Ph.D. PRELIMINARY EXAMINATION

MICROECONOMIC THEORY

Applied Economics Graduate Program

May 2014

The time limit for this exam is four hours. The exam has four sections. Each section includes two questions. Answer one question from each section. Before turning in your exam, number each page of your answers in sequential order, and identify the question (for example, III.2 for section III and question 2). Indicate, by circling below, the questions you completed. If you answer more than one question in a section and do not circle a question corresponding to that section, the first of the two that appears in your solution paper will be graded.

NOTE: THIS EXAM SHOULD HAVE 13 PAGES INCLUDING THIS COVER PAGE. IF IT DOESN'T, PLEASE REQUEST A REPLACEMENT FROM THE PROCTOR.

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STUDENT ID LETTER: ______ (Fill in your code letter)

Answer one question from each section. Indicate the one you answered by circling:

Section I: Question 1 Question 2
Section II: Question 1 Question 2
Section III: Question 1 Question 2
Section IV: Question 1 Question 2

*** TURN IN THIS SHEET WITH YOUR ANSWER PAGES***
Part I

Answer at most one question from Part I.
Question I.1

Microeconomic Analysis of Child Labor: A household with one parent and one child has a utility function that depends on only two goods, a general consumption good denoted by \( x_1 \) and the number of years that the child spends in school, denoted by \( x_2 \). In this problem, you will use microeconomic theory to analyze the choice the household faces between sending the child to school and having the child work to earn wealth for the household.

a) The first thing to analyze is the household’s budget constraint. The household has a wealth of \( w \) that can be spent on the two goods. The general consumption good, \( x_1 \), has a price of \( p_1 \). The cost of education is \( p_2 \) per year of education (assume that it is possible to purchase fractions of a year of education, so that education is not “lumpy”). Assume that education is free, so that \( p_2 = 0 \). Finally, note that there is a maximum amount of years of child time, denoted by \( T_c \), that can be allocated to either child education, \( x_2 \), or to child labor (child labor can be denoted by \( T_c - x_2 \)). The “wage” for each year of child labor is denoted by “wage”.

Draw the budget constraint faced by this household, with \( x_1 \) on the horizontal axis and \( x_2 \) on the vertical axis. Be sure to account for the fact that \( x_2 \) has an upper bound of \( T_c \), and assume that both \( x_1 \geq 0 \) and \( x_2 \geq 0 \). If there are any “kinks” in the budget constraint, label the values of \( x_1 \) and \( x_2 \) that correspond to those kinks. Also indicate the slope of the budget constraint. Finally, indicate the value of \( x_2 \) at the point on the budget constraint at which \( x_1 = 0 \), and the value of \( x_1 \) at the point on the budget constraint at which \( x_2 = 0 \).

b) Suppose that the wage for child labor, \( wage_c \), increases. Is it possible that this will decrease the utility of the household? Refer to your diagram for part a) to explain your answer. Your answer should be relatively brief.

c) Consider again the increase in the child wage that occurred in part b). Suppose that the household has a continuous utility function that is quasi-concave. Is it possible that the increase in the child wage has no effect on the household’s utility? If it is not possible, explain why. If it is possible, indicate where on the budget set the household chooses to consume. Does your answer depend on whether preferences are strictly quasi-concave? You need not answer in mathematical terms. Drawing a well-labeled diagram is sufficient.

d) Return to the original budget constraint (budget set) in part a). Suppose that the government is worried that parents are sending their children to school for too much time, and so depriving their children of a fun childhood. The government decrees that the maximum amount of time that a child can spend in school is \( x_{2,\text{max}} \), which is less than \( T_c \). Show how the budget constraint changes. Is it possible that, in response to this policy, that a parent who chooses \( x_2 = T_c \) before this policy change will decide to choose \( x_2 = 0 \) after the change? Does your answer depend on whether the utility function is quasi-concave? As in c), you can provide an answer using a diagram.
Question 1.2

Edgeworth Box for 2 Consumers: There are two goods, $x_1$ and $x_2$, and 2 consumers, A and B. Consumers A and B have the following utility functions:

Consumer A: $u_A(x_{1A}, x_{2A}) = x_{1A}^{0.5}x_{2A}^{0.5}$

Consumer B: $u_B(x_{1B}, x_{2B}) = \min(x_{1B}, x_{2B})$

a) Verify whether Consumer A has a quasi-concave utility function. Check for strict concavity as well. [Hint: Consider a fixed level of utility, which can be depicted as an indifference curve in a “space” where $x_{1A}$ is the horizontal axis and $x_{2A}$ is the vertical axis. For this fixed level of utility, which you can denote by $u_{A0}$, rearrange the expression of utility so that $x_{2A}$ is a function of $x_{1A}$. Check whether this indifference curve has the shape associated with a quasi-concave utility functions.]

b) Suppose that each consumer has an endowment of $(1/2, 1/2)$. Draw an Edgeworth box with Consumer A’s “origin” at the lower left and Consumer B’s origin at the upper right. Show the dimensions of the box. Show where the endowment is. Show the utility curves of both consumers.

c) Show the set of Pareto optimal points in the Edgeworth box.

d) Derive the offer curves of Consumer A for both goods. Denote the prices of goods 1 and 2 by $p_1$ and $p_2$, respectively.

e) For Consumer B’s endowment, what is his or her offer curve. You do not have to derive it mathematically, just describe it in terms of your diagram. Also, show where the Walrasian equilibrium is in your diagram. Is it Pareto optimal? What (relative) prices correspond to that Walrasian equilibrium? Refer to your answer to part d) to justify your claim regarding the prices.
Part II

Answer at most one question from Part II.
Question II.1

Consider the production possibility set (PPS):

\[
PPS = \left\{ (q, -z) \in \mathbb{R}_+ \times \mathbb{R}^2_+ : (z_1^\rho + z_2^\rho)^{1/\rho} \geq q^\alpha \right\}
\]

where \( q \) is output; \( z_1 \) and \( z_2 \) are inputs; and \( \alpha > 0 \) and \( \rho > 0 \) are constant parameters. This PPS is nonempty, strictly convex, closed, and satisfies weak free disposal in output and inputs.

a) Under what conditions on \( \alpha > 0 \) and \( \rho > 0 \), if any, will this PPS exhibit non-increasing returns to scale? Justify your answer.

b) Derive the input distance function assuming \( q > 0 \).

c) Assuming \( \alpha = 2 \), the profit function for a competitive firm given this PPS is

\[
\pi(p, r_1, r_2) = \frac{p^2}{4 \left( r_1^{\rho - 1} + r_2^{\rho - 1} \right)^{\rho - 1 / \rho}}
\]

where \( p > 0 \) is the price of \( q \), \( r_1 > 0 \) is the price of \( z_1 \) and \( r_2 > 0 \) is the price of \( z_2 \). Use this profit function to derive the firm’s unconditional supply and unconditional input demands.

d) It is easy to verify that unconditional supply and input demands you derived in part c) are homogeneous of degree zero in \( p, r_1 \), and \( r_2 \), and that the profit function in part c) is homogeneous of degree one in \( p, r_1 \), and \( r_2 \). When there are \( L \) commodities that can be used as either outputs or inputs, show that in general the unconditional supplies \( y_l(p) \) for \( l = 1, \ldots, L \) will be homogeneous of degree zero in \( p \) and profit function \( \pi(p) \) will be homogenous degree one in \( p \) for a profit maximizing competitive firm given any price vector \( p \in \mathbb{R}_+^L \).
Question II.2

Consider a world with only two states denoted by $a$ and $b$. In state $a$, the farmer experiences an insect infestation, while in state $b$ he does not. A farmer’s profits in the state $a$ and $b$ are

$$\pi_a = \left(p + \frac{\varepsilon}{\phi_a}\right)(y - L(z)) - wz \quad \text{and} \quad \pi_b = \left(p - \frac{\varepsilon}{\phi_b}\right)y - wz$$

where $y > 0$ is the farmer’s crop yield in bushels without an insect infestation and $y - L(z)$ is his yield with an infestation; $z \geq 0$ is the amount of insecticide the farmer uses in gallons; $w > 0$ is the competitive price for insecticide per gallon; $p > 0$ is the average price the farmer receives for his yield per bushel; $\varepsilon$ is a random price shock with opposite effects in state $a$ and $b$; and $\phi_a > 0$ and $\phi_b > 0$ are the farmer’s subjective beliefs about the probability of states $a$ and $b$ such that $\phi_a + \phi_b = 1$. Assume the yield loss function $L(z)$ is continuous and twice differentiable such that $y \geq L(z) > 0$, $L'(z) < 0$ and $L''(z) > 0$.

Consider two types of farmers. The first type seeks to maximize expected profit. The second type seeks to maximize the utility of profit $W(\pi_a, \pi_b)$ where $\pi_s$ is profit in state $s$ and $W_s = \frac{\partial W(\pi_a, \pi_b)}{\partial \pi_s} > 0$ for $s = a, b$. Also assume that the second type’s preferences are risk averse, generalized Schur-concave with respect to $\phi_a$ and $\phi_b$.

a) Set up the profit maximizing farmer’s optimization problem and derive his first-order conditions (note that the second-order conditions will be satisfied). Under what condition will the farmer find it optimal to apply at least some insecticide (i.e., choose $z > 0$)? What is the economic intuition of this condition?

b) Set up the utility maximizing farmer’s optimization problem and derive the first-order conditions assuming the second-order conditions are satisfied.

i) Assuming $\varepsilon = 0$ so that there is no price risk, show that a utility maximizing farmer will use more insecticide than the expected profit maximizing farmer [You may assume both have an interior solution]? What is the economic intuition for this result?

ii) Assuming $\varepsilon \neq 0$, derive the condition under which a utility maximizing farmer uses less insecticide than an expected profit maximizing farmer [You may again assume both have an interior solution]? What has to be true of $\varepsilon$ for this result to hold? What is the relationship between price and yield risk in this circumstance and how does this relationship change the intuition of the result in part ii)?
Part III

Answer at most one question from Part III.
**Question III.1**

Two neighbors each produce good $X$. Let $x_i$ be the production of person $i$ ($i = 1, 2$). Production of good $X$ generates net benefits for person $i$ of $B_i(x_i) = a_i - x_i^2/2$. Production of good $X$ also generates pollution. Let $C_i(x_1, x_2)$ be the pollution cost to person $i$.

a) Suppose that the two neighbors choose production simultaneously and that pollution costs are $C_i(x_1, x_2) = (c/2)(x_1 + x_2)^2$. Find a Nash equilibrium for this game. Is this Nash equilibrium unique?

b) Assuming the same pollution cost function as in part a), suppose that person 1 moves first followed by person 2. Solve for the subgame perfect equilibrium in this game.

c) How does the equilibrium you found in part b) compare to the equilibrium you found in part a) for the case where $c_1 = c_2 = 1$? Explain the intuition for the result.

d) Now suppose that pollution costs are only a function of the maximum level of production by either person: $C_i(x_1, x_2) = (c/2)[\text{Max}(x_1, x_2)]^2$. Assume that $c_1 = c_2 = c$ and that $a_2/(1+c) < a_1 < a_2$. Find a Nash equilibrium for this game. Is this Nash equilibrium unique?

e) Finally, suppose that the pollution cost function and parameter values are the same as in part d) but person 1 moves first followed by person 2. Solve for the subgame perfect equilibrium for this game.
Question III.2

A monopolist can produce good \( X \) at constant marginal cost of \( c \) per unit. The monopolist has two types of customers: high types (\( H \)) and low types (\( L \)). The value of good \( X \) to an individual consumer of each type is: \( V_H(x) = ax - \frac{x^2}{2} \); \( V_L(x) = bx - \frac{x^2}{2} \), with \( a > b > c > 0 \). The probability that a given consumer is of type \( H \) or type \( L \) is: \( \Pr(t = H) = \lambda, \Pr(t = L) = 1 - \lambda \).

a) Suppose the monopolist knows each individual consumer’s type. Find the profit maximizing price and quantity to sell to an individual consumer of each type.

For the remaining parts of this problem, assume that the monopolist does not know the type of any individual consumer but only knows the probabilities of each type. Suppose that the monopolist offers contracts \((T_H, x_H), (T_L, x_L)\), where \( T_i \) is the amount of money needed to buy quantity \( x_i, i = H \) or \( L \). The monopolist designs the contracts such that a high type individual will choose \((T_H, x_H)\) and a low type individual will choose \((T_L, x_L)\).

b) Define the individual rationality constraints and the incentive compatibility constraints for both the high type and the low type (this will define four equations).

c) Set up the monopolist’s profit maximization problem with choice variables \( T_H, x_H, T_L, x_L \).

d) Assuming it is most profitable to sell to both types, solve for the profit maximizing contracts \((T_H, x_H)\) and \((T_L, x_L)\).

e) How does your answer in part (d) compare to your answer in part (a)? Explain the intuition for any differences.
Part IV

Answer at most one question from Part IV.
Question IV.1

Consider a $2 \times 2$ competitive exchange economy, with consumers $j = 1, 2$ and goods $x$ and $y$. Preferences are represented by

$$U_1(x_1, y_1) = x_1 y_1^2 \quad \text{and} \quad U_2(x_2, y_2) = x_2^2 y_2$$

where $x_j$ and $y_j$ are consumer $j$'s consumption of goods $x$ and $y$. The aggregate endowment is $(10, 20)$.

a) Compute the set of Pareto-optimal allocations (the contract curve) for this economy and represent the set in an Edgeworth box.

b) The transfer paradox occurs when one consumer, say consumer 1, makes a gift to another consumer, say consumer 2, and the consumer who receives the gift is worse off, in equilibrium, after the gift than before. That is, for some $a > 0$, agent 2 is worse off at the equilibrium relative to endowment $\omega' = (\omega_1 - a, \omega_2 + a)$ than at the equilibrium relative to $\omega = (\omega_1, \omega_2)$ where $\omega_j$ is consumer $j$'s endowment of goods $x$ and $y$. Use a carefully drawn Edgeworth box to show that if the transfer paradox occurs, the equilibrium budget line through $\omega'$ and the equilibrium budget line through $\omega$ must cross at a point on the same side of the contract curve as $\omega$ and $\omega'$. The diagram you use to describe this point may resemble the one in part a), but it should not be the same.

c) Take the following fact as given: A necessary condition for the transfer paradox to occur in a given exchange economy is that there are initial endowments at which there are multiple equilibria. Now compute the competitive equilibrium for any distribution of the initial endowment $\omega$ so that the aggregate endowment remains at $(10, 20)$. Show that the equilibrium is unique, and therefore that the transfer paradox cannot occur in this economy.
Question IV.2

Two roommates found a place to live rent free. Now they are deciding whether to buy a car together. They have a total income of $1,000 and a car would cost $400. Roommate 1’s utility is given by $U_1(x_1, y) = x_1(1 + y)$ and 2’s by $U_2(x_2, y) = x_2(3 + y)$, where $y = 1$ if they buy a car and $y = 0$ if they do not, and where $x_1$ and $x_2$ are the amounts they spend on private goods.

a) Write an equation for the utility possibility frontier if they are not allowed to buy a car and another equation for the utility possibility frontier if they must buy a car.

b) Graph these two utility possibility frontiers and shade in the utility possibility set if they are free to decide whether or not to buy a car.

c) Suppose that if they don’t purchase the car, the roommates split their money equally. Could they achieve a Pareto improvement by buying the car? Show your answer in two ways:

i) Compare the sum of willingnesses to pay to the cost of the car.

ii) Show whether or not the outcome where each has $500 and they have no car produces a point on the overall utility possibility frontier.

d) Suppose that if they don’t purchase the car, income would be divided so that $x_1 = 650$ and $x_2 = 350$. Could they achieve a Pareto improvement by buying the car? Show your answer in two ways:

i) Compare the sum of willingnesses to pay to the cost of the car.

ii) Show whether or not the situation where $x_1 = 650$ and $x_2 = 350$ and they have no car is on the overall utility possibility frontier.