WRITTEN PRELIMINARY Ph.D. EXAMINATION

Department of Applied Economics

June 20, 2014

Consumer Behavior and Household Economics

Instructions

• Identify yourself by your code letter, not your name, on each question

• Start each question’s answer at the top of a new page

• There are SIX questions in this exam

• You are to answer a total of FOUR questions

• Among Questions 1 and 2, you MUST answer at least one

• You have four hours to complete the examination
Attention: Of the FOUR questions that you must answer, ONE of them MUST be either Question 1 or Question 2. (You can also choose to answer both of them.)

QUESTION 1. Making Comparisons.

   a) Demographic shifters in demand functions are said to have “price like effects”. Using a simple model of utility, show what you understand by this statement.

   b) Prove that, in general, an equivalence scale cannot be identified from expenditure data.

   c) Pendakur (1999) defines shape invariant equivalence scales. What do you understand by this? How does shape invariance relate to base independence?

   d) When will a single cost of living index measure changes in the cost of living for an entire population? Show a Stone Price index satisfies these restrictions.

   e) What challenges arise when you wish to compare costs of living between countries as opposed to between time periods? Discuss approaches that are used to deal with these challenges.
Attention: Of the FOUR questions that you must answer, ONE of them MUST be either Question 1 or Question 2. (You can also choose to answer both of them.)

QUESTION 2. Semiparametric Methods.

a) Propose a method for testing the a standard Working-Leser Engel curve:

\[ w_i = \alpha_0 + \beta \ln x_i + \epsilon_i \]

against a nonparametric alternative.

b) Your able research assistant tells you that half of your data was collected on weekends and the other half was collected on week days. Propose a nonparametric test as to whether pooling data from weekends and week days is justified.

c) Consider the following semiparametric extension of the standard form above,

\[ w_i = \alpha Z_i + g(\ln x_i) + \epsilon_i \]

where \( Z \) is a vector of demographic characteristics for household \( i \). Propose an algorithm for estimating this model and explain the relative advantages of estimating this model over a fully nonparametric model of the form, \( w_i = g(\ln x_i, Z_i) + \epsilon_i \)

d) What restrictions does the semiparametric function described above place on the underlying preferences? Hint: Recall Shephard’s Lemma tells us that budget shares can be written as the derivative of the log cost function with respect to log prices.
A consumer has the following utility function for two goods, x and y, for T time periods:

\[ U = \sum_{t=1}^{T} v_t(x_t, y_t) \]

For simplicity, there is no discounting over time. The price of y always equals 1 and does not change over time, while the price of x at time t, denoted by \( p_t \), does change over time.

a) Assume that the consumer makes a life cycle plan at time \( t = 1 \), and that prices for all time periods are known at time \( t = 1 \). The consumer has an initial wealth of W and does not receive any other income at any time period. Write the consumer’s life cycle budget constraint.

b) From now on, assume that \( v_t(x_t, y_t) \) has the following functional form:

\[ v_t(x_t, y_t) = a_t \frac{x_t^{1-1/\epsilon}}{1-1/\epsilon} + y_t \]

Assume that \( \epsilon \neq 1 \) and \( \epsilon \neq 0 \), Use standard constrained optimization methods to calculate \( x_t \), the consumer’s (Marshallian, or Walrasian) demand for good x at time t. You can assume an interior solution. Do you notice anything unusual about the demand for x at time t?

c) Is the consumer’s (Marshallian) demand for \( y_t \) well determined? You can assume that the demand for y will be \( > 0 \) for all time periods. If not, is there anything about the demand for \( y_t \), perhaps related to the demand for y in other time periods, that is well determined?

d) Use your answer to part b) to derive the (Marshallian) price elasticity of \( x_t \). Also, derive the wealth (life cycle income) elasticity of \( x_t \).

e) Using your answers to b) and c), what is the consumer’s indirect utility function? What is the consumer’s cost (expenditure) function? You can denote the indirect utility function by V and the cost function by C.

f) Use your answer to e) to obtain the Hicksian demand for \( x_t \). Compare it to your answer for part b). What interpretation can you give to this comparison?
QUESTION 4: Rational Addiction.

Rational addiction is explained in economic models of consumer behavior. A basic model of rational addiction is:

\[ u(t) = u[y(t), c(t), S(t)] \]

a) How is this model used to explain rational addiction? Define the arguments in the equation and explain how the depreciation rate and time discount rate impact the consumption of the potential addictive good.

b) Show the steady states of the model graphically. Illustrate the existence of the stable steady state and unstable steady state.

c) Use the graph to illustrate how favorable/unfavorable events would impact the consumption of addictive goods in real world situations.

d) Use the graph to show how the decrease in price of the addictive good affects the consumption of the addictive good in the short run and long run.

Consider a two-person household that must decide how to purchase one private good, denoted by $q$, and two public goods denoted by $G$ and $H$. Denote the two persons in the household by A and B. Let $q_A$ be the consumption of the private good by person A, and $q_B$ be the consumption of the private good by person B. The utility functions of these two persons are given by:

$$u_A = q_A G^a H^o, \quad a > 0 \text{ and } o > 0$$
$$u_B = q_B G^b H^o, \quad b > 0 \text{ and } o > 0$$

For simplicity, assume that the total income of this household is equal to 1, and it is divided into $\rho$ for person A and $1 - \rho$ for person B, where $0 < \rho < 1$. Throughout this problem each person will spend his or her own money on consumption of the private good for his or her own consumption, and may spend some money for one or both of the two public goods. Also for simplicity, assume that the prices of all three goods are equal to 1, which is simply a normalization of the units of each of the goods.

a) Assuming that both persons want to maximize their utility, is it ever possible that $q_A = 0$ or $q_B = 0$? Is it ever possible that person A or person B will not spend any money on good $G$? Is it every possible that person A or person B will not spend any money on good $H$? Do not use any math to answer this question.

b) Derive the first order conditions for person A’s Marshallian demand functions for $q_A$, $G$ and $H$. Using your answer for a), indicate whether you can assume an interior solution for any or all of those three goods.

c) Derive the first order conditions for person B’s Marshallian demand functions for $q_B$, $G$ and $H$. Using your answer for a), indicate whether you can assume an interior solution for any or all of those three goods.

d) Given your answers to b) and c), is it ever possible that both persons A and B spend some of their own money on good $G$ and good $H$? (That is, both people buy both public goods.) Explain your answer, and state very clearly when, if ever, this is possible.

e) Consider the case where person A spends some of her money on both of the public goods ($G$ and $H$), while person B spends all of his money on the consumption of the private good. Solve for $q_A$, $q_B$, $G$ and $H$ in terms of $\rho$, $a$, $o$, $b$ and $o$. What is the range of $\rho$ that corresponds to this situation? [Hint for range $\rho$: Look at person B’s FOC for good $G$ or $H$.]

f) Is the solution for part e) efficient or inefficient? Demonstrate your answer. What is the intuition for this result? [Hint: What happens to each person’s utility if a small amount of money is diverted from the private good to the public good $G$? The contribution of each person can be proportional to each person’s share of total income. Total differentiation is useful.]
QUESTION 6: Noisy Information.

Consider the following hypothetical facts:

“Two percent of people in the world are deficient in Vitamin D. We have a test for Vitamin D deficiency. If someone is deficient in Vitamin D, that person has an 80% chance of having a positive test result. If someone is not deficient in Vitamin D, that person has a 20% chance of having a positive test result. Mary was just given the test, and she had a positive test result.”

   a) Assume that Mary was drawn randomly from the world population. What is the probability that she is truly deficient in Vitamin D?

   b) Predict the responses of a population of naive subjects who are asked to estimate the probability of Mary’s Vitamin D deficiency, given the information above. Justify your answer. Describe the kinds of errors that they are likely to make.