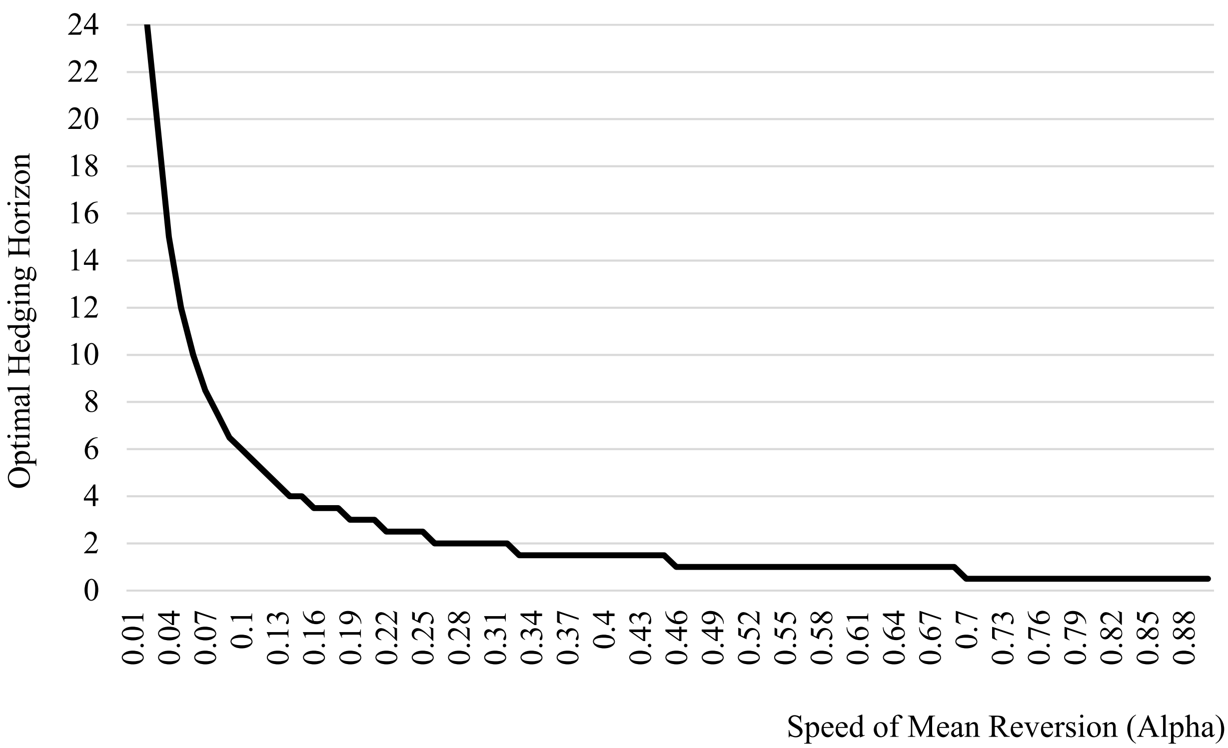
For α values higher than 0.6, the hedging horizon is not a critical aspect of dairy risk management program, as the futures prices suggest near full recovery within 1-3 months. On the other end of the spectrum, at α value of 0.01, the hedging horizon is again irrelevant, since price recovery is too slow for hedging with deferred contracts to guarantee a reasonable price. In the above scenario, where $\alpha = 0.01$ and spot price is \$8.00, even the most distant contract listed on CME (24 months out) would have price just marginally higher, at \$9.57. This is critically below costs of production, and locking in such low price would make no difference to the ability of a dairy farm to survive a low price period. Finally, for α less than 0.3, hedging horizon seems to matter a lot. Figure 2 helps us further build the intuition about this issue by presenting the unconditional standard deviation of futures prices at each horizon, for the same set of values of α .

Figure 2. Unconditional variance of futures prices at hedging horizon 1 to 24 months, for various speeds of mean-reversion of spot prices.



The lower the variance of futures prices, the higher is the probability the hedgeable price will exceed the critical threshold. Next, Figure 3 presents the results of a simulation of minimum necessary hedging horizon needed to guarantee the hedgeable price of $14.00/cwt with 95% probability, as a function of the speed of mean reversion.

Figure 3. Optimal hedging horizon as a function of the speed of mean reversion.



Note: Class III milk spot price mean: $15.27, standard deviation: $3.62/cwt. Desired threshold price: $14.00. Desired probability of successful hedging: 95%.

Hedging horizon will be shorter for lower price thresholds, as observed in Figure 4. When the critical threshold is reduced from $14.00/cwt to $13.00/cwt and further down to $12.00/cwt, the optimal hedging horizon, at $\alpha = 0.08$ is reduced from 19, to 12, and finally to 7.5 months.

Figure 4. Optimal hedging horizon as a function of the speed of mean reversion, for various threshold prices.

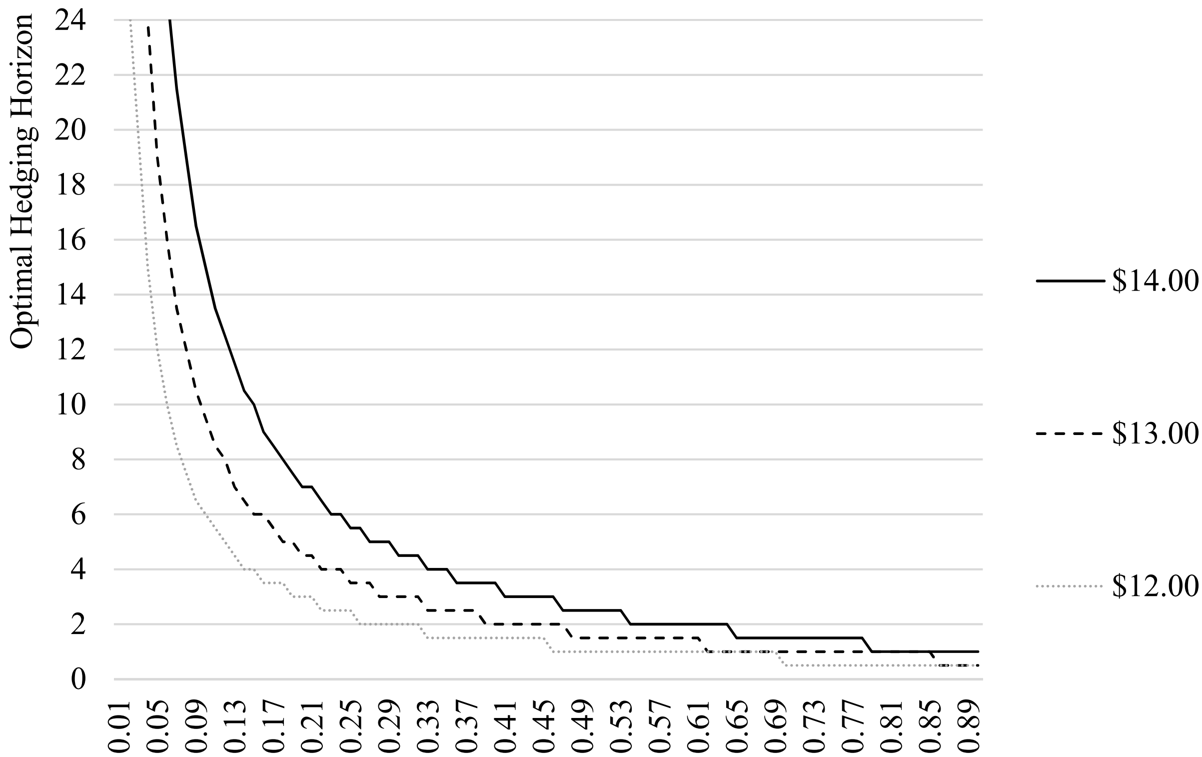


Table 1 summarizes the same point in a tabular form.

Table 1. Optimal horizon as a function of the speed of mean reversion and threshold price.

	Threshold Price		
Alpha	$14.00/cwt	$13.00/cwt	$12.00/cwt
0.10	15	9.5	6
0.15	10	6	4
0.20	7	4.5	3
0.25	5.5	3.5	2.5

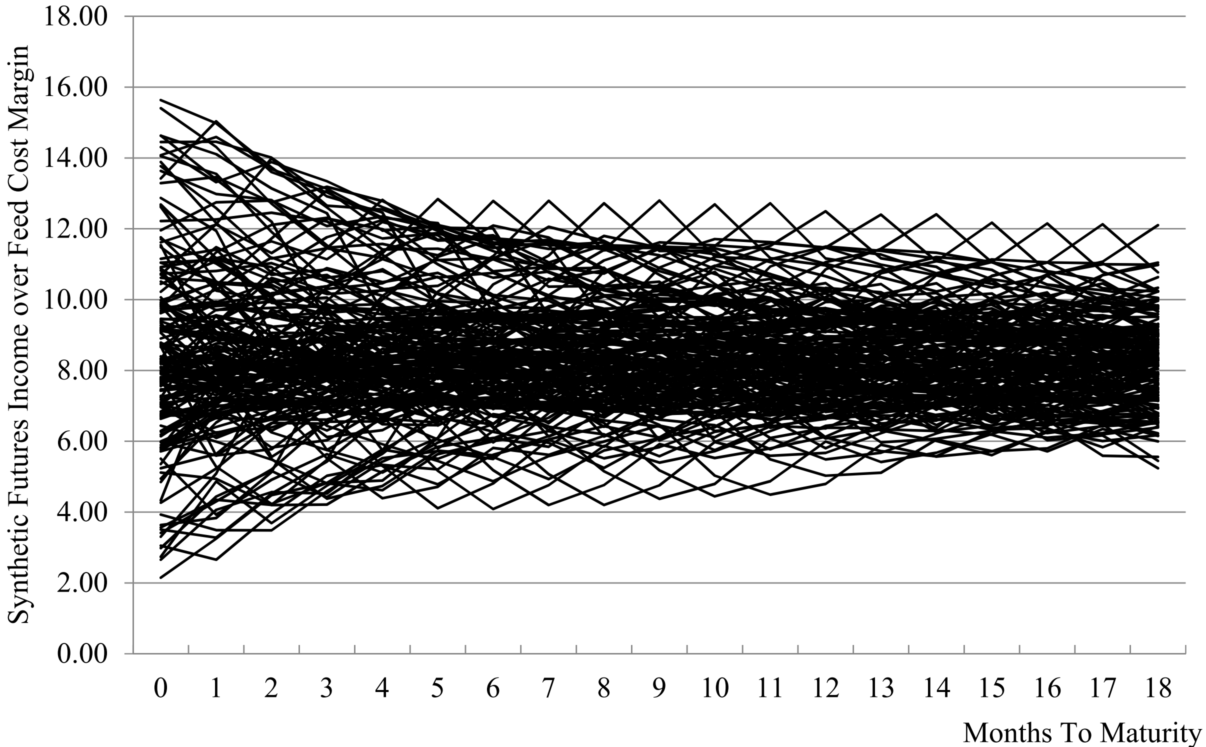
In summary, the hedging horizon must be chosen such that major shocks to the cash prices, which have accrued before the hedge initiation, will have little bearing on the expected profit margins for the time period an agent seeks to hedge. This idea is further explored in Appendix 1 where California mailbox milk prices are hedged using Class III milk futures and options on futures. While hedging program with the horizon of 1 month reduces semivariance of prices by only 8-10%, hedging with futures and puts at 11 month horizon reduces semivariance by 81% and 58%, respectively.

2. Are income over feed cost margins mean-reverting?

In section 1, we have stipulated, but not proven that the milk prices and income over feed cost margins are mean-reverting. In this section we examine if statistical evidence corroborates our stipulation. Figure 5 presents term structures of dairy income over feed costs (IOFC) margins, observed once per month from May 2002 through June 2016. Each line on the chart presents one term structure of synthetic futures IOFC margins.

What is immediately obvious is that when spot margins are above the long-term average, the term structure of futures margins slopes downward. Vice versa, when spot margins are below the long-term average, the term structure of futures margins tends to slope upward. This stylized fact matches findings of Bessembinder et al. (1995) who developed a test for mean-reversion of spot prices using the slopes of the term structure of futures prices. They found that futures in many asset classes, especially in agricultural commodities and crude oil, are mean reverting. This means that if the prices rise higher than the mean, they are expected to converge back to the mean, and, on the other hand, if prices drop too low relative to the mean, they are expected to increase back up to the level of the mean.

Figure 5. Term structures of synthetic futures income over feed cost margins, 2002-2016



In this section we modify and extend the Bessembinder et al. to test if dairy income over feed cost margins are mean-reverting. Consider the following regression:

$$F_{t,t+k+1} - F_{t,t+k} = \alpha_k + \beta_k \left[y_t - \overline{y_t} \right] + \varepsilon_t \tag{1.13}$$

$F_{t,t+k}$ and $F_{t,t+k+1}$ are futures prices, observed at time t , for contracts expiring in k and $k+1$ months, respectively. The difference between these futures prices is thus the slope of the term structure between two consecutive futures prices at horizon k . With $\overline{y_t}$ we denote the moving 5-year average of spot prices. The characteristic of the spot prices that are expected by futures market participants to be mean reverting is that when the difference between the current spot price and the 5-year average price is positive, the slope of the term structure is negative. Thus, our first testable hypothesis is:

$$\beta_k < 0, \forall k \tag{1.14}$$

Furthermore, if prices are mean-reverting, then cumulative past shocks, materialized as the deviation of the current spot price from the long-term average price, are going to affect nearby futures prices more than deferred futures prices. Mathematically, coefficient β_k should be closer to zero for higher values of k:

$$k_1 < k_2 \Rightarrow \beta_{k_1} < \beta_{k_2} < 0 \tag{1.15}$$

We estimate equation (1.13) using synthetic income over feed cost futures prices for period 2002-2016. The results are provided in the Table 2.

Table 2. Term structure-based stationarity test for dairy income over feed cost margins.

Hedging Horizon (k)	β_k	R^2
1	-0.16*** (0.02)	0.26
2	-0.13*** (0.02)	0.20
3	-0.11*** (0.02)	0.15
4	-0.08*** (0.02)	0.11
5	-0.06** (0.02)	0.06
6	-0.05* (0.02)	0.03
7	-0.03 (0.02)	0.01

Standard errors in parentheses
* p<0.05, ** p<0.01, *** pα <0.001

3. Estimating the speed of mean reversion

Given unconditional variance of cash prices, σ^2 , and unconditional variance of futures prices at k - month horizon $\sigma_k = (1-\alpha)^{2k}\sigma^2$, we can glean insight about the speed of the mean reversion α by taking the ratio of the sample variance of spot prices and k-horizon futures prices.

$$\frac{Var(y_t)}{Var(F_{t,t+k})} = \frac{\sigma^2}{(1-\alpha)^{2k}\sigma^2} = \frac{1}{(1-\alpha)^{2k}} \quad (1.16)$$

It follows that

$$\alpha = 1 - \left[\frac{Var(F_{t,t+k})}{Var(y_t)}\right]^{\frac{1}{k}} \quad (1.17)$$

We can obtain sample estimates of spot and futures price variances from observed spot and futures prices over 2002-2016 period. Using equation (1.17) for $k = 1,...,12$, estimates of α are provided in Table 3.

Table 3. Estimated coefficients of speed of mean reversion

k	$Var(F_{t,t+k})$	α
Spot	13.18	N/A
1	10.40	0.11
2	8.55	0.10
3	7.44	0.09
4	6.70	0.08
5	6.15	0.07
6	5.73	0.07
7	5.38	0.06
8	5.09	0.06
9	5.02	0.05
10	5.05	0.05
11	4.98	0.04
12	4.93	0.04

The average estimated $\alpha = 0.07$. Using observed mean of spot prices of $15.49/cwt, and variance of 13.18, we can calculate what is the highest futures price that can be guaranteed at hedging horizon k with 95% probability.

k	$y^{*}_{0.05}$	k	$y^{*}_{0.05}$
Spot			
1	9.94	13	13.17
2	10.32	14	13.33
3	10.69	15	13.48
4	11.02	16	13.62
5	11.33	17	13.75
6	11.63	18	13.87
7	11.90	19	13.99
8	12.15	20	14.09
9	12.38	21	14.19
10	12.60	22	14.28
11	12.80	23	14.36
12	12.99	24	14.44

Conclusions

The theory of competitive markets predicts that profits in excess of opportunity costs cannot be sustained in the long run. This implies profit margins in competitive agricultural sectors should be mean-reverting. Expected time series properties of spot prices and margins can be inferred from the behavior of the term structure of futures prices. When spot prices or margins are mean-reverting, effective risk management will require that hedges be executed at a horizon that is inversely related to the speed of the mean reversion. Optimal hedging horizon is the minimum contract time-to-maturity at hedge initiation necessary to guarantee the futures price will be above a set threshold level with desired level of certainty.

We illustrate these principles in the U.S. dairy sector, and find that dairy income over feed cost margins are mean-reverting, with at least about 40-60% of shocks to spot milk prices resolved within a year. Hedging at distant horizons substantially improves hedging effectiveness. The main challenge with implementing the insights of this research by dairy producers is that dairy futures and options markets are not liquid at distant horizons needed for effective risk management. To the extent this is due to the lack of supply of short (producer-based) hedges, it may not be a critical problem. If, on the other hand, increase in hedging demand by dairy producers would not be met by more processors wiling to hedge their input prices at long horizons, such developments would provide a justification for provision of dairy risk management tools by the federal government.

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Appendix I: Hedging Horizon and Hedging Effectiveness: A Case Study with a Representative California Milk Producer

To study the impact of hedging horizons on hedging effectiveness, we put ourselves in the shoes of a hypothetical dairy producer in California who receives mailbox price and hedges her production with Class III milk contracts on a monthly basis. We devise two simple hedging strategies and see how they performed over the 2007-2016 period. The first strategy is a short futures hedge. The second strategy involves a long position with puts with the highest available strike price subject to premium costs being 50 cents or lower. Without loss of generality, the position size is set at one contract.

Each hedge position starts on a CME trading day that is closest to the 15th of a month. The particular month in which the hedge is set up depends on the hedging horizon. We experiment with different hedging horizons that encompass all odd numbers of months below 12. Once the hedge is set up, we hold the position until the hedging instrument expires. For example, to hedge April 2015 milk production with a hedging horizon of 3 months, we set up the hedge on Jan 15th of the same year and hold the position until Apr 29th when the Class III milk price is announced. We repeat this for each production month from December 2007 to July 2016.

After the repetition, we looked at the distribution of the effective hedged price, defined as the sum of the California mailbox milk price plus the hedging gains or losses.

Figure A.1 to Figure A.6 illustrate how the futures hedge compares to the baseline case of no hedge. The blue area represents the lowest price in both cases. The red area represents the price mark-downs due to negative hedging P&L. The combined levels of the blue and red areas signal the mailbox price which the producer would have had without any hedge. Finally, the green area shows price compensation due to positive hedging P&L. Therefore, the combined green and blue areas represent the effective price levels after hedge. Figure A.7 to A.12**Error! Reference source not found.** depict the effective milk price under option-based hedge strategy that utilizes puts.

Figure A.1 Effective milk price under 1-month ahead futures hedge

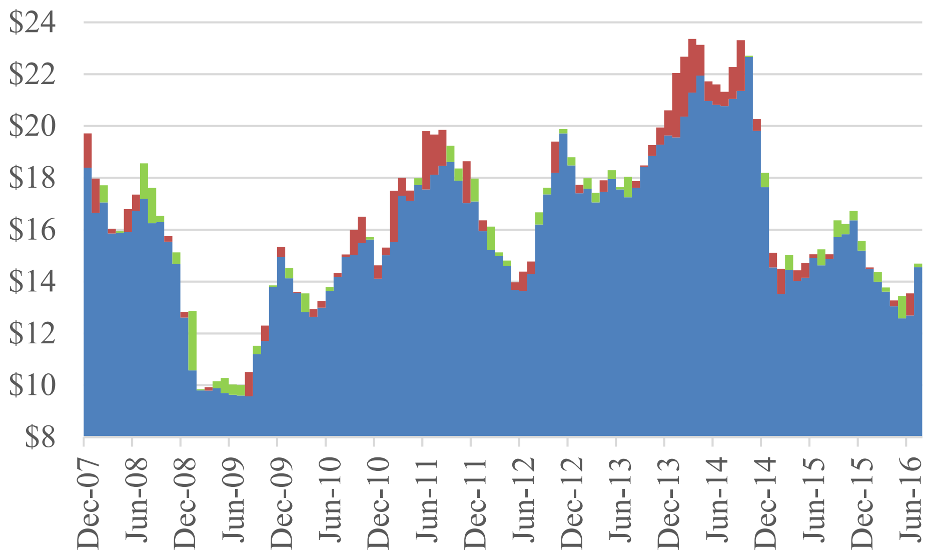


Figure A.2 Effective milk price under 3-month ahead futures hedge

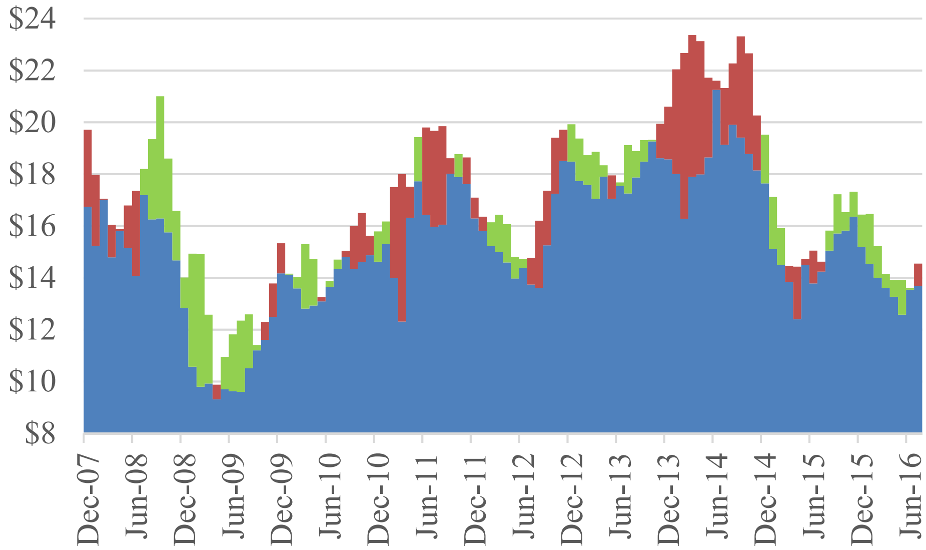


Figure A.3 Effective milk price under 5-month ahead futures hedge

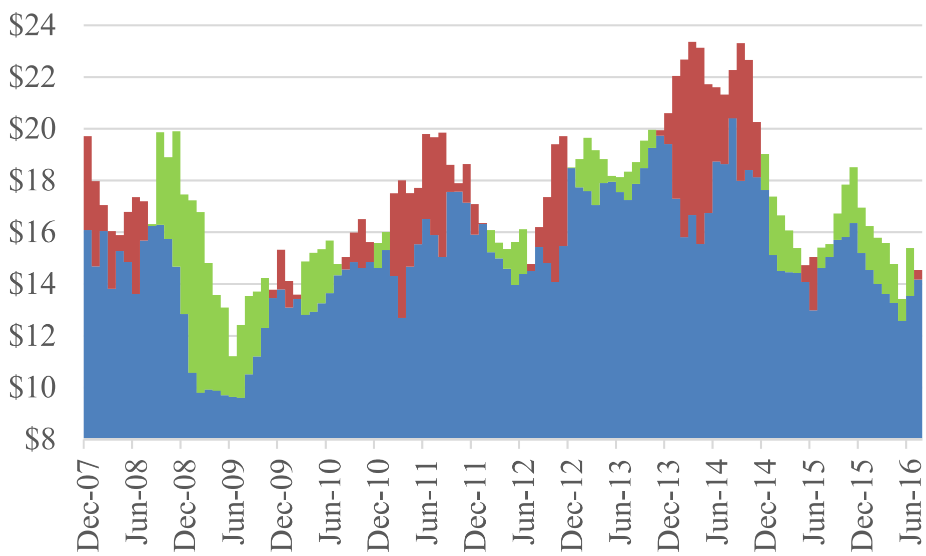


Figure A.4 Effective milk price under 7-month ahead futures hedge

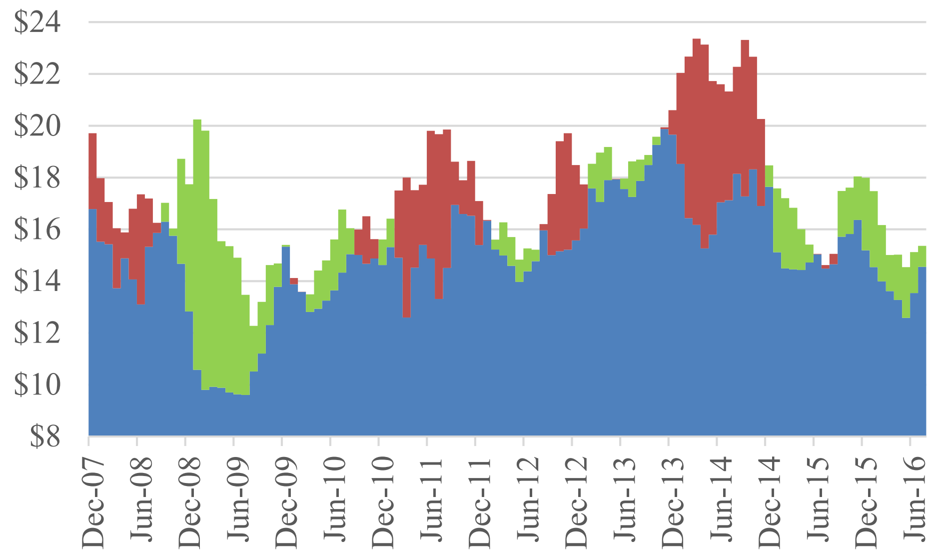


Figure A.5 Effective milk price under 9-month ahead futures hedge

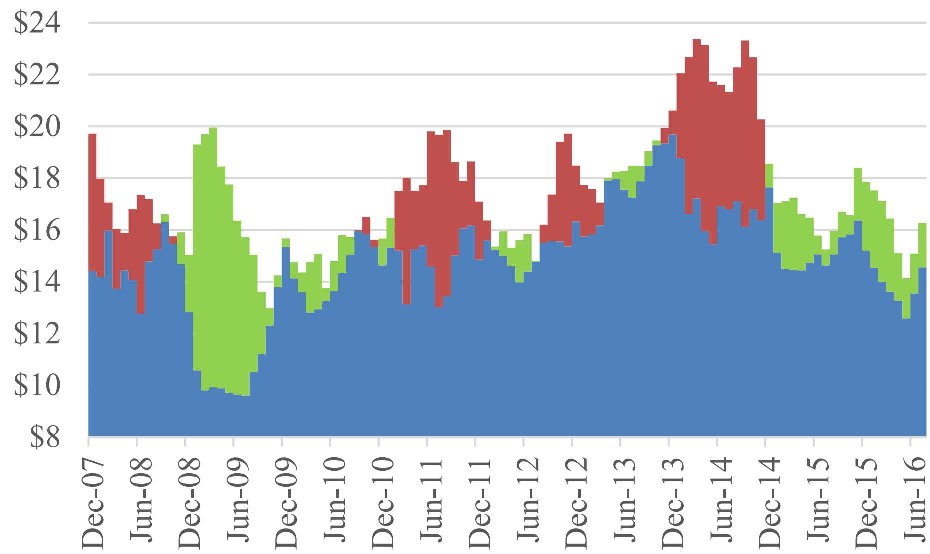


Figure A.6 Effective milk price under 11-month ahead futures hedge

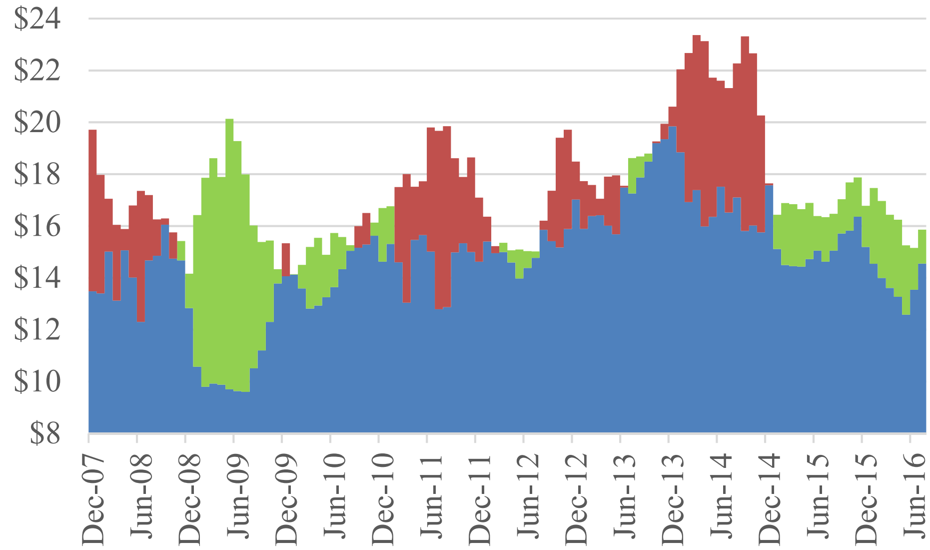


Figure A.7 Effective milk price under 1-month ahead put hedge

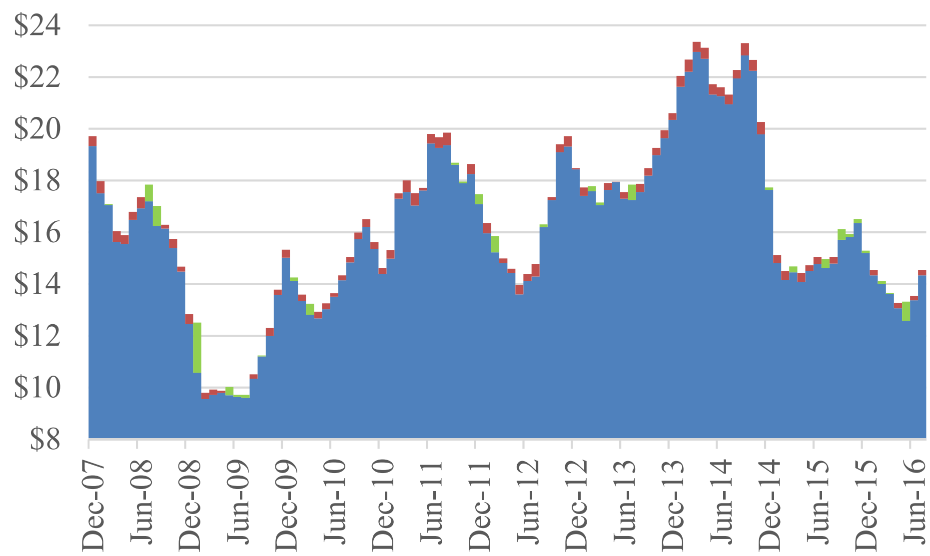


Figure A.8 Effective milk price under 3-month ahead put hedge

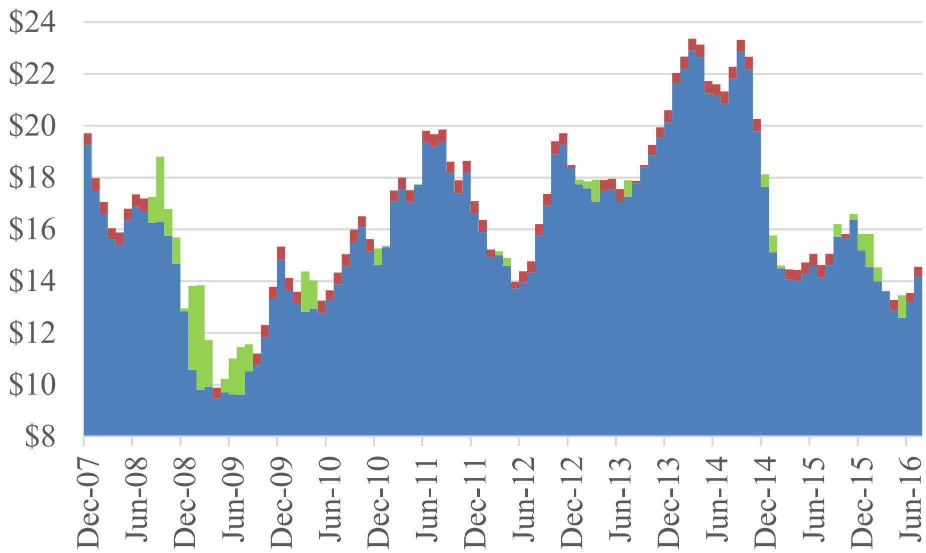


Figure A.9 Effective milk price under 5-month ahead put hedge

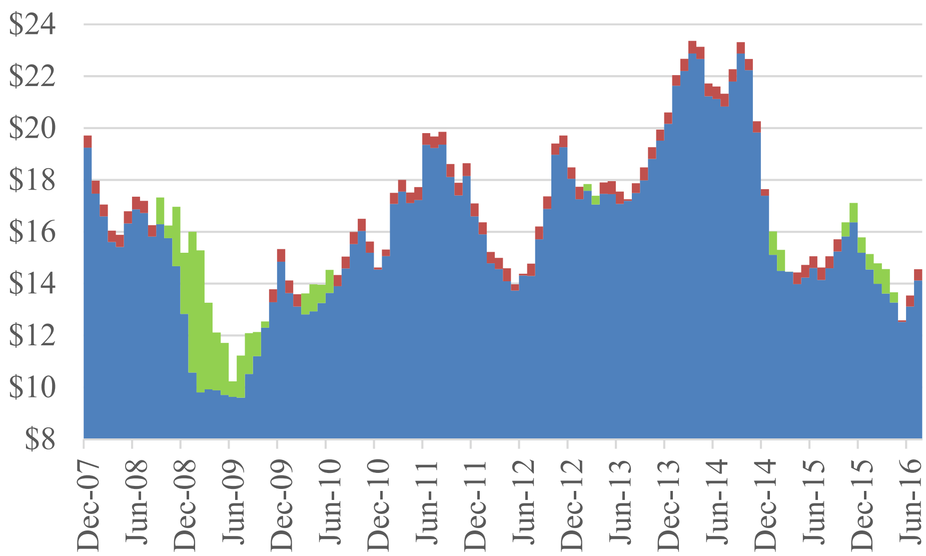


Figure A.10 Effective milk price under 7-month ahead put hedge

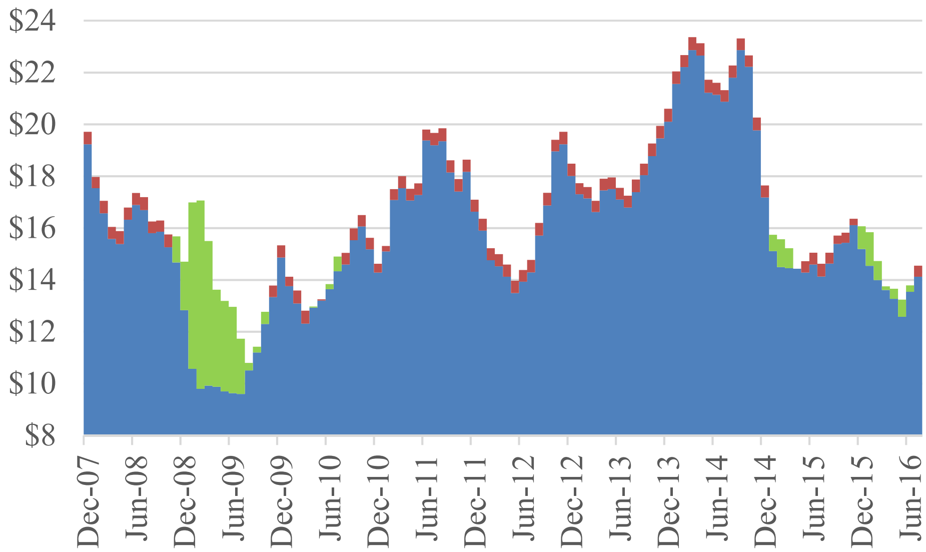


Figure A.11 Effective milk price under 9-month ahead put hedge

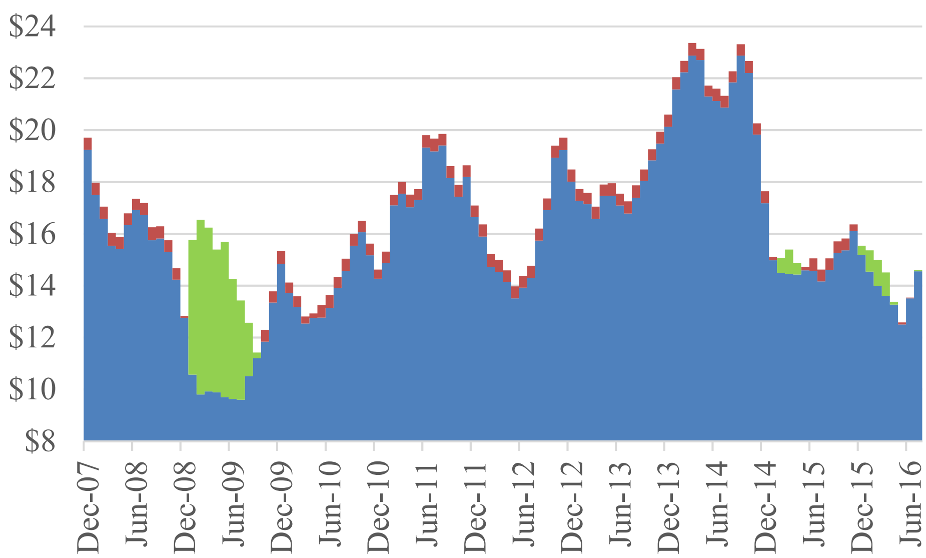
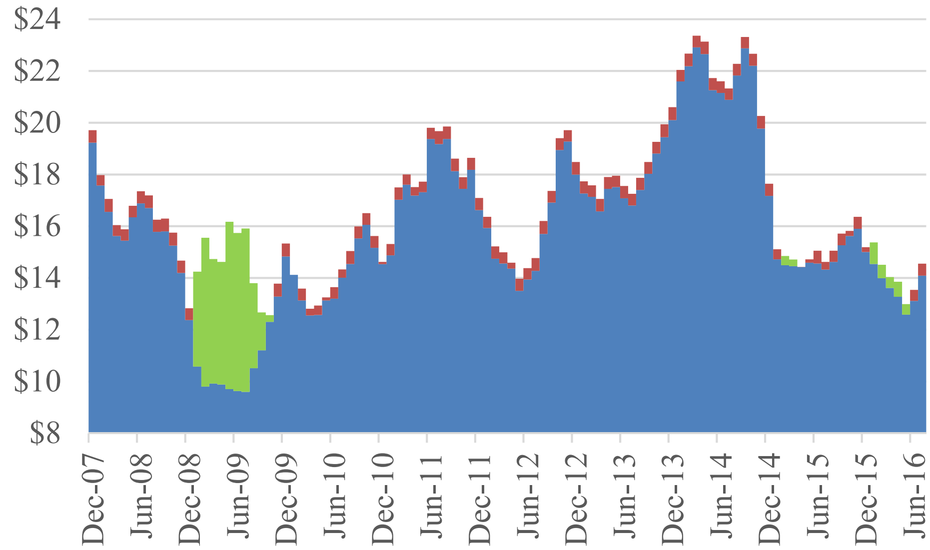


Figure A.12 Effective milk price under 11-month ahead put hedge



Under either strategy, the producer receives considerable price boosts in 2009. Due to the fact that hedging with futures curbs upside potential, the futures positions incur significant losses in 2012.

Another way of looking at hedging effectiveness is to measure the amount of variance being reduced. Here we focus on both the variance and the semivariance of the effective hedged prices. While variance treats both positive and negative price shocks symmetrically, semivariance measures only the down-side risk. Table A.1. and Table A.2 show the sample variance, semivariance and threshold semivariance of the effective milk prices and their comparison to the mailbox prices. Under all three measures, the futures-based hedging strategy reduces price dispersion more than the option-based hedging strategy. As the hedging horizon becomes longer, the reduction measured by variance and semivariance converges under futures hedge. On the other hand, the overall variance reduction under the put hedge is not as high as the futures hedge, possibly due to the premiums charged in holding the put positions. But the semivariance reduction becomes more pronounced under the put hedge strategy and approaches the reduction level of the futures hedge strategy at longer horizon. This observation seems in concert with the fact that futures cannot remove downside risks.

Table A.1. Hedging Effectiveness of the futures-based hedging strategy

	Variance		Semivariance (Threshold: 0.0)		Semivariance (Threshold: 1.5)	
	Value	Reduction	Value	Reduction	Value	Reduction
Spot	10.64	--	9.70	--	6.44	--
1 month	8.59	19%	6.42	8%	6.33	2%
3 months	5.93	44%	3.69	47%	3.28	49%
5 months	4.00	62%	2.05	71%	1.79	72%
7 months	3.03	71%	1.29	82%	1.36	79%
9 months	2.55	76%	1.29	81%	1.03	84%
11 months	2.44	77%	1.61	77%	1.47	77%

Table A.2. Variance reduction of effective hedged price with puts

	Variance		Semivariance (Threshold: 0.0)		Semivariance (Threshold: 1.5)	
	Value	Reduction	Value	Reduction	Value	Reduction
Spot	10.64	--	9.70	--	6.44	--
1 month	9.89	7%	6.54	6%	6.16	4%
3 months	8.64	19%	5.33	24%	4.24	34%
5 months	7.52	29%	3.55	49%	2.82	56%
7 months	7.15	33%	3.36	52%	2.45	62%
9 months	7.01	34%	2.95	58%	2.27	65%
11 months	6.77	36%	2.29	67%	1.44	78%