Ph.D. PRELIMINARY EXAMINATION

MICROECONOMIC THEORY

Applied Economics Graduate Program

May 2013

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COVER SHEET
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STUDENT ID LETTER: ______ (Fill in your code letter)

INSTRUCTIONS

The time limit for this exam is four hours. The exam has four sections. Each section includes two questions. Answer one question from each section. Before turning in your exam, number each page of your answers in sequential order, and identify the question (for example, III.2 for section III and question 2). Indicate, by circling below, the questions you completed. If you answer more than one question in a section and do not circle a question corresponding to that section, the first of the two that appears in your solution paper will be graded.

Answer one question from each section. Indicate the one you answered by circling:

Section I: Question 1 Question 2
Section II: Question 1 Question 2
Section III: Question 1 Question 2
Section IV: Question 1 Question 2

*** TURN IN THIS SHEET WITH YOUR ANSWER PAGES***
Ph.D. PRELIMINARY EXAMINATION

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NOTE: The exam should have 15 pages including this cover page.
Part I

Answer at most one question from Part I.
Question I.1

An individual who faces a risky environment has the following (Bernoulli) utility function:

\[ u(x) = \alpha x + \beta x^2. \]

a) What restrictions on \( \alpha \) and \( \beta \) are needed for this individual to be risk averse?

b) Assume that this individual is strictly risk averse. Calculate the Arrow-Pratt coefficient of absolute risk aversion for this person. How does this change when \( x \) increases? Based on your answer, discuss whether you think that this is a plausible (Bernoulli) utility function.

c) Show that the expected utility of an individual with this (Bernoulli) utility function depends only on the mean (\( E[x] \)) and variance of \( x \). [Hint 1: \( \text{Var}(x) = E[(x - E[x])^2] \). Hint 2: The expectation of the product of a constant and a variable equals the constant multiplied by the expectation of the variable.]

d) Suppose that this person has a total wealth of \( w \), and he or she wants to divide this wealth into an amount of money, \( m \), that has no risk but also yields no interest, and a financial asset \( f \), that has a (risky) return of \( r \) with a mean of \( E[r] \) and a variance of \( \text{Var}(r) \). Thus \( w = m + f \), and the individual wants to choose the optimal division of \( w \) into \( m \) and \( f \). Assuming that this individual maximizes expected utility, what is the optimal amount of \( m \) as a function of \( w, E[r], \text{Var}(r) \) and the parameters of the (Bernoulli) utility function? [Hint: Do not use your answer to part (c), instead note that the “income” of this individual from the risky asset will be \( (w - m)(1 + r) \).]
Question I.2

1. A consumer has a utility function that depends on only two goods, $x_1$ and $x_2$. In fact, the consumer does not like $x_1$ at all, because it is oatmeal and the consumer does not like oatmeal. This implies that the consumer’s utility decreases as consumption of $x_1$ increases. Fortunately, the consumer does like $x_2$, because it is pizza. The consumer’s utility increases as consumption of $x_2$ increases.

The following diagram shows the consumer’s preferences for the two goods. Copy this into your blue book to answer parts a) and c) of this question.

![Diagram showing consumer preferences for $x_1$ and $x_2$]

a) Suppose that the prices of both $x_1$ and $x_2$, denoted by $p_1$ and $p_2$, respectively, are strictly positive. Assume that the consumer has a strictly positive amount of wealth, denoted by $w$. Draw the associated budget set (budget line) in this diagram, assuming that Walras’ law holds. Show the consumer’s demand for both goods in the diagram, assuming that the consumer wants to maximize his or her utility.

b) Suppose that the government decides to increase consumption of oatmeal, which is much healthier than pizza, by taxing pizza (which increases the price of pizza) and subsidizing oatmeal (which decreases the price of oatmeal). Will a tax on pizza alone increase consumption of oatmeal? Will a subsidy that decreases the price of oatmeal to 0 increase the consumption of oatmeal? Just explain your answers in words, without doing anything to the diagram.

(Question I.2 is continued on the next page)
(Question 1.2 is continued)

c) Can a subsidy that leads to a negative price of oatmeal, that is a policy that gives consumers a certain amount of money for each unit of oatmeal consumed, increase the consumption of oatmeal? Explain your answer, either by adding to the figure in your answer for a) or drawing a new figure.

d) Assume the following specific form for the consumer’s utility function:

\[ u(x_1, x_2) = x_1^{-\alpha} x_2^\beta, \quad \text{where } \alpha > 0, \beta > 0, \text{ and } \beta > \alpha \]

Note the negative sign in front of \( \alpha \)!

The price of \( x_2 \) is \( p_2 \), with \( p_2 > 0 \). The government subsidizes oatmeal heavily, so that its price is negative, so that \( p_1 < 0 \). Assuming that \( w \) equals wealth, and that the consumer spends his or her entire wealth on goods \( x_1 \) and \( x_2 \), use constrained optimization to solve for the Walrasian demands for \( x_1 \) and \( x_2 \). You can assume that the solution is an interior one.

e) Are these demand functions homogenous of degree zero? Do they satisfy Walras’ law? Show your results for both questions.
Part II

Answer at most one question from Part II.
Consider a firm’s cost function $C(r, q) = r_1^{0.5}r_2^{0.5}q$, where $q \in \mathbb{R}_+$ is output, and $r_1 > 0$ and $r_2 > 0$ are the prices of inputs $z_1 \geq 0$ and $z_2 \geq 0$. This cost function is derived from an input requirement set that is nonempty, closed, strictly convex, and satisfies weak free disposal.

a) Derive the conditional input demands for $z_1$ and $z_2$.

b) Derive the input distance function assuming $q > 0$.

c) Let $p = a - q$ be the market demand where $p$ is the price of output and $a > 0$. Assume the firm acts as a profit maximizing monopolist given this cost function. How much will the firm produce, what price will it charge, and what are its unconditional input demands?

d) Show that the general revenue function $R(p, z)$ with output prices $p \in \mathbb{R}_+^{M}$ and inputs $z \in \mathbb{R}_+^{N}$ is convex in output prices.
Question II.2

Consider a world with only two states denoted by $g$ for good and $b$ for bad. A firm’s profits in the good and bad state are

$$\pi_g = \mu(z) + \sigma(z) - wz$$
$$\pi_b = \mu(z) - \frac{\phi_g}{\phi_b} \sigma(z) - wz$$

where $z \in \mathbb{R}_+$ is an input, $\mu(z) > 0$ is certain revenue given $z$; $\sigma(z) > 0$ is additional revenue in the good state given $z$; $\phi_g > 0$ and $\phi_b > 0$ are a firm’s subjective beliefs about the probability of the good and bad state such that $\phi_g + \phi_b = 1$; $\frac{\phi_g}{\phi_b} \sigma(z)$ is the lost revenue in the bad state; and $w > 0$ is the competitive price of the input. Assume $\mu(z)$ and $\sigma(z)$ are continuous and twice differentiable for all $z$. Also assume $\mu'(z) \geq 0$ and $\mu''(z) \leq 0$ for all $z$.

Consider two types of firms. The first type seeks to maximize expected profit. The second type seeks to maximize the utility of profit $W(\pi_g, \pi_b)$ where $\pi_s$ is profit in state $s$ and $W_s = \frac{\partial W(\pi_g, \pi_b)}{\partial \pi_s} > 0$ for $s = g, b$. Also assume that the second type of firm’s preferences are risk averse with respect to $\phi_g$ and $\phi_b$, and generalized Schur-concave.

a) Set up the profit maximizing firm’s optimization problem and derive its first-order conditions assuming second-order conditions are satisfied. If $\mu'(z) > 0$ and $\mu''(z) < 0$ for all $z$, under what condition will it be optimal for this firm to choose $z > 0$? What is the economic intuition of this condition?

b) Set up the utility maximizing firm’s optimization problem and derive the first-order conditions assuming the second-order conditions are satisfied.

i) Assuming $\mu'(z) = \mu''(z) = 0$ for all $z$, what condition must hold for it to be optimal for the utility maximizing firm to choose $z > 0$? What is the economic intuition of this condition [Hint: The variance of profit is increasing in $\sigma(z)$]?

ii) Assuming $\mu'(z) > 0$ and $\mu''(z) < 0$ for all $z$ and it is optimal for the expected profit maximizing firm to choose $z > 0$, what condition must hold for it to be optimal for the utility maximizing firm to use less of the input than the expected profit maximizing firm?
Part III

Answer at most one question from Part III.
Question III.1

Suppose there are $N$ countries that use fossil-fuels. Each country $i$, $i = 1, 2, \ldots, N$, uses fossil-fuels to provide energy to its citizens. Suppose that the use of fossil-fuels generates benefits of $B(q_{it}) = \alpha q_{it} - \frac{(q_{it})^2}{2}$ for $0 \leq q_{it} \leq 2\alpha$, and 0 otherwise, where $q_{it}$ is the use of fossil-fuels by country $i$ at time $t$. Use of fossil-fuels also results in emissions of greenhouse gases that lead to climate change. Damages from climate change to country $i$ at time $t$ are given by: $D(S_i) = \frac{(S_i)^2}{2}$, where $S_i$ is the stock of greenhouse gases at time $t$. The stock of greenhouse gases evolves according to: $S_t = \phi S_{t-1} + Q_t$, where $Q_t = \sum_{i=1}^{N} q_{it}$, and $0 \leq \phi \leq 1$. For simplicity let the initial level of greenhouse gases be $S_0 = 0$. Let the discount factor between periods be $\delta$, $0 < \delta < 1$.

a) Suppose there is a single time period ($t = 1$). Solve for the Nash equilibrium level of fossil-fuel use for each country.

b) Again suppose there is only a single time period. Solve the efficient level of fossil-fuel use for each country that maximizes the sum of net benefits of all $N$ countries. How does this level compare with the aggregate level of fossil-fuel use found in part (a)? Explain the economic intuition for your result.

c) Now suppose there are two time periods, $t = 1, 2$. For this part, assume that $N = 3$. Solve for the subgame perfect Nash equilibrium level of output in period 1. How does it compare to the level of output in the one period problem solved in part (a)?
Question III.2

Equilibria in Games.

a) Find all Nash equilibria for the game shown in Figure 1.

b) Show whether there is a dominant strategy equilibrium or an iterated strict dominance solution to the game shown in Figure 2.

c) Show that a dominant strategy equilibrium must be a Nash equilibrium but that a Nash equilibrium need not be a dominant strategy equilibrium.

d) Suppose that player 2 moves first and player 1 moves second for the game shown in Figure 2. Solve for the subgame perfect Nash equilibrium for this dynamic game of complete information.

e) Find all pure strategy perfect Bayesian equilibrium for the game shown in Figure 3.

f) For all pure strategy perfect Bayesian equilibria found in part (e), show which of these equilibria satisfy the Intuitive Criterion.

Figure 1: Player 1 chooses $U$ or $D$. Player 2 chooses $L$ or $R$.

<table>
<thead>
<tr>
<th></th>
<th>$L$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>0,1</td>
<td>4, 2</td>
</tr>
<tr>
<td>$D$</td>
<td>2, 4</td>
<td>1, 0</td>
</tr>
</tbody>
</table>

Figure 2: Player 1 chooses the row. Player 2 chooses the column.

<table>
<thead>
<tr>
<th></th>
<th>$W$</th>
<th>$X$</th>
<th>$Y$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>4, 3</td>
<td>1, 2</td>
<td>2, 4</td>
<td>0, 0</td>
</tr>
<tr>
<td>$B$</td>
<td>6, 2</td>
<td>0, 1</td>
<td>3, 3</td>
<td>3, 0</td>
</tr>
<tr>
<td>$C$</td>
<td>3, 5</td>
<td>2, 6</td>
<td>1, 4</td>
<td>5, 2</td>
</tr>
<tr>
<td>$D$</td>
<td>5, 3</td>
<td>9, 4</td>
<td>2, 6</td>
<td>4, 3</td>
</tr>
</tbody>
</table>

(Figure 3 on the next page)
Figure 3: Dynamic game of incomplete information. Player 1 can be of two types \((t_1, t_2)\). Player 1 moves first and chooses \(L\) or \(R\). Player 2 observes player 1’s choice but not its type and then chooses \(U\) or \(D\).
Part IV

Answer at most one question from Part IV.
Question IV.1

Suppose a $2 \times 2$ competitive exchange economy has a strictly positive total endowment $(\omega_1, \omega_2)$ and the agents’ preferences are represented by the utility functions $U_1(x_1) = \min\{x_1^1, x_1^2\}$ and $U_2(x_2) = \min\{x_2^1, x_2^2\}$

a) Characterize the set of Pareto-optimal allocations when $\omega_1 = \omega_2 = e > 0$. Justify your answer.

b) Characterize the set of Walrasian equilibria when the agents' endowments are given by $\omega_1 = (3, 5)$ and $\omega_2 = (5, 3)$. Justify your answer.

c) Show that any Walrasian equilibrium allocation for the economy with endowments as in part (b) is Pareto optimal.

d) Characterize the core for the economy with endowments as in part (b).

e) Characterize the set of Pareto-optimal allocations when $\omega_1 = e > 0$ and $\omega_2 = 2e$. Justify your answer.
Question IV.2

Consider an economy with two consumers, $i = 1, 2$, and two goods, $x$ and $y$. One good, $y$, is public and the other, $x$, is private. Consumers are endowed with the private good in amounts $\omega_1 = 20$ and $\omega_2 = 10$. Each consumer can choose to contribute some of this endowment, $z_i$, to the provision of the public good, which is produced according to the linear production technology $y = (z_1 + z_2)/2$. The rest, $x_i = \omega_i - z_i$, is consumed. Preferences are given by $U_1(x_1, y) = x_1 y$ and $U_2(x_2, y) = x_2 y^2$.

a) Find conditions for Pareto optimality of an allocation $(x_1, x_2, y)$ for this economy.

b) Assume that each consumer takes the other's contribution as given and then maximizes her own utility. What is the Nash equilibrium outcome? Is it Pareto optimal?

c) Now assume that consumer 1 is a Stackelberg leader in the choice of $z_1$, and consumer 2 is a follower. Find the Stackelberg equilibrium. Is it Pareto optimal?