WRITTEN PRELIMINARY Ph.D EXAMINATION

Department of Applied Economics
Jan./Feb. - 2011
Trade, Development and Growth

For students electing
Macro (8701/Prof. Roe) & Micro (8703/Prof. Glewwe) option

Instructions

• Identify yourself by your code letter, not your name, on each question
• Start each question’s answer at the top of a new page
• You are requested to answer a total of FOUR questions
• Answer ONE question from Set One
• Answer THREE questions from Set Two
• You have four hours to complete this examination
I. Child labor and consumer boycotts.

Assume that firms in a developing country can use either child labor or adult labor, or both, to produce soccer balls. Let \( A \) by the number of adults hired by a firm, and let \( C \) be the number of children hired. Labor is the only input, and the production technology has constant returns to scale, so let the production function be:

\[
Y = F(A, S) = b[A + \gamma C], \text{ where } b > 0 \text{ and } 0 < \gamma < 1
\]

The restriction that \( 0 < \gamma < 1 \) simply indicates that the productivity of child labor is less than the productivity of adult labor.

Questions

1. Assume that the price for soccer balls faced by the firm equals 1 if the firm does not use child labor. However, there is a boycott on soccer balls produced by child labor, so that a firm that sells soccer balls produced by child labor receives a price of \( \alpha \) where \( \alpha < 1 \). Denote child wages by \( w_C \) and adult wages by \( w_A \), and assume that \( w_A > w_C \). What are the profits of a firm that hires only adult labor? What are the profits of a firm that hires child labor (and maybe some adult labor as well)? These expressions for profits should be functions of \( A, C, \alpha, \gamma, b, w_A \) and \( w_C \).

2. Consider a firm that hires both adult and child labor. Clearly the amount of money required to hire one more adult, \( w_A \), could be used to hire \( w_A/w_C \) more child laborers. Using your answer to part (1.), under what conditions is it more profitable for this type of firm to hire an adult worker, as opposed to hiring more child workers?

3. Consider your answer to (2.), in particular the condition that indicates when it is more profitable to hire another adult than it is to hire another child. What does this imply about the replacing current child laborers with adult laborers? More generally, under what conditions would you
expect to find the following types of firms: (a) All labor is adult labor; (b) All labor is child labor; and (c) Both child and adult labor are hired. You do not have to show a lot of derivations in your answer: just use economic reasoning.

4. Assume that an increase in the number of consumers who boycott soccer balls produced using child labor decreases the equilibrium wage rate for child labor, that is decreases $w_C$. To see what impact the boycott has on poor households in developing countries, assume that these households have the utility function $U = c^\alpha \ell^\beta$, where $c$ is household consumption, $\ell$ is child leisure (adults are assumed not to have leisure), and $\alpha > 0$ and $\beta > 0$. Let the household budget constraint be $c = I + (T_C - \ell)w_C$, where $I$ is adult income (assumed to be fixed) and $T_C$ is total child time. The household chooses $c$ and $\ell$ to maximize utility. Show the first order conditions for this household’s constrained maximization problem.

5. Use your answer to part (4) to find the utility maximizing values for child leisure and for household consumption, assuming an interior solution. What is the impact of the increase in the consumer boycott on the amount of time that children spend working (which is $T_C - \ell$) and household consumption? Comment on whether this model is sufficient for making policy decisions regarding child labor.
II. Theory: characterizing equilibrium

Consider the three sector growth model in which there are two traded goods sectors, industry and agriculture, and a non-internationally traded good, services. Index these goods as \( j = m \) (industry), \( a \) (agriculture), \( s \) (service). Households own the economy’s factor endowments, labor, \( L(t) \), capital \( K(t) \), and land \( H \). Households rent the services of these resources in exchange for factor payments \( w(t) \), \( r_k(t) \) and \( r_H(t) \) respectively, which they in turn allocate to savings and expenditures on industrial \( Q_m(t) \), food \( Q_a(t) \) and service goods \( Q_s(t) \).

The representative household seeks to maximize

\[
\int_0^\infty \frac{q^{1-\theta} - 1}{\theta} e^{(n-\rho)t} dt
\]

given the flow budget constraint

\[
\dot{k} = w + k(r - n) + \pi H - \epsilon
\]

where

\[
\epsilon = \mathcal{E}(p_a, p_m, p_s) q \equiv \min_{q_a,q_m,q_s} \{p_a q_a + p_m q_m + p_s q_s : q \leq u(q_a, q_m, q_s)\}
\]

and \( p_m = 1 \).

Given, in effective unites, the behavioral rules for households

\[
\dot{\epsilon} = \frac{1}{\theta} \left( r^k - \rho - \delta - \theta x + \lambda_s \frac{\dot{p}_s}{p_s} (\theta - 1) \right)
\]

where expenditures per effective worker are

\[
\dot{\epsilon} = \mathcal{E}(p_m, p_a, p_s) \tilde{q}
\]

The behavioral rules for firms in the manufacturing and the service sector are

\[
C^j(\hat{w}, r^k) \hat{y}_j \equiv \min_{l_j,k_j} \left\{ \hat{w} l_j + w^k \hat{k}_j : \hat{y}_j \leq f^j(l_j, \hat{k}_j) \right\}, \ j = m, s
\]

and agriculture (in units per effective worker) they are

\[
\pi^a(p_a, \hat{w}, r^k) H \equiv \max_{l_a,k_a} \left\{ p_a f^a(l_a, \hat{k}_a, H) - \hat{w} l_a - r^k \hat{k} \right\}
\]
**Question:**
State any necessary additional assumptions and conditions that may be necessary, then

1. Characterize *intra-temporal* equilibrium
2. Derive the reduced form functions for this model
3. Show "how" you solve for the model’s steady-state values
4. Derive the models two differential equations
SET TWO:

Answer THREE of the following four questions (III to VI)

III. Policy Application: Financial crises

Imbalances in the world economy have received increased attention since the recession that struck many countries beginning in 2007. A typical feature can be seen from the imbalances in Europe (attached figure). The negative bar includes the countries of Ireland, Greece, Spain and Portugal.

By a simple accounting identity

\[ S - I = \underbrace{\text{Gov. Sur/Def.}}_{\text{Current Account}} + \underbrace{(G - T) + (X - M)}_{\text{Current Account}} \]

a current account deficit (exports \(X\) less than imports \(M\)) implies a capital inflow that can cause savings \(S\) to be less than investment \(I\) and/or for government expenditures \(G\) to exceed tax revenues \(T\), or both. Some emphasize that a current account deficit implies that the returns to resources in the trade surplus country are in part due to the excess demand in the trade deficit country. While a country may incur a current account deficit, even in the long-run, the growth in the deficits of many countries (Greece in particular) have led to a painful restructuring, of government deficits.

Question

Consider an economy (say Greece) having the following key characteristics.
Start with the supposition that:

- The country has accumulated a large stock of government debt (i.e., $G > T$ for some extended period)
- As a member of the European Union, it cannot devalue its currency (the Euro) (if it cannot devalue the Euro, adjustments must come in terms of home good prices, wages and other variables rather than the nominal value of the currency)
- Assume debt repayment implies the need for $T > G$ for some extended period into the future
- For simplicity, presume the rate of return to capital $r^k$ is determined by the productivity of capital in Greece only.

1. Provide a graphical depiction and accompanying explanation of the adjustments brought about by a "financial collapse" and the need to accommodate payments on principle and interest for the case of $T > G$. In this depiction, you should also discuss briefly the likely dynamic effects on saving, investment and transition growth.

2. Now, for the purpose of this question, narrow you focus to agriculture. Be more analytical, and explain the adjustment in agricultural factor (labor, capital) allocations associated with readjustment ($T > G$). To answer this question, you need to take into account the links between the evolution of the price of home goods, wages, and returns to capital $r^k$. 

<table>
<thead>
<tr>
<th>Share: Labor</th>
<th>Agric.</th>
<th>Mnf.</th>
<th>Serv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>0.5871</td>
<td>0.5191</td>
<td>0.5885</td>
</tr>
<tr>
<td>Land</td>
<td>0.3175</td>
<td>0.4809</td>
<td>0.4115</td>
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<tr>
<td>Sec. share in econ.</td>
<td>0.2158</td>
<td>0.2116</td>
<td>0.5726</td>
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</tbody>
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IV. Measuring Productivity

1. Using a Laspeyres indexing procedure to aggregate inputs will cause measured multi-factor productivity to increase in response to changes in relative factor prices, even in the absence of technical change. True or false? Illustrate graphically and explain in a one output, two input world.

2. Setting aside the problem of aggregation bias, identify and carefully discuss at least two additional reasons for measured multi-factor productivity growth to deviate from zero.

3. Measuring the Welfare Effects of R&D: Using a clearly labeled figure of a multi-market model and associated explanation, evaluate the statement: “Research spillovers to the rest of the world increase the national welfare of an innovating, exporting country compared with a situation of no spillovers, true or false.
V. Landlord and tenant contracts

(This is a variation of a model discussed in Apec 8703.)

A landlord leases a plot of land to a tenant. Tenant effort, denoted by $e$, cannot be observed by the landlord. Actual output, which is random, equals either 0 or 1. The probability that output (denoted by $y$) equals either 0 or 1 is a function of $e$:

$$\text{Prob}[y = 1] = e$$

$$\text{Prob}[y = 0] = 1 - e$$

Production requires a fixed amount of inputs, denoted by $B$. Assume that the contract is a sharecropping contract, where the shares of the output ($y$) going to the tenant and the landlord are the same as the shares that each must contribute to the inputs.

Finally, contracts take the following form. The landlord selects $\alpha$, which is the share of the output given to the tenant (and the share of $B$ that must be paid by the tenant). In addition, the tenant must pay "$r$" to the landlord as an upfront "fee". (However, if $r < 0$ then the landlord provides the tenant with an upfront payment.) This fee must be paid both if $y = 1$ and if $y = 0$, and it does not depend on $y$.

Questions

1. Based on the above description of the contract, what is the tenant’s income if $y = 1$? What is it if $y = 0$? [Note: income could be $< 0$ if $y = 0$.] What is the tenant’s expected income, given the above probabilities for $y = 1$ and $y = 0$?

2. Write out the expression for the landlord’s expected profits as a function of $\alpha$, $e$, $B$ and $r$.

3. Assume that, in addition to expected income, tenants “suffer” a disutility of $-e^2/2$ for the effort they put into working the plot. Assume that the units of this disutility are the same as those for expected income. Given $\alpha$ and $r$, and using your answer to (1.), what is the tenant’s optimal level of effort?
4. The landlord knows that the tenant will choose the level of effort as given in your answer to (3.). He also knows that the tenant will not agree to the contract if his (the tenant’s) expected utility is <0. In effect, that means that the landlord faces a constraint that the tenant’s expected utility should be equal to zero. Given this situation, and your answer to (2.), what is the landlord’s optimal choice for the share of output that should go to the tenant (optimal \( \alpha \))? What is the intuition for this result?

5. Given your answer to (4.), what value of \( r \) maximizes expected profits for the landlord?
VI. Growth Accounting

Consider a competitive economy producing two goods \( Y_1(t) \), \( Y_2(t) \) by employing the following neoclassical technology (meaning CRS) and factors of production

\[
Y_1 = \min \left\{ F^1(K_1, AL_1), \frac{Y_{11}}{\sigma_{11}}, \frac{Y_{21}}{\sigma_{21}} \right\}
\]

\[
Y_2 = \min \left\{ F^2(K_2, AL_2), \frac{Y_{12}}{\sigma_{12}}, \frac{Y_{22}}{\sigma_{22}} \right\}
\]

where \( K_j, L_j \) are stocks of capital and labor employed in the \( j \)-th sector, and \( \sigma_{ij} \) are input-output coefficients. \( A \) is the assumed augmentation effect of Harrod neutral technological change. Gross output is \( Y_j \) and \( Y_{ij} \) is the amount of output from sector \( i \) that is employed as an intermediate factor of production in sector \( j \). The amount of a final good produced is thus \( Y_{i}^{final} = Y_i - \sum_i Y_{ij} \). If we assume the economy is competitive and technologies are neoclassical, a competitive market equilibrium should lead to the following GDP function (which we assume is smooth-continuous and at least once differentiable in its arguments)

\[
GDP = G(p_1, p_2, K, AL) =
\]

\[
\max_{K_1, K_2, L_1, L_2} \left\{ (p_1 - \sigma_{11}p_1 - \sigma_{21}p_2) F^1(K_1, AL_1) + (p_2 - \sigma_{12}p_1 - \sigma_{22}p_2) F^2(K_2, AL_2) \right\}, \text{ s.t.}
\]

\[
K \geq K_1 + K_2, \quad L \geq L_1 + L_2
\]

where \( K \) and \( L \) are the country's stock of labor and capital at each point in time \( t \) (\( t \) is omitted for notational purposes) and we assume the capital and labor constraints are binding at each \( t \).

1. Given the assumption that \( G(p_1, p_2, K, AL) \) exists, conduct a "growth accounting exercise", i.e, perform the operations that would, in principle,

   (a) Allow you to calculate the sources of growth in \textbf{real} \( GDP^{\text{real}} \).

   (b) Use this result to isolate/solve for \( \dot{A}/A \)

2. What are "typical" numerical measures of \( K \), and \( L \)?
3. Growth accounting exercises often begin with the assumption of a "production function," for example

\[ Y = F(K, AL) \]

Using calculus, derive Solow’s Residual using \( F(K, AL) \)

4. Show that the result obtained 3 (Solow’s Residual) can be identical to the same result obtained from \( GDP = G(p_1, p_2, K, AL) \), and then comment briefly, why \( G(p_1, p_2, K, AL) \) might or might not be conceptually preferable to \( F(K, AL) \).

5. If: the economy is in long-run balanced growth-path equilibrium, the rate of growth of \( L \) is constant at rate \( n \), and the rate of Harrod neutral technological change is also constant at rate \( x \), what is the contribution of the growth in \( K \), technological change \( A \), and labor force \( L \) to the long-run growth in GDP?