WRITTEN PRELIMINARY Ph.D. EXAMINATION

Department of Applied Economics
University of Minnesota

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PRODUCTION ECONOMICS

Instructions

- Identify yourself by your code letter, not by your name, on each question.
- Start each question’s answer at the top of a new page.
- Answer the first question.
- Answer three of the remaining questions.
- You have four hours to complete this examination.
1. Subsistence farmers in a West African country grow sorghum (a food grain) and groundnuts (a cash crop) using family labor and land. The farmers live in an area that is not densely populated and in a society where there is no land ownership. Therefore, land is available in essentially unlimited quantities.

You work for the Ministry of Agriculture and have been asked to study farmer responses to changes in the price of groundnuts. That price is set by the government. In your analysis, you will assume that farmers grow enough sorghum to meet their family consumption needs but do not buy or sell sorghum. Farmers are assumed to maximize the expected utility of cash income from groundnut production, \( \pi \). Cash income is a random variable, because yields are random. Expected cash income, \( E(\pi) \), is defined by the following expression:

\[
E(\pi) = (PE(Y) - VC)A,
\]

where \( P \) is the price of groundnuts, \( E(Y) \) is expected yield per hectare, \( VC \) is variable cash cost per hectare, and \( A \) is the number of hectares of land planted in groundnuts. The variance of cash income, \( V(\pi) \), is defined by the expression:

\[
V(\pi) = P^2A^2V(Y),
\]

where \( V(Y) \) is the variance of groundnut yield per hectare.

Assuming a representative farmer’s risk preferences can be represented by a constant absolute risk aversion utility function, that farmer can be said to maximize the certainty equivalent of cash income, CE, which is defined by the following expression:

\[
EU = E(\pi) - (\lambda/2)V(\pi),
\]

where \( \lambda \geq 0 \) is the constant level of absolute risk aversion.

(a) Farmers affect cash income by choosing how many hectares of groundnuts to plant. Derive the first-order necessary condition for the certainty equivalent maximizing level of \( A \) and check the second order conditions to ensure that this condition does identify a maximum. You do not need to rearrange the first order condition to get an explicit expression for \( A \).

(b) Derive an expression for \( dA/d\lambda \) and determine its sign or the conditions when it is positive or negative.

(c) Derive an expression for \( dA/dP \) and determine its sign or the conditions when it is positive or negative. What do your results imply for the effectiveness of price increases as a policy tool for increasing groundnut production?
2. Consider the revenue function defined by

\[ R(p, x) = \max_{y} \{ p \cdot y : y \in Y(x) \} \]

Prove the following properties of the revenue function:

(a) \( R(p, x) \) is nonnegative; \( R(p, x) \geq 0, p, y \in \mathbb{R}_+^M \).

(b) \( R(p, x) \) is nondecreasing in prices, \( p, p' \geq p \rightarrow R(p', x) \geq R(p, x) \).

(c) \( R(p, x) \) is convex in \( p \), with \( p > 0 \).

(d) Consider the profit function defined as

\[ \pi(p, w) = \max_{(x, y) \geq 0} \{ p \cdot y - w \cdot x : (x, y) \in T \} \]

Show that

\[ \pi(p, w) = \max_x \{ R(p, x) - w \cdot x \} \]

3. A firm produces scalar output \( Y \) with two inputs – labor and capital. The firm either purchases capital on the market or uses labor to produce it in-house (i.e., produces its own capital). Let \( K_M \) represent the level of capital purchased on the market and let \( K_H \) represent the level of capital produced in-house. In-house capital is produced according to the technology

\[ K_H = g(L_K) \]

where \( L_K \) is the amount of labor devoted to in-house capital production. Assume \( g \) is strictly concave, i.e., \( g'(L_K) > 0 \) and \( g''(L_K) < 0 \), where \( g' \) and \( g'' \) are the first and second derivative of \( g \) respectively. The (perfectly competitive) market price of labor and capital are denoted \( w \) and \( r \) respectively. Let \( L_Y \) denote labor used to produce output directly and define \( K = K_H + K_M \). Output is produced according to the technology \( Y = f(L_Y, K) \) where \( f_i(L_Y, K) = \frac{\partial f}{\partial i} > 0 \), and \( f_{ii} (L_Y, K) = \frac{\partial^2 f}{\partial i^2} < 0 \), \( i = L_Y, K \).

(a) Set up the firm's cost minimization problem.

(b) Derive the first-order conditions and interpret them.

(c) Derive the condition(s) under which the firm chooses to produce all capital in-house.

(d) Explain how the firm's ability to produce capital in house affects its elasticity of demand for \( K_M \)? (Intuition is preferred to derivation)

(e) Explain how the firm's ability to produce \( K_L \) in house affects its elasticity of supply for \( Y \)? (Intuition is preferred to derivation)
4. The vertical linkage between farmers and food processors is an important one in the food system. In many cases, this linkage is "governed" by market-based relationships between independent firms - either spot markets or marketing contracts. Examples of agricultural products for which market-based vertical linkages are common include: soybeans for oil, soy sauce, and other foods; fresh fruits and vegetables; and wheat for flour. In other cases, farmers form cooperatives that own the processing plants. Examples of products for which this form of vertical linkage is common include: sugar beets, corn for corn sweetener, and vegetables for canning or freezing. Finally, in other cases, processing firms vertically integrate upstream to own farm production facilities. This kind of vertical linkage is observed for plantation crops such as coffee, sugar cane, and pineapple. In addition, poultry processors achieve a form of virtual vertical integration through long term production contracts under which they retain ownership of birds placed in buildings owned by farmers.

Transaction cost economics (Williamson) and incomplete contract theory (Hart) both provide insights on when and why each of these governance/ownership patterns is likely to occur.

(a) Briefly summarize the key features (assumptions, unit of analysis, general predictions, etc.) for each of these two theories.

(b) Use each theory to develop explanations for the fact that farmer/processor linkages for some products are governed by market-based relationships, while upstream or downstream vertical integration is common for other products. Keep in mind that there are two production processes involved in this setting: farm production and food processing.

(c) Outline the basic methodology for a study designed to assess the power of these two theories in explaining the governance of the farmer/processor relationship. In your discussion, identify the unit of analysis for your study, key variables for which you would collect data, and the basic structure of a statistical model you would use in analyzing your data.
5. A farmer has $A$ acres to plant to corn. He can plant these acres to any combination of two different hybrids. The first hybrid is a conventional with revenue equal to $r - d$ where $r$ and $d$ are the random revenue potential and revenue loss due to pests. Average revenue potential and loss are $m_r$ and $m_d$. The variance of revenue potential and loss are $\sigma^2_r$ and $\sigma^2_d$. The covariance is $\sigma_{rd}$. The second hybrid the farmer can use is resistant to pest such that revenue is equal to potential, $r$. Common production costs are $\$C$ per acre, which is normalized to 0. The resistant hybrid has additional seed cost equal to $\$T$ per acre. The farmer’s expected utility of profit from planting corn is $EU = E(\pi) - (\lambda/2) V(\pi)$ where $\pi$ is the net return, $E(\pi)$ is the expected return, $V(\pi)$ is the variance of expected returns, and $\lambda > 0$ is the coefficient of constant absolute risk aversion. For $\lambda = 0$, the farmer is risk neutral. Let $\alpha$ denote the proportion of land devoted to the conventional hybrid and let $1 - \alpha$ denote the proportion of acres planted to the resistant hybrid.

(a) Set up the farmer’s constrained optimization problem and derive the first-order necessary conditions.

(b) Assuming $m_d > T$, how much conventional hybrid should a risk neutral farmer plant?

(c) Assuming $m_d > T$, how much conventional hybrid should a risk averse farmer plant?

(d) Discuss the implication of your findings in parts b and c in terms of whether the resistant hybrid is a risk increasing or risk decreasing input.

6. You have farm level data from farm/household units from seven villages in Zhejiang Province, China. For each farm the data include price and level data for five major inputs: fertilizer, land, seed, labor, and machinery. The data also includes price and level data for four major outputs: rice, wheat, vegetables, and tea.

Below are two tasks. **You are to discuss only one of them.** Your discussion should include at least the following points: (i) the function(s) you would estimate or calibrate, (ii) the specific functional form(s) you would use, (iii) the restrictions you would place on the parameters of the function(s), and (iv) a description of the parametric or nonparametric approach to your analysis (e.g., what first-order conditions you would be estimating or calibrating, etc.).

(a) Discuss how you would calculate the effect of a 10% increase in the price of irrigation water on rice and vegetable production.

(b) Discuss how you would test for the presence of financial constraints in farmer production.