WRITTEN PRELIMINARY Ph.D. EXAMINATION

Department of Applied Economics

Winter 2013

Natural Resource and Environmental Economics

Instructions:

- Identify yourself by your code letter, not your name, on each question.
- Start each question’s answer at the top of a new page.
- Answer two of the first three questions I–III: Natural Resource Economics.
- Answer two of the last three questions IV–VI: Environmental Economics.
- You have four hours to complete this examination.
Natural Resource Economics

Answer two of questions I–III

I. A community that owns a resource is solving the following optimization problem:

\[
\max_{\{h(t)\}} \int_0^\infty e^{-rt} \left\{ [P - c(x(t))] h(t) + M(x(t)) \right\} dt
\]

s.t. \( \dot{x}(t) = F(x(t)) - h(t); \ x(0) = x_0, \)

where \( x(t) \) denotes the stock of the resource. Harvest, \( h(t) \in [0, \bar{h}] \), can be sold in the market at a constant unit price \( P \). The discount rate is \( r \). \( c(x(t)) \) is the unit cost of harvest with \( c'(x) < 0 \). \( F(x(t)) \) is the growth function with \( F''(x(t)) < 0 \). The resource also provides a continuous flow of non-market value, \( M(x(t)) \), with \( M'(x(t)) > 0 \) and \( M''(x(t)) < 0 \).

A. Write down the Necessary conditions in Current Value Form.

B. Derive the equation that characterizes the steady state (no \( h \) or costate variable in it). Put marginal benefit of one more unit of \( h(t) \) on the left-hand side. Define \( R(x) = F(x)[P - c(x)] \) as the sustainable rent. The right-hand side of the equation contains two terms, one related to the present value of \( R(x) \) and the other related to the present value of non-market value. Clearly show your steps.

Fact 1: \( \frac{d}{dx} \{F(x)[P - c(x)]\} = F'(x)[P - c(x)] - c'(x)F(x). \)

Fact 2: \( \frac{1}{r} = \int_0^\infty e^{-rt} dt. \)

C. Does the presence of \( M(x) \) increase or decrease the optimal steady-state level of \( x \)?

D. Assume a logistic growth function: \( F(x) = \gamma x(1 - x/K) \), where \( \gamma \) is the intrinsic growth rate and \( K \) is the carrying capacity. If \( M'(x) \) is so large that \( M'(x) \gg p - c(x) \) for a wide range of \( x \) values including \( x_0 \), what is the optimal steady-state level of \( h \) and \( x \)?

II. A community of \( N \) users owns a lake. The lake is valued for the water that can be pumped from it and its amenity value. The amenity value depends on the level of water stock in the lake, \( X_t \). The net benefit enjoyed by user \( i \) is given by \( U_i(X_t, Y_{it}) \), where \( Y_{it} \) is the volume of water pumped from the lake at time \( t \) by user \( i \). The discount rate is \( r \). The equation of motion is \( X_{t+1} - X_t = R - \sum_{i=1}^N Y_{it} \), where \( R \) is the constant rate of recharge to the lake. Users are not identical. Users are non-cooperative and not myopic.

A. Assume an infinite horizon. Set up user \( i \)'s optimization problem as an optimal-control problem.

B. Assume an infinite horizon. Set up user \( i \)'s optimization problem as a Bellman equation.

C. Is the solution to the problem in (B) an Open Loop or Feedback strategy?

D. Continuation of (B). Derive an equation analogous to the canonical equation \( \dot{\lambda} - r\lambda = -H_x \) in optimal control problems.
III. A society is extracting a non-renewable resource to maximize (expected) discounted utilities from consumption, \( q(t) \), which is also the extraction rate. Cost of extraction is zero. The utility function \( U(q) \) has the usual properties: \( U(0) = 0; U'(q) > 0, U''(q) < 0 \), and \( \lim_{q \to 0} U'(q) = \infty \).

At each instant \( t \), the extraction generates air pollution in the amount of \( \alpha q(t) \), where \( \alpha \) is a positive constant. Pollution will NOT cause any damage before the cumulative pollution, \( \alpha Q(t) \), reaches a threshold level, \( Z \). Let \( Q(t) \) denote the cumulative extraction. \( Z \) is a random variable with a probability density function \( f(Z) \). Once \( Z \) is reached, the society will collapse (\( U = 0 \)). The discount rate is \( r \) and

\[
\frac{U(q)}{U'(q)} - q > 0.
\]

A. Write down the society’s optimization problem.

B. Derive the necessary conditions (ignore the transversality conditions).

C. Derive the expression of \( \dot{q} \) that does not involve the shadow value.

D. First write down the expression of \( \dot{q} \) without uncertainty. Then determine how the uncertainty affects the extraction path. You only need to answer “tilt extraction towards now” or “tilt extraction towards the future.”
IV. Consider a 2-good economy with $J$ identical agents, each of whose utility is determined by consumption of a single marketed good $x_j$ and a single public bad $s$, which we may call smoke. Consumer $j$’s utility is given by

$$U_j(x_j, s) = \ln x_j - \gamma \ln s,$$

where $\gamma \in (0, 1)$. Each $j$ has endowment of the marketed good given by $\omega_j = \omega/J$, where $\omega > 0$ is the aggregate endowment. In the beginning, smoke is present at the uncontrolled level $s^0$. Consumers can dedicate a portion $z_j$ of their endowment to mitigation of smoke, consuming the remainder. Thus, $x_j = \omega_j - z_j$. Mitigation is linear in $z$, so that for a given vector $z = (z_1, \ldots, z_J)$ of contributions, the level of smoke after regulation is $s = s^0 - \sum_j z_j$. Assume that $\omega > s^0$.

A. Letting the social welfare function be $W = \sum_j U_j$, determine the Pareto-optimal level of smoke $s$ and of the marketed good $x_j$ for each $j$. Let the optimal solution be denoted $(\hat{x}_1, \ldots, \hat{x}_J, \hat{s})$.

B. Now determine the voluntary-contribution equilibrium, at which each consumer chooses the level of $z_j$ taking $z_{-j}$ as given. Let the equilibrium solution be denoted $(x^*_1, \ldots, x^*_J, s^*)$. Determine whether $s^*$ is greater than or less than the optimal $\hat{s}$.

C. Suppose now that the regulator has decided to impose a uniform lump-sum tax $t$, measured in the same units as $x$ or $\omega$, on all agents. Thus, for each $j$ we have $\omega_j = x_j + z_j + t$. The taxes are applied to mitigation of pollution, so that now $s = s^0 - \sum_j z_j - Jt$. Solve again for the VCE and show that, for $J > 1$, the resulting equilibrium consumption level $x^*_j$ is equal to $x^*_j$. That is, a lump-sum tax cannot correct the free-rider problem.

V. A consumer obtains utility from two goods, a private market good $x$ and a fixed environmental good $q$. Her utility for these two goods is described by the function $U(x, q) = \ln x + \gamma q$, with $\gamma > 0$.

A. The consumer has income of $M = 250$ and the initial level of $q$ is $q^0 = 10$. If the price of $x$ is $p = 10$ and the price of $q$ is $r = 0$, determine her willingness to pay (define this term carefully) for a change in $q$ to $q^1 = 12$.

B. Now suppose that the new level of the environmental good, $q^1 = 12$, is the status quo and determine the consumer’s willingness to accept for a change back to $q^0 = 10$. Discuss the relationship between WTP and WTA.

C. Suppose that, instead of $r = 0$, the price of $q$ is $r = 3$. Solve the problem given in part a. using this price and keeping $p = 10$, $q^0 = 10$, $q^1 = 12$, and $M = 250$. 


VI. Two firms are the only sources of pollution in their airshed. Marginal cost of abatement for the two firms is given by

\[ MC_1(a_1) = 20 + 4a_1 \quad \text{and} \quad MC_2(a_2) = 10 + 8a_2. \]

The marginal benefit of aggregate abatement is \( MB(a) = 400 - 4a \), where \( a = a_1 + a_2 \).

A. Determine the socially optimal level of abatement. Compute the welfare loss, relative to this optimum, that results when the regulator requires equal abatement from each firm: \( a_i = a^*/2 \).

B. Compute the Pigouvian tax that will yield the optimal level of abatement. If initial emissions are \( \tilde{e}_1 = 70 \) and \( \tilde{e}_2 = 30 \), compare the welfare outcome under the tax to the optimum from part a.

C. Suppose now that the benefits of abatement are uncertain, with \( MB(a) = 400 - 4a + u \), where \( u \sim U[-10, 10] \). Find the level of abatement, aggregate and for each firm, that maximizes expected social welfare.

D. Finally, suppose that marginal benefits are upward sloping, with

\[ MB(a) = 100 + \frac{3}{2}a + u. \]

Again find the optimal level of abatement, aggregate and for each firm.