Ph.D. PRELIMINARY EXAMINATION

MICROECONOMIC THEORY

Applied Economics Graduate Program

August 2013

The time limit for this exam is four hours. The exam has four sections. Each section includes two questions. Answer one question from each section. Before turning in your exam, number each page of your answers in sequential order, and identify the question (for example, III.2 for section III and question 2). Indicate, by circling below, the questions you completed. If you answer more than one question in a section and do not circle a question corresponding to that section, the first of the two that appears in your solution paper will be graded.

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STUDENT ID LETTER: _______ (Fill in your code letter)

Answer one question from each section. Indicate the one you answered by circling:

Section I:  Question 1  Question 2
Section II:  Question 1  Question 2
Section III:  Question 1  Question 2
Section IV:  Question 1  Question 2

*** TURN IN THIS SHEET WITH YOUR ANSWER PAGES***
Part I

Answer at most one question from Part I.
Question I.1

Walras Law.

a. State Walras’ law using the notation used in class (\( w = \) wealth, \( p = \) a vector of prices, and \( x \) is a vector of commodities (goods)). What is the intuition for why this “law” is likely to hold? Please be brief.

b. Suppose that a consumer’s demand for each good, \( x_i \), is such that the following expression holds for consumer spending on each good (there are \( L \) goods):

\[
p_i x_i = \alpha_i + \beta_i w + \sum_{j=1}^{L} \gamma_{ij} p_j
\]

Suppose that this consumer’s demand satisfies Walras’ law. What restrictions does this imply for the \( \alpha \), \( \beta \) and \( \gamma \) parameters? To answer this question, it is useful to express Walras’ law using a summation sign instead of vector notation.

c. Now assume that the consumer’s demand is such that the budget share for each good \( x_i \), which is defined as \( \frac{p_i x_i}{w} \), always satisfies the following equation:

\[
\frac{p_i x_i}{w} = \alpha_i + \beta_i \log(w) + \sum_{j=1}^{L} \gamma_{ij} \log(p_j)
\]

For Walras law to hold, what restrictions must hold for the \( \alpha \), \( \beta \) and \( \gamma \) parameters?

d. Finally, assume that the consumer’s demand implies that spending for each good \( x_i \) satisfies the following expression:

\[
p_i x_i = \alpha_i + \beta_i w + \gamma_i w^2
\]

For Walras law to hold, what restrictions must hold for the \( \alpha \), \( \beta \) and \( \gamma \) parameters?
Question I.2

An individual has a utility function that depends on 2 goods: \( u(x) = (x_1)^2x_2 \).

a. Derive the (Walrasian) demands for both goods as functions of their prices \( (p_1 \text{ and } p_2) \) and total wealth \( (w) \). You can assume an interior solution.

b. Use your answers from part (a) to derive the expenditure function that is implied by this utility function.

c. Suppose that \( p_1 = 8 \) and \( p_2 = 4 \). Using your answer to part (b) how much wealth is required for this consumer to reach a utility of 1000? Note that these numbers are chosen so that your answer should be an integer, not a complicated fraction.

d. Good one is tobacco. To reduce tobacco consumption and increase government revenue, the government adds a tax of 2 to the price of tobacco. Suppose that the person has the amount of money given in your answer for part (c), and that the price of good 2 remains at 4. What utility level will the consumer reach? [Again, your answer should be an integer.] How much will tobacco consumption be reduced by the tax?

e. Suppose that the government is simply interested in raising money, not in reducing tobacco consumption. How much money did it raise from this consumer with the tax imposed in part (d)? One government official claims that the consumer would be better off, and the government would raise more money, if the tobacco tax were eliminated and the consumer were taxed a lump sum of 20. Is that correct? [Hint: Go back to your derivations to part (b) and find the indirect utility function. Also, a useful calculation at the very last step is that \( 1/(27\times64)=0.0005787 \).]
Part II

Answer at most one question from Part II.
Question II.1

Consider the production possibility set (PPS):

\[ PPS = \left\{ (q_1, z) \in \mathbb{R}_+^2 \times \mathbb{R}_+ : z^\tau \geq q_1^{1/3} q_2^{2/3} \right\} \]

where \( q_1 \) and \( q_2 \) are outputs; \( z \) is an input; and \( \tau > 0 \) is constant parameter. This PPS is nonempty, strictly convex, closed, and satisfies weak free disposal in inputs and outputs.

a) What other condition on \( \tau \) is required for this PPS to ensure it exhibits non-increasing returns to scale? Justify your answer.

b) Derive the output distance function for this PPS assuming \( z > 0 \).

c) This output distance function will yield the revenue function, \( R(p_1, p_2, z) = \theta p_1^{1/3} p_2^{2/3} z^\tau \), where \( \theta > 0 \) is a constant parameter, and \( p_1 > 0 \) and \( p_2 > 0 \) are output prices. Use this revenue function to derive the conditional supplies.

d) Let \( r > 0 \) represent the competitive price of \( z \). Using the revenue function in c) and assuming an interior solution exists, find the profit maximizing unconditional input demand. Also, derive the unconditional supplies (Hint: This can be accomplished most rapidly using duality results).

e) It is easy to verify that the conditional supplies in part (c) are non-decreasing in their own price. Show that this result holds in general for a revenue function derived from a closed and nonempty production possibility set with \( N \) inputs and \( M \) outputs.
Question II.2

Consider a competitive firm with the cost function $c(q)$ where $q \in \mathbb{R}^+$, $c'(q) > 0$, and $c''(q) > 0$. Furthermore suppose there are two states of the world denoted by $g$ for good and $b$ for bad. In the good state, the firm experiences a positive random shock to output or price, while in the bad state a negative random shock to output or price occurs. The firm’s profits in the good and bad state are

$$\pi_g = (p + \sigma \phi_g)(q + \epsilon \phi_g) - c(q) \quad \text{and} \quad \pi_b = (p - \sigma \phi_b)(q - \epsilon \phi_b) - c(q)$$

where $p > 0$ is the expected price received by the firm for its output; $\phi_g > 0$ and $\phi_b > 0$ are a firm’s subjective beliefs about the probability of the good and bad state such that $\phi_g + \phi_b = 1$; $\sigma \phi_g \geq 0$ is the price increase and $\epsilon \phi_g \geq 0$ is the output increase in the good state; and $\sigma \phi_b \geq 0$ is the price decrease and $\epsilon \phi_b \geq 0$ is output decrease in the bad state.

Consider two types of firms. The first type seeks to maximize expected profit. The second type seeks to maximize the utility of profit, $W(\pi_g, \pi_b)$, where $\pi_s$ is profit in state $s$ and

$$W_s = \frac{\partial W(\pi_g, \pi_b)}{\partial \pi_s} > 0$$

for $s = g, b$. Also assume that the second type of firm’s preferences are risk averse with respect to $\phi_g$ and $\phi_b$, and generalized Schur-concave.

a) Set up the profit maximizing firm’s optimization problem and derive its first-order condition assuming the second-order condition is satisfied and an interior solution exists. What is the economic intuition of this condition?

b) Set up the utility maximizing firm’s optimization problem and derive the first-order condition assuming the second-order condition is satisfied and an interior solution exists.

i) Suppose $\sigma = 0$ and $\epsilon > 0$. Will the utility maximizing firm produce more or less output than the expected profit maximizing firm? Justify your answer and provide the economic intuition for your result.

ii) Now suppose $\sigma > 0$ and $\epsilon = 0$. Will the utility maximizing firm produce more or less output than the expected profit maximizing firm? Justify your answer and provide the economic intuition for your result.
Part III

Answer at most one question from Part III.
Question III.1

Wall-E-Mart is currently the only retailer of waste disposal systems in the town of Futuredale. Wall-E-Mart buys waste disposal systems from a wholesale company for $c per unit. The inverse demand curve for waste disposal systems in Futuredale is \( P(q) = a - bq \), for \( q \leq \frac{a}{b} \), and 0 otherwise, where \( P \) is the price and \( q \) is the quantity of waste disposal systems.

a. Solve for the profit maximizing quantity and price for Wall-E-Mart when it acts as a monopolist. Solve for the profit for Wall-E-Mart.

b. Now suppose the residents of Futuredale form a Consumer Union that bargains directly with Wall-E-Mart. The goal of the Consumer Union is to maximize consumer surplus for residents of Futuredale. Suppose that the Consumer Union gets to make a take-it-or-leave it offer to Wall-E-Mart that specifies the quantity \( q \) the Consumer Union will purchase and the total amount they will pay \( R \). Wall-E-Mart can then either accept or reject the offer. If the offer is accepted, trade occurs as specified in the offer. If the offer is rejected, the situation will revert to monopoly and Wall-E-Mart will earn monopoly profits as in part (a). Find the subgame perfect equilibrium.

c. Now suppose that there is no Consumer Union but that there is a potential entrant, Bulls-Eye Corporation, who is considering opening a store that sells waste disposal systems in Futuredale. Bulls-Eye Corporation can buy waste disposal systems from a wholesale company at a cost of $c. Bulls-Eye Corporation has a fixed cost of entry of $F. Solve for the Cournot equilibrium level of outputs for each firm \((q_W, q_B)\) if Bulls-Eye Corporation enters. Under what conditions will it be profitable for Bulls-Eye Corporation to enter?

d. Finally suppose that Bulls-Eye Corporation does not know the true state of demand prior to entry but that Wall-E-Mart does. Formally, Nature chooses high demand \((H)\) or low demand \((L)\) with equal probability and reveals the selection to Wall-E-Mart. Suppose that Wall-E-Mart supplies Futuredale for one period prior to potential entry and that Bulls-Eye observes price, \( P_H \) or \( P_L \) (but not the true state of demand). Bulls-Eye then decides whether or not to enter \((E \text{ or } N)\). This game of potential entry is shown on the next page. The first number is the payoff for Wall-E-Mart and the second number is the payoff for Bulls-Eye Corporation. Find all pure strategy perfect Bayesian equilibria for this game.
Game for part d

(2,1) E

(6,0) N

(2,-2) E

(4,0) N

(4,2) E

(8,0) N

(1,-1) E

(2,0) N

PL

H

PH

p_{PH} = 0.5

p_{PL} = 0.5
Question III.2

Vegetables can be produced either using organic production (O) or conventional production (C). The cost of organic production is $4 per unit. The cost of conventional production is $3 per unit. There are many vegetable producers and each producer can produce one unit. Consumers are willing-to-pay $8 per unit for organic vegetables and $5 per unit for conventional vegetables and consume as much as the producers can produce.

a. Suppose that consumers can distinguish between organic and conventional vegetables. Suppose that a regulatory authority sets the price of organic vegetables at $6 per unit and the price of conventional vegetables at $4 per unit. Producers move first and choose to produce organic vegetables or conventional vegetables. Consumers then choose whether to purchase organic vegetables, conventional vegetables, or not buy vegetables. Assume the payoff for each consumer is the willingness-to-pay minus price and the payoff for each producer is price minus production cost. Solve for subgame perfect equilibrium. (Note: this part is relatively straight-forward – don’t “over think” it.)

b. Now suppose consumers cannot distinguish between organic and conventional vegetables. Producers can claim that their vegetables are organic (SO) or that their vegetables are conventional (SC). (Note: a producer’s claim does not need to be accurate.) Consumers observe the claim by a producer but not the actual method of production by that producer. Assume that the price consumers are willing-to-pay is based on expected quality: \( P(S_i) = 8 \times \mu(O \mid S_i) + 5 \times (1 - \mu(O \mid S_i)) \), where \( P(S_i) \) is the price given claim \( i \), \( i = O \) or \( C \), and \( \mu(O \mid S_i) \) is the probability that consumers give to vegetables being organic given they have observed signal \( S_i \). Solve for perfect Bayesian equilibrium in which first producers choose production method and claim, and consumers pay for vegetables given a claim based on expected quality given the claim.

c. The regulatory authority is concerned that producers may claim they produce organic vegetables when they do not do so. The regulator initiates an inspection program for producers who claim their vegetables are organic. The regulatory authority gets positive payoffs from honest claims, and very negative returns from fraudulent claims that go undetected, but it must also pay for inspection costs. The regulatory authority imposes a fine of $10 on producers it inspects who make fraudulent claims. Producers’ payoffs depend on the price of vegetables given they claim organic (\( P_O \)), minus production cost, and minus a fine if imposed. Payoffs for a producer and for the regulatory authority are shown in the figure on the next page. Solve for all Nash equilibria for this game.

d. Finally, suppose there are two types of producers: A-types, who can only grow organic vegetables and B-types that can only grow conventional vegetables. Nature chooses the type for each producer. Assume the probability of an A-type is 0.25 and the probability of a B-type is 0.75. Each producer decides whether to certify their vegetables as organic or not. It costs much less for A-types to certify than B-types.
(because the B-types have to bribe the certification firm). Consumers will gladly pay a high price for organic vegetables and a low price for conventional vegetables but get a low payoff if they pay a high price for conventional vegetables or a low price for organic vegetables. This game is shown below. The first number is the payoff for the producer and the second number is the payoff for the consumer. Find all pure strategy perfect Bayesian equilibrium for this game.

**Game for part c**

<table>
<thead>
<tr>
<th></th>
<th>Regulator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inspect</td>
</tr>
<tr>
<td>Producer</td>
<td></td>
</tr>
<tr>
<td>Organic</td>
<td>( P(S_0) - 4, 0 )</td>
</tr>
<tr>
<td>Conventional</td>
<td>( P(S_0) - 3 - 10, -2 )</td>
</tr>
</tbody>
</table>

**Game for part d**

```
(10, 2)                  H                  Not                  A                  Certify
                    H
(6, 0)  L
(8, 0)  H
(4, 2)  L

Pr_A = 0.25
Pr_B = 0.75

(8, 2)                  H
(4, 0)                  L
(3, 0)                  H
(-1, 2)                 L
```

Part IV

Answer at most one question from Part IV.
Consider a 2 x 2 competitive exchange economy. Answer all three questions.

a. Show that a Walrasian equilibrium is weakly Pareto optimal: it is impossible to make everyone strictly better off. Give an example of a Walrasian equilibrium that is weakly Pareto optimal but not strongly Pareto optimal: it is possible to make some people strictly better off without making others worse off. Your answer may be in the form of a mathematical statement or a carefully labeled Edgeworth box.

b. Suppose that consumer 1 has utility function $U_1(x_1) = (x_1^{1/4})(x_2^{3/4})$ and consumer 2 has utility function $U_2(x_2) = (x_2^{1/8})(x_2^{7/8})$. Consumer 1's endowment is $(\alpha, \alpha)$ and consumer 2's is $(1 - \alpha, 1 - \alpha)$. Compute the Walrasian equilibrium. Show that the price of good 2 relative to good 1 is decreasing in $\alpha$. Interpret the result.

c. Suppose that the preferences of both consumers are strictly convex. True or false: The utility possibility set is a convex set in $(U_1, U_2)$-space. Provide a proof if true or a counterexample if false.
Question IV.2

Consider Arrow's social-choice problem, with individuals $J = \{1, 2, \ldots, n\}$, alternatives $X$, individual orderings $P_j$ over $X$, and the social welfare function $f(\{P_j\}_j^{n})$. Recall that the set of individuals $D \subset I$ is semi-decisive for alternatives $x$ and $y$ if $\forall j \in D, xP_j y$ and $\forall j \notin D, yP_j x$ together imply $xP y$. Recall also that the set of individuals $D \subset I$ is decisive for alternatives $x$ and $y$ if $\forall j \in D, xP_j y$ implies $xP y$.

a. Provide a proof of the first step of Arrow's theorem: there is some pair $a, b \in X$ such that some person $j \in I$ is semi-decisive (SD) over $a$ and $b$.

b. A committee of five voters is to rank five alternatives, $A$ through $E$. The following table shows individual rankings, with the number of voters holding each ranking given in the column headings and the (strict) rankings from most to least preferred reading down the column.

<table>
<thead>
<tr>
<th>3 Voters</th>
<th>1 Voter</th>
<th>1 Voter</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>B</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>E</td>
<td>D</td>
</tr>
<tr>
<td>D</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>E</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

The committee will use the Borda count to rank the alternatives. Each voter assigns points from zero (for that voter's least-preferred alternative) to four (for the most-preferred alternative). The points are added up and the committee's ranking is assigned according to the points received by each alternative. What ranking will result if the Borda count is used and all voters vote sincerely? Use the example to show that the Borda count violates the independence of irrelevant alternatives.