CHAPTER 10: COST

For next time please read ch 11

COSTS IN THE LONG RUN

- In the long run the firm can adjust the levels of all factor of production.
- The question that we want to address is therefore the following:
- The manager of the firm wants to produce a given level of output at the lowest possible cost. She is free to choose any input combination she pleases. Which one should she choose?

COST MINIMIZATION IN THE LONG RUN

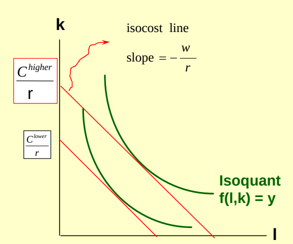
- Suppose that there are two factors of production that have prices w and r, let *l* and *k* measure the amounts used of the two factors and let f(*l*,*k*) be the production function for the firm, we can write this problem as:

$$\min_{\{l,k\}} \; w\,l + r\,k$$
$$\text{such that} \quad f(l,k) = y$$

- The solution to this cost-minimization problem -- **the minimum cost necessary to achieve the desired level of output** -- will depend on **w, r, and y**.
- Thus, we write it as **c(w, r, y)**. This function is the **cost function** which measures the **minimal costs** of producing **y** units of output when factor prices are **(w, r)**.

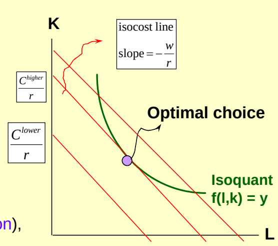
- To understand the solution to the minimization problem, let us depict the cost and the technological constraints facing the firm on the same diagram.
  - The **isoquants** give us the technological constraints -- all the combinations of l and k that can produce y.
  - Now, we want to plot all the combinations of l and k that have some given level of cost, C. We write:

$$w\,l + r\,k = C,$$
which gives:
$$k = \frac{C}{r} - \frac{w}{r}\,l$$



- Thus, it is a straight line with a slope of **- w/r** and a vertical intercept of **C/r**.
- As we let the number C vary we get a whole family of **isocost lines**. Every point on an isocost line has the same cost, C, and higher isocost lines are associated with higher costs.

- Thus, our cost-minimization problem can be rephrased as:
  - **find the point on the isoquant that has the lowest possible isocost line associated with it.**
- Thus, if the optimal solution involves using some of each factor (i.e., not a corner solution), and if the isoquant is a nice smooth curve (i.e., no kink), then the cost-minimizing point will be characterized by a tangency condition:



The slope of the isoquant must be equal to the slope of the isocost line.

The marginal rate of technical substitution must equal the factor price ratio:

$$-\frac{MP_L(l,k)}{MP_K(l,k)} \equiv MRTS(l,k)$$
$$= -\frac{w}{r}$$

Note that if we have a corner solution where one of the two factors isn't used, the tangency condition need not be met. Similarly, if the isoquant has kinks, the tangency condition has no meaning. These exceptions are just like the situation with consumer theory.

- Consider any change in the pattern of production $(\Delta l, \Delta k)$ that keeps output constant. Such a change must satisfy:

$$MP_L(l,k)\;\Delta l \;+\; MP_K(l.k)\;\Delta k \;=\; 0$$

- So along the isoquant

$$\frac{\Delta k}{\Delta l} = -\frac{MP_L}{MP_K}$$

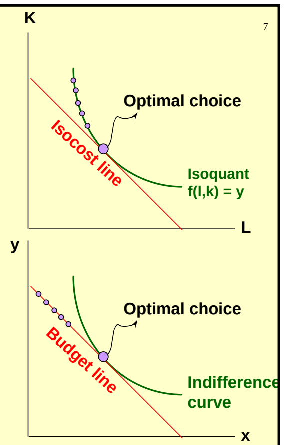
- Now consider the change $\frac{\Delta k}{\Delta l}$ along the isocost line

$$\frac{\Delta k}{\Delta l} = -\frac{P_L}{P_K}$$

- Putting the two expressions together gives us the condition for tangency:

$$\frac{\Delta k}{\Delta l} = -\frac{MP_L(l,k)}{MP_K(l,k)} = -\frac{w}{r}$$

- Note that the graph bears a certain resemblance to the solution to the consumer-choice problem depicted earlier.
- Although the solutions look the same, they really aren't the same kind of problem.
  - In the producer problem, the isoquant is the technological constraint and the producer **moves along the isoquant** to find the optimal position.
  - In the consumer problem, the straight line was the budget constraint, and the consumer **moved along the budget constraint** to find the most-preferred position.



How to find minimum cost of production?

- From the graph we can deduce the two conditions needed for cost minimization:

1. There is a tangency between the isocost curve and the isoquant. This implies:

$$\frac{\Delta k}{\Delta l} = -\frac{MP_L(l,k)}{MP_K(l,k)} = -\frac{w}{r}$$

2. The technological constraint must hold: $f(l,k)=Q_0$

Cost minimization: Example

- Output for a simple production process is given by Q = 2KL, where K denotes capital, and L denotes labor. The price of capital is $20 per unit. The price of labor is $5 per unit. What is the total cost of producing 32 units of output?

- $MP_L=2k$, $MP_K=2L \Rightarrow MP_L / MP_K = k/l$
- At the minimum cost $MP_L / MP_K = P_L/P_K \Rightarrow$ k/l =5/ 20
- In addition we know that 32=Q=2kl

- Combine these two peaces of information: k = 0.25l
  2(0.25l)l=32 $\Rightarrow$ l=8, k=2.

Total cost to produce Q=80 is thus, 8X5 + 2X20 = $56

LONG-RUN COSTS

- In the above analysis, we have regarded the firm's fixed costs as being the costs that involve payments to factors that it is unable to adjust in the short run. In the long run a firm can choose the level of its "fixed" factors - they are no longer fixed.

- Just to be specific, let's think of the fixed factor as being plant size and denote it by **k**. The firm's **short-run** cost function, given that it has a plant of k square feet, will be denoted by **TC_s(y, k)**, where the s subscript stands for "short run."

- Clearly, for any given level of output there will be some plant size that is the optimal size to produce that level of output. Let us denote this plant size by **k(y)**. This is the firm's **conditional factor demand** for plant size as a function of output y. [Of course, it also depends on the prices of plant size and other factors of production, but we have suppressed these arguments.]

- Then, the long-run cost function of the firm will be given by:
  c_s (y, k(y)).
  - This is the total cost of producing an output level y, given that the firm is allowed to adjust its plant size optimally.
  - That is, the long-run cost function of the firm is just the short-run cost function **evaluated at the optimal choice of the fixed factors**:

    c(y) = **c_s(y, k(y))**

- Let us see how this looks graphically when it is represented as a function of **output**.
  - Pick some level of output $y^\#$, and let $k^\# = k(y^\#)$ be the optimal plant size for that level of output.
  - The short-run cost function for a plant of size $k^\#$ will be given by **$c_s(y, k^\#)$**, and the long-run cost function will be given by **$c(y) = c_s(y, k(y))$**, just as above.

- Now, note the important fact that the **short-run** cost to produce output y must always be **at least as large as** the **long-run** cost to produce y.

That is, $c(y) \le c_s(y, k^\#)$ for all levels of y.

- Why? In the short run the firm has a fixed plant size, while in the long run the firm is free to adjust its plant size optimally. Since one of its long-run choices is always to choose the plant size $k^\#$, its optimal choice to produce y units of output must have costs at least as small as $c(y, k^\#)$.

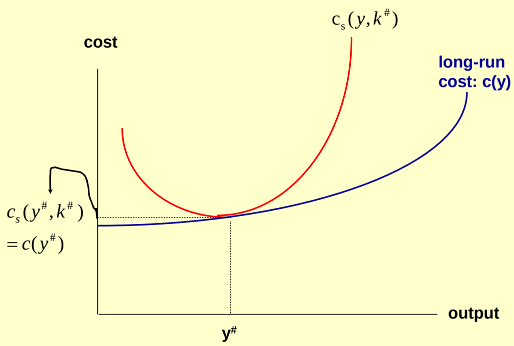
A given short-run cost curve [e.g., $c_s(y, k^\#)$] must lie above the long-run cost curve and must be tangent to the long-run cost curve at a point.

- In fact, at one particular level of y, namely $y^\#$, we know that

$$c(y^\#) = c_s(y^\#, k^\#)$$

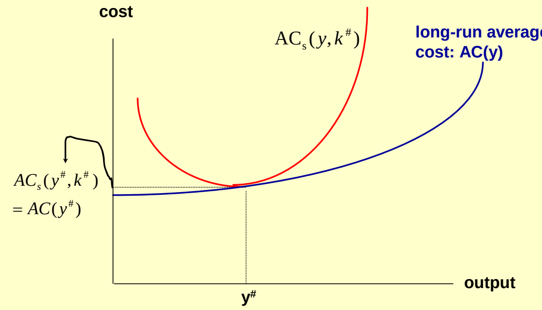
  - Why? Because at $y^\#$ the optimal choice of plant size is $k^\#$. So at $y^\#$, the long-run costs and the short-run costs are the same.

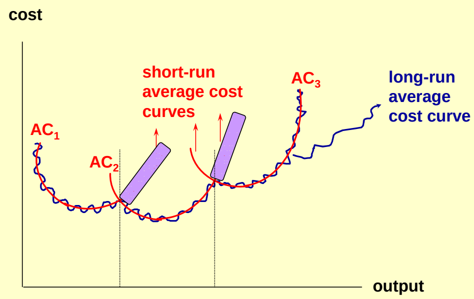
The short-run cost curve must be tangent to the long-run cost curve.



- Thus, the short-run cost is always greater than the long-run cost and they are equal at one level of output.
- The same property applies to the short-run and the long-run **average** costs:

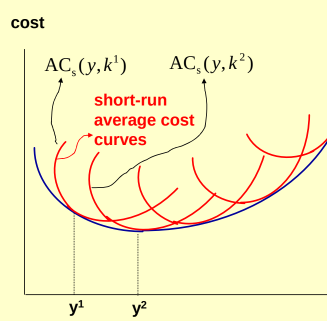
$$AC(y) \le AC_s(y,k^\#) \quad and \quad AC(y^\#) = AC_s(y^\#,k^\#)$$





Note that the key result is still the same as before. That is, the short-run average costs always are at least as large as the long-run average costs, and they are the same at the level of output where the long-run demand for the fixed factor equals the amount of the fixed factor that you have in the short run.

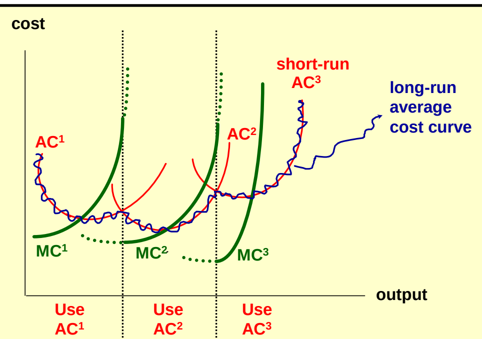
- We can do the same sort of construction for levels of output other than $y^\#$. Suppose we pick outputs $y^1, y^2, \ldots, y^n$ and accompanying plant sizes $k^1 = k(y^1), k^2 = k(y^2), \ldots, k^n = k(y^n)$. Then we get a picture like the one below:



Thus, the long-run average cost curve is the **lower envelope** of the many short-run average cost curves.

LONG RUN MARGINAL COSTS

- We've seen that the long-run average cost curve is the lower envelope of the short-run average cost curves.
- What are the implications of this for **long-run marginal costs**?
  - Consider first the case where there are discrete levels of plant size.
  - In this case, the long-run marginal cost curve consists of **the appropriate pieces** of the short-run marginal cost curves, as depicted in the next slide.
  - For each level of output, we see which short-run average cost curve we are operating on and then look at the marginal cost associated with that curve.



When there are discrete levels of the fixed factor, the firm will choose the amount of the fixed factor to minimize average costs. Thus, the long-run marginal cost curve will consist of the various segments of the short-run marginal cost curves associated with each different level of the fixed factor.

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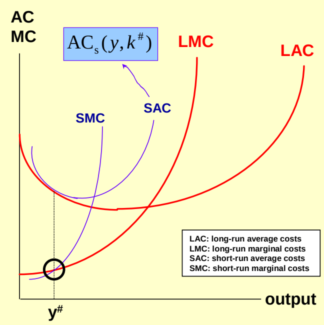
- In fact, the above result has to hold true no matter how many different plant sizes there are.
  - The long-run marginal cost at any output level **y** has to equal the short-run marginal cost associated with the optimal level of plant size to produce **y**.



Summery: cost

- Short run cost:
  - Description of TC
  - AC, AVC, MC
  - The relationship between cost curves and production function
  - Production with more than one plant
- Long run cost:
- Cost minimization
- Isocost curve
- Conditions for cost minimization

Summery: cost

- Long run cost: Choice of plant size
  - Relationship between AC in the short run and long run AC
  - Sort run and long run MC
  - Optimal plant size