Ph.D. PRELIMINARY EXAMINATION

MICROECONOMIC THEORY

Applied Economics Graduate Program

August 2014

The time limit for this exam is four hours. The exam has four sections. Each section includes two questions. Answer one question from each section. Before turning in your exam, number each page of your answers in sequential order, and identify the question (for example, III.2 for section III and question 2). Indicate, by circling below, the questions you completed. If you answer more than one question in a section and do not circle a question corresponding to that section, the first of the two that appears in your solution paper will be graded.

**********************************************

STUDENT ID LETTER: _______ (Fill in your code letter)

Answer one question from each section. Indicate the one you answered by circling:

Section I: Question 1 Question 2
Section II: Question 1 Question 2
Section III: Question 1 Question 2
Section IV: Question 1 Question 2

*** TURN IN THIS SHEET WITH YOUR ANSWER PAGES***

NOTE: The exam should have 13 pages including this cover page.
Part I

Answer at most one question from Part I.
Consider a consumer whose utility is a function of two goods, $x_1$ and $x_2$. The consumer’s utility function is:

$$u(x) = x_1 \times e^{x_2}$$

a) Let $w$ be the consumer’s wealth, and denote the price of good 1 as $p_1$ and the price of good 2 as $p_2$. Assuming that the consumer spends his or her entire wealth on goods $x_1$ and $x_2$, use constrained optimization to solve for the Walrasian demands for $x_1$ and $x_2$.

b) Use your answer to a) to derive the indirect utility function of this consumer. Then, use the indirect utility function to obtain the expenditure function.

c) Use your answer to b) to obtain the Hicksian demands for $x_1$ and $x_2$.

d) Write down the Slutsky equation for a single good. For both $x_1$ and $x_2$, show that your answers for a) and c) satisfy the Slutsky equation for own-price derivatives (the change in the demand for a good when its own price changes).
Question I.2

Expected Utility Behavior. Consider 3 people, A, B and C. They have the following (Bernoulli) utility functions:

Person A: \( u_A(x) = x^{0.5} \)

Person B: \( u_B(x) = x \)

Person C: \( u_C(x) = x^2 \)

a) For each of these three people, is he or she risk neutral, risk averse, or “risk loving” (neither risk neutral nor risk averse). You do not need to demonstrate, just give a very brief answer.

b) There are 3 possible lotteries that these people can choose from. Each lottery has only two possible outcomes (amounts of money). For each lottery, both outcomes have a probability of 0.5. The 3 lotteries are:

\( L_1: 49 \) with probability 0.5, or \( 49 \) with probability 0.5 (no risk)

\( L_2: 64 \) with probability 0.5, or \( 36 \) with probability 0.5

\( L_3: 81 \) with probability 0.5, or \( 16 \) with probability 0.5

Assume that all three persons maximize expected utility. Each can choose one lottery. Which lottery does each person choose? Show the calculations needed to justify your answer.

c) For Person A, what are the certainty equivalents of \( L_2 \) and \( L_3 \)?

d) Suppose that lotteries are not “all or nothing” but instead can be purchased in terms of “shares” that sum to 1. For example, instead of choosing between \( L_2 \) and \( L_3 \) a person can choose a “lottery portfolio” that is, say, 0.4 “shares” of \( L_2 \) and 0.6 “shares” of \( L_3 \). For any consumer, let \( s_1 \) be the share of \( L_1 \), \( s_2 \) be the share of \( L_2 \), and \( s_3 \) be the share of \( L_3 \), where \( s_1 + s_2 + s_3 = 1 \). Assume also that the three lotteries are perfectly correlated, so that there is a 0.5 probability that they all have their lower returns and a 0.5 probability that they all have their higher returns. Give an intuitive reason for why Consumer A will never purchase any shares of \( L_3 \). That is, \( s_3 = 0 \) for Consumer A. You should be able to answer this in 2-3 sentences.

e) Using the fact that Consumer A will never purchase shares of \( L_3 \), show whether it is the case that a “mixture” of \( L_1 \) and \( L_2 \) is better for Consumer A than purchasing either all of \( L_1 \) or all of \( L_2 \). [Hint: Express \( s_2 \) as \( 1 - s_1 \) in Consumer A’s expected utility. Calculate Consumer A’s marginal expected utility with respect to \( s_1 \) and evaluate it at \( s_1 = 0 \) and \( s_1 = 1 \). You do not need to solve for the optimal \( s_1 \).]
Part II

Answer at most one question from Part II.
Question II.1

Consider the revenue function for a two output, single input technology:

\[ R(p_1, p_2, z) = (p_1^\alpha + p_2^\alpha)\frac{1}{\alpha} z^\tau \]

where \( z \in \mathbb{R}_+ \) is the input; \( p_1 > 0 \) and \( p_2 > 0 \) are the competitive output prices; and \( \alpha > 0 \) and \( \tau > 0 \) are constant parameters. The PPS used to derive this revenue function is nonempty, strictly convex, closed, and satisfies weak free disposal of outputs and inputs.

a) Derive the conditional supplies for this revenue function.

b) Assuming the competitive price of the input is \( r > 0 \) and \( \tau = 0.5 \), find the profit maximizing unconditional input demand. Show that this unconditional input demand is homogeneous of degree zero in \( p_1, p_2 \) and \( r \), and non-increasing in \( r \).

c) Derive the unconditional supplies using duality results.

d) It is easy to verify that the revenue function above is homogeneous of degree one in \( p_1 \) and \( p_2 \) and that the conditional supplies derived in part a) (assuming they are correct) are homogenous of degree zero in \( p_1 \) and \( p_2 \). Show that these homogeneity properties hold in general for any revenue function and conditional supplies derived from a production possibility set with \( N \) inputs and \( M \) outputs.
Question II.2

Consider a world with only two states denoted by $a$ and $b$. Assume a firm can produce multiple outputs in these two states using two inputs denoted by $z_1 \geq 0$ and $z_2 \geq 0$ and a production possibility set that is nonempty, strictly convex, closed, and satisfies weak free disposal of outputs and inputs. The revenue cost function derived from this production possibility set is

$$C(R_a, R_b) = \frac{1}{r_1 r_2^2} \left( \frac{R_a^2}{2} + \frac{R_b^2}{3} \right)$$

where $r_1 > 0$ and $r_2 > 0$ are the competitive prices for $z_1$ and $z_2$, and $R_a \geq 0$ and $R_b \geq 0$ are the revenue produced in state $a$ and $b$. The risk averse firm’s state contingent utility of profit is

$$W(\pi_a, \pi_b) = \pi_a^\alpha \pi_b^\beta$$

where $\pi_a = R_a - C(R_a, R_b)$ and $\pi_b = R_b - C(R_a, R_b)$ are profits in state $a$ and $b$, and $\alpha > 0$ and $\beta > 0$ are constant parameters.

a) Given the firm’s state contingent utility function,

(i) find its subjective beliefs regarding the probability $\phi_a > 0$ of state $a$ and $\phi_b > 0$ of state $b$ where $\phi_a + \phi_b = 1$.

(ii) derive its relative risk premium and show that it has constant relative risk aversion.

b) Given the firm’s revenue cost function, derive its production-risk premium.

c) It is straightforward to show that the revenue cost function above is homogeneous of degree one in $r_1$ and $r_2$. Show that this must be true in general when there are $S$ states of the world, and the production possibility set has $M$ outputs in each state and $N$ inputs.
Part III

Answer at most one question from Part III.
Question III.1

There are two countries, $A$ and $B$. Suppose that good $X$ is consumed only in country $B$. The inverse demand function in country $B$ is $P(x) = a - x$, where $x$ is the total output produced and sold by firms in countries $A$ and $B$. Let $c$ denote the constant marginal cost of production that is the same for all firms, with $0 < c < a$.

(a) Suppose there are $N (> 1)$ firms in the two countries that are Cournot competitors. Solve for the Nash equilibrium.

(b) Now suppose that there are two firms, one in each country, and that the game has two periods. In period 1, the government of country $A$ chooses an export tax or subsidy per unit of exports. In period 2, the two firms, which have observed government $A$’s choice, simultaneously choose quantities. The objective of country $A$’s government is to maximize the sum of its own receipts and the profit of its firm. Find the optimal tax or subsidy policy for government $A$. Explain the economic intuition for the result.

(c) Finally suppose that all $N$ firms are located in country $A$ (country $B$ does not produce good $X$). Assume a general inverse demand curve $P(x)$, i.e., do not assume a linear inverse demand curve as in parts (a) and (b). Show that an optimal policy for the government of country $A$ is to levy a unit export tax equal to

$$-P(x_m)(N-1)x_m/N$$

where $x_m$ is the monopoly output ($x_m$ maximizes $x(P(x)-c)$). Give an explanation for the results in terms of externalities.
There are two people who live in a community that is considering whether or not to provide a public good. Each person chooses whether or not to contribute to providing the public good. The benefit of the public good to person 1 is $b_1$ and to person 2 is $b_2$. The cost of contributing to provide the public good is 1. The public good is provided if at least one person contributes. The public good is not provided if neither person contributes in which case both people get a payoff of 0.

a) Suppose that both people know that $b_1 > 1$ and $b_2 > 1$ and that both people simultaneously choose whether or not to contribute to the public good. Write down a 2 x 2 normal form game and find all Nash equilibria for this game. Do these Nash equilibria achieve an outcome that maximizes the sum of the payoffs to both players?

b) Now suppose that both people know that $b_1 < 1$ and $b_2 < 1$ and, as in part (a), that both people simultaneously choose whether or not to contribute to the public good. Write down a 2 x 2 normal form game and find all Nash equilibria for this game. Do these Nash equilibria achieve an outcome that maximizes the sum of the payoffs to both players?

c) Now suppose that the people choose whether or not to contribute in a sequential manner with person 1 choosing first. Find all subgame perfect equilibria for the case where both people know that $b_1 > 1$ and $b_2 > 1$.

d) Finally, suppose that the benefit for each person is drawn from a uniform probability distribution on (0, 2). Each person knows their own benefit but not the benefit for the other player. Find a Bayesian Nash equilibrium for this game.
Part IV

Answer at most one question from Part IV.
Question IV.1

Consider a $2 \times 2$ competitive exchange economy, where subscript $j = 1, 2$ indexes consumers and superscript $i = 1, 2$ indexes goods. Preferences are represented by the utility functions

$$U_1(x_1^1, x_1^2) = \begin{cases} 2x_1^1 + x_1^2 & \text{if } x_1^1 \leq x_1^2 \\ x_1^1 + 2x_1^2 & \text{if } x_1^1 \geq x_1^2 \end{cases} \quad \text{and} \quad U_2(x_2^1, x_2^2) = \min\{x_2^1, x_2^2\}.$$ 

The initial endowment vectors are $\omega_1 = (2, 2)$ and $\omega_2 = (2, 0)$.

(a) In a carefully labeled Edgeworth box diagram, indicate the endowment $\omega$ and draw at least one indifference curve for each consumer. Also draw the contract curve, the set of Pareto-optimal allocations. Be careful to distinguish the set of allocations that are strongly PO from those that are weakly PO.

(b) Write down the offer curves, the $x_j^i(p, \omega_j)$, for the two consumers. This problem may be solved more easily using a careful diagram than formal optimization techniques. Construct a new Edgeworth-box diagram depicting the offer curves.

(c) Derive the Walrasian equilibrium price-allocation vector $(p^*, x^*)$. Indicate and label the equilibrium in your Edgeworth box from part b.
Question IV.2

Consider a 2-person economy with one private good \( x \) and one public good \( y \). Consumers are indexed by the subscript \( j = 1, 2 \). Their utility functions are Cobb-Douglas and quasi-linear respectively:

\[
U_1(x_1, y) = \ln x_1 + \ln y \quad \text{and} \quad U_2(x_2, y) = x_2 + \ln y.
\]

Each consumer is endowed with \( w_j = 1.5 \) units of the private good. Let \( x_j \) denote the consumed portion of \( w_j \) and let \( z_j = w_j - x_j \) denote the portion contributed to the provision of \( y \). The technology for producing the public good is linear, given by \( y = z_1 + z_2 \).

(a) Determine the Pareto-optimal outcome \((\hat{y}, \hat{x}_1, \hat{x}_2)\). Confirm that the Samuelson condition is satisfied at your solution.

(b) Now determine the outcome that results at the voluntary-contribution Nash equilibrium \((y^*, x_1^*, x_2^*)\). Your solution should include the BBV-style reaction functions, the \( z_j(z_{-j}) \), and their joint solution. Be sure to consider the possibility that \( z_j = 0 \) for one or both consumers.

(c) Now suppose that the total endowment remains the same, but the division changes so that \( w_1 = 5/2 \) and \( w_2 = 1/2 \). Determine the new equilibrium outcome. Interpret your result in light of the BBV theorem regarding crowding out of contributions to a public good.

(d) Finally, suppose the endowment shifts in the other direction, so that \( w_1 = 1/2 \) and \( w_2 = 5/2 \). Determine the new equilibrium outcome and once again interpret your result in light of BBV.