Natural Resource and Environmental Economics

Instructions:

- Identify yourself by your code letter, not your name, on each question.
- Start each question's answer at the top of a new page.
- Answer two of the first three questions
  (choose two among questions I-III: Environmental Econ).
- Answer two of the second three questions
  (choose two among questions IV-VI: Natural Resource Econ).
I. An isolated woodlot in the midst of farm country contains four species of squirrels. Call them species 1, 2, 3, and 4. The genetic distances between the species are: $d(1, 2) = 100$, $d(1, 3) = 20$, $d(1, 4) = 70$, $d(2, 3) = 110$, $d(2, 4) = 80$, and $d(3, 4) = 40$. Their abundances are $n_1 = 300, n_2 = 45, n_3 = 120$, and $n_4 = 78$.

A. Are these genetic distances ultrametric? Calculate Weitzman’s diversity index for the four species, using

$$ V(S) = \max_{i \in S} [V(S \setminus i) + d(i, S \setminus i)], $$

where $d(j, Q) = \min_{i \in Q} d(j, i)$ is the distance between any species $j$ and the set $Q$. Determine which species, if allowed to go extinct, would lead to the smallest reduction in diversity.

B. The Shannon index is a popular measure of biodiversity in the scientific community. This index is $H = -\sum P_j \ln P_j$, where $j$ indexes the species and $P_j = n_j/N$ is relative abundance of species $j$, $N$ equal to the total number of individuals. As with $V(S)$, for Shannon bigger is better. $H$ is maximized when all species occur in equal numbers. Determine which species, if allowed to go extinct, would lead to the smallest reduction in diversity.

C. Discuss briefly which of these measures you think best captures biological diversity.

II. Two firms, indexed by $j = 1, 2$, are the only sources of pollution in an airshed. The airshed is divided into two adjacent zones, also indexed by $j$. Firm $j$ is located in zone $j$, with zone 1 to the west of zone 2. Emissions by firm $j$ are denoted $e_j$; initial uncontrolled emissions levels are $e_1^0 = 8$ and $e_2^0 = 6$. Assume that pollution is perfectly mixed within each zone, but not across zones. Denote firm $j$’s abatement by $a_j$. Let $C_j(a_j)$ denote firm $j$’s abatement cost function, where

$$ C_1(a_1) = 0.5a_1^2 \quad \text{and} \quad C_2(a_2) = a_2^2. $$

Let $t_{ij}$ denote the transfer coefficient capturing the portion of $i$’s emissions that land in zone $j$. Their values are $t_{11} = 0.5$, $t_{12} = 0.3$, $t_{21} = 0.1$, and $t_{22} = 0.6$. Pollution landing in zone $j$ is $P_j = \sum_k e_k t_{kj}$ and aggregate abatement experienced in zone $j$ is $A_j = \sum_k a_k t_{kj}$. Benefits from abatement in the two zones are

$$ B_1(A_1) = 10A_1 - A_1^2 \quad \text{and} \quad B_2(A_2) = 10A_2 - 0.5A_2^2. $$

A. From the perspective of a social planner, solve for the optimal level of emissions in each zone.

B. Assuming that firms behave competitively in the permit market, describe how an emissions-permit system of the Montgomery sort would be established in this airshed. Do you think this scheme could achieve the social optimum?

C. An optimum might not treat citizens equally. Suppose now that the population in zone 1 is half that of zone 2. Compute the social optimum under the constraint that each citizen of the entire airshed must be exposed to the same level of pollution. Could a Montgomery EPS achieve this outcome? If so, how; if not, why not?

III. Consider a permit-trading situation in which banking is allowed but borrowing is prohibited. A permit issued in period $t$ may be banked until period $t+1$ or later, at which time it is interchangeable with a permit issued at that date. Each of $n$ competitive firms has a well-behaved abatement cost function $C(a)$ and each is given the same initial allocation of permits in each period. Any firm whose emissions in a period exceed its holdings of permits in that period is assessed a fine $F > p$, where $p$ is the equilibrium price of permits.

A. True or False: From the perspective of period $t$, a permit issued at $t$ is worth more than a permit that will be issued at $t+1$.

B. Suppose that at time $t$ the number of banked permits from previous periods is zero. Firm 1 is a monopoly seller of permits. Do you expect that the equilibrium price of permits in this case will approach (or equal) $F$? Why or why not?
IV. Let’s say fish can be sold at a constant price, P. The discount rate is \( r \). The biological growth function is the standard logistic model, \( F(x) \). Cost is described by \( C(h, x) \), where \( x \) is the fish stock. The first derivatives of the cost function are as follows: \( C_h > 0, C_x < 0 \). The second derivatives of the cost function are as follows: \( C_{hh} > 0 \) and \( C_{hx} \) is a positive constant (no \( h \) or \( x \) in it), \( C_{xx} = C_{hx} = 0 \). For the sake of simplicity, let’s assume \( F_x < r \) and \( C_h < P \) and the steady state level of stock, \( x^* \), is higher than the level of \( x \) corresponding to maximum sustainable yield (MSY).

A. Suppose a firm’s objective is to maximize the present value of the stream of profit flows from selling fish. Set up the optimal control problem, assuming an infinite time horizon and initial stock \( x(0) = x_0 > 0 \).

B. Write down the current value Hamiltonian. Derive the necessary conditions.

C. First derive the isocline for \( \dot{h} \), derive the slope of the isoclines \( \frac{\partial h}{\partial x} \) and judge whether the slope is positive or negative. Then construct the phase diagram in the space of \( h \) and \( x \) (put \( h \) on the vertical axis and \( x \) on the horizontal axis). Be sure to include a full set of sample trajectories.

V. A. Derive the Faustmann formula for the forestry rotation problem. The growth function of wood volume is \( w(t) \), where \( t \) is the age and \( w(t) \) has the usual shape: it rises at first, reaches a peak, and then falls. \( P \) is the constant price of timber. The cost of harvesting the timber is \( C \) and the discount rate is \( r \). At the end of each rotation, a profit, \( Pw(T) - C \), is obtained. The objective is to maximize the present value of profit from harvest over the infinite horizon. Denote the optimal length of one rotation as \( T^* \).

B. Derive the formula that determines the optimal timing to replant a vineyard. The vineyard returns a revenue flow of \( \pi(t) = Pw(t) \) at each instant within a rotation, where \( t \) refers to the vineyard age and \( w(t) \) is the same as in part (A). Assume a constant cost of replanting vineyard, \( C \). So within each rotation, a profit of \( \int_{0}^{T} e^{-\tau} Pw(t) dt - C \) is obtained. The objective function is to maximize the present value of profit over the infinite horizon. Denote the optimal length of one rotation as \( \hat{T}^* \).

C. Is \( \hat{T}^* \) longer than \( T^* \)? Briefly explain why.

VI. Consider a monopoly extracts a non-renewable resource with a known amount of reserves \( X_0 \), zero cost, and a common discount rate of \( r \). The monopoly also faces a periodic demand curve of \( Q^D(t) = AP_t^{-\varepsilon}, \ A > 0, \varepsilon > 0 \). There is a known backstop technology that can supply the resource at a constant price \( P_B \).

A. It will take \( T_1 \) for the firm to fully exhaust the resource base. That is, \( \int_{0}^{T_1} Q(t) dt = X_0 \). Derive an expression for \( T_1 \). You can get a closed-form solution here.

B. A bill is passed that subsidizes the backstop technology. Now the backstop technology can supply the resource at a lower price, \( P_B - \tau \). It takes \( T_2 \) for the firm to fully exhaust the resource base under the backstop technology price drop. Derive an expression for \( T_2 \). You can get a closed-form solution here. Is \( T_2 \) larger or smaller than \( T_1 \)?

C. Sketch the price path over time under part (A) and (B) on the same graph. Mark \( T_1, T_2, P_B \), and \( P_B - \tau \) on the graph. Let’s say the nonrenewable resource is a fossil fuel that generates greenhouse gases. The backstop technology is a much cleaner renewable energy that emits fewer greenhouse gases. Draw some implications (up to 5 sentences) for climate policies from your analysis here.