Carbon Trading, Carbon Taxes and Social Discounting

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Abstract

This paper considers the optimal design of policies to carbon emissions in an economy where an externality arises from the consumption of an exhaustible resource (oil) and a technology exists to mitigate the externality. I focus on the implications for policy design of assuming social preferences differ from private preferences regarding future generations. In particular, I consider a welfare function that places direct Pareto weights on unborn generations, as opposed to future generations receiving weight only through the altruism of their ancestors. This specification delivers a social discount rate which is lower than that of private individuals. I first show that standard policies, such as price or quantity controls on the net emissions of carbon, are insufficient to achieve the social optimum: When social and private discount rates differ, more sophisticated policies are necessary. The main results of the paper characterize these sophisticated policies. I show that an optimal tax scheme requires subsidizing the mitigation technology and taxing carbon emissions, but each at different rates: the optimal subsidy for removing a ton of carbon from the atmosphere will in general not equal the optimal tax for creating a ton of carbon. I also show that an optimal cap and trade system must include a cap on carbon offset allowances.

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1 Introduction

This paper studies optimal policies to control carbon emissions in an environment where social preferences differ from private preferences regarding future generations. A welfare function that places direct Pareto weights to unborn generations delivers a social discount rate that is lower than that of private individuals. I argue that standard policies to control carbon emissions, such as taxes or caps on the net emissions of carbon, are then insufficient to implement the social optimum. More sophisticated carbon policies have to be designed.

I develop the argument using a simple model of climate change in which an externality (“carbon in the atmosphere”) arises from the consumption of an exhaustible resource (“fossil fuel”). The externality can be mitigated by using an available technology (“sequestration”). In this model, households decide how fast to eat up the resource and how much effort to devote to mitigation activities. Both decisions affect the aggregate stock of carbon but, since households do not take this effect into account, the allocation in the decentralized environment is suboptimal and there is a need for policy intervention.

To address intergenerational issues, I set up an economy composed of an infinite sequence of generations. Each generation consists of a continuum of altruistically linked individuals who live for only one period. I consider a welfare function that places direct weights on current as well as future unborn generations. When only the first generation receives positive weight in the welfare function, social and private discount rates coincide and the planning problem corresponds to that of a representative infinitely lived individual. Even though their welfare is not directly valued, future generations are still valued through the altruism of earlier ones. On the other hand, when the social welfare function places positive weights on future generations, the allocation that solves the planning problem will not in general coincide with the one that the first generation would have chosen as a representative infinitely lived individual. Moreover, this welfare criterion is formally equivalent to solving a social planning problem where the social discount rate is lower than the one of the individuals in the society.

The main results of the paper focus on the decentralization of the optimal allocation and on how the design of policy instruments is affected by the choice of alternative welfare
criteria. One special such criteria is when only the current generation receives positive weight in the social welfare. In this case, the social discount rate coincides with the private one and some well known features of optimal taxation of carbon emissions hold. In particular, carbon taxes and sequestration subsidies are equal to the value of the externality, which I refer to as the “social cost of carbon”. It is equal to the discounted sum of future marginal damages from carbon emissions. It follows that optimal taxes are sensitive to the discount rate and, for that reason, this last one has been the subject of much controversy. Alternatively, in a cap and trade system in which the government sets a cap on the net emissions of carbon, the social cost of carbon arises as the equilibrium price of permits.

I refer to these policies as “standard policies” since they regulate the net emissions of carbon (emissions net of sequestration). That is, a regulator would typically not care about how the optimal level of carbon is reached, if by reducing emissions or by offsetting them through sequestration. In order to reach the social optimum, it is enough to control the net amount of emissions. Carbon taxes provide incentives for eating up fuel resources at the optimal rate and tax credits on sequestration induce households to devote the right amount of effort to remove the emissions generated by their consumption. As a result, total emissions follow the optimal path. Alternatively, in a cap and trade system, firms that are willing to extract fossil fuels must either buy permits to pollute or pay households for sequestration. The equilibrium price of carbon permits induces the right amount of sequestration and rate of fossil fuel extraction in the decentralized environment.

However, the social optimum associated with valuing future generations only through the altruism of the present generation is only one among many other efficient allocations. These alternative efficient allocations are each associated with positive direct weights on unborn generations in the welfare criterion and each corresponds to a point on the Pareto frontier. Moreover, this welfare criterion implies a social discount factor that, at any point in time, is higher than that of the individuals in the society.

The central contribution of this paper is to show that the standard policies are insufficient to optimally control carbon emissions when social and private discount rates differ. First, the direct link between the social cost of carbon in the planning problem and the policy
instruments in the decentralized environment no longer holds. And second, it is not enough
to control the net emissions of carbon. A social planner that cares about future generations
wants to treat differently the emissions of carbon from the emissions offsets.

The first main result of the paper is that, when social and private discounting differ, the
tax rate on emissions should be different from the tax credit on sequestration. This is true
even though both have the same effect on the overall externality. The reason is that taxes on
carbon emissions depend now not only on the shadow cost of carbon but also on the private
and the social discount factors. Both show up as an extra term in the tax formula. This
result is important because it implies that it is not possible to solve for the path of carbon
emissions based on a social planner’s problem and then associate taxes to the shadow cost
of carbon in that problem. When social and private discounting differ, particular attention
need to be posed on the decentralized environment and the design of optimal policies.

The second main result of the paper is that an optimal cap and trade system must include
trading on sequestration licenses. If the government sets net emissions caps to those which
 correspond to the carbon emitted in the optimal allocation, the economy with a cap and trade
system will exhibit too fast a depletion of fossil fuels and too much sequestration. Hence, a
cap on carbon sequestration allowances has to be coupled with the cap on net emissions of
carbon in order to implement the optimal allocation.

The basic intuition behind the two main results of the paper is as follows. There are two
ways of delivering the optimal level of carbon to the next generation. One requires the current
generation to sequester more; the other to consume less. Even though both are equivalent in
terms of the amount of the externality (carbon in the atmosphere) that the future generation
will inherit, there is a higher social return in doing so through less consumption. The higher
social return comes from the fact that future generations will not only inherit less carbon but
also more reserves of fossil fuels, which they also value. This extra return counts directly, as
well as through the altruism of current generations, when a social welfare function that assigns
direct weights on future generations welfare is considered. In this sense, a low social discount
rate could be thought of as a kind of externality which optimal policies need to correct for.
Overall, the results derive from the simple rule in public finance by which optimal policies
require as many instruments as margins need to be corrected. Standard policies provide only one instrument. If climate policies are meant to deal not only with the environmental externality itself but also with intergenerational equity, an extra lever is missing.

One important contribution of this paper is to provide a unified framework that nests Stern’s (“normative”) and Nordhaus’s (“positive”) approach to discounting. When only the current generation receives positive weight in the social criterion, social and private discounting coincide and the problem corresponds to the one solved in positive studies, as in Nordhaus and Boyer (2003). When future unborn generations receive positive direct weight in the social welfare function, the social discount rate is lower than the private one as advocated by Stern (2007). I argue in this paper that a comparison between these two climate proposals can not ignore the policies that are needed to implement the optimal path of carbon each of them propose. This piece of information is currently missing in the literature. The main contribution of this paper is to fill this gap by characterizing these policies. The paper shows that if a social discount rate different from that of private individuals is used to obtain the optimal path of carbon emissions, then standard price or quantity controls are insufficient to implement the social optimum. Therefore, while Nordhaus’s path of carbon can always be implemented through a standard carbon tax, Stern’s one requires sophisticated policies as the ones I characterize in this paper.

This paper also considers cases where the social discount rate is not only different from the private one but also varies over time. This case is more complicated since it renders the objective of the social planning problem dynamically inconsistent, thereby making the notion of optimality itself become problematic. To analyze this case, I first derive a weighting scheme which delivers a particular time varying social discount rate: hyperbolic discounting. I then use numerical methods to characterize the carbon policies that implement the social optimal allocation under commitment and the ones that implement the Markov Perfect (time-consistent) allocation. In this last case, the planning problem takes the form of a dynamic game between subsequent planners. I discuss all these issues in the last section of the paper.

The paper is related to the vast literature on discounting in climate change, specially Stern (2007), Nordhaus (2007) and Weitzman (2001) and also Arrow et al. (1996) and Dasgupta
(2008) who provide good summaries of the controversy on discounting. The paper is also related to the literature on optimal taxation of fossil fuels with a climate externality as Sinn (2008) and Golosov et al. (2011). This paper differs from those in that they do not study the interaction between social discounting and optimal taxation. The approach to social discounting adopted in this paper corresponds to the one in Bernheim (1989), Phelan (2006) and Farhi and Werning (2007), although they work in a different environment. In dealing with the time inconsistency problems that arise from using a time-varying discount rate, the paper is related to the literature on hyperbolic discounting, Cropper and Laibson (1998) and Krusell et al. (2002).

The remainder of the paper is organized as follows. Section 2 sets up the basic model. Section 3 solves the social planning problem. Section 4 proposes two alternative market economies. In the first one, the policy instrument is a tax scheme while in the second, it is a cap and trade system. Section 5 characterizes optimal policies and presents the main results of the paper. Section 6 contains a numerical example illustrating the results. Section 7 presents an extension of the model which contemplates a time varying social discount rate. Section 8 provides some conclusions from the analysis. Finally, all proofs and a discussion of the social welfare function used in the paper are included in the Appendix.

2 The Basic Model

Consider the following economy. At any point in time, \( t \in \{0, ..., \infty\} \), the economy is populated by a unit mass continuum of identical individuals, who live for one period and constitute generation \( t \). There is a single consumption good \( k_t \). The good (fossil fuel) is exhaustible and thus, at any point in time, must satisfy

\[
k_{t+1} \in [0, k_t]
\]  

(1)

The economy starts with an initial endowment equal to \( k_0 \). Resource feasibility requires

\[
c_t + k_{t+1} = k_t
\]  

(2)
for every period \( t \), where \( c_t \) represents fossil fuel consumption. Further, the amount of carbon in the atmosphere, \( S_t \) increases with consumption and decreases if individuals exert an effort level \( z_t \) (sequestration). In particular, the amount of carbon in the atmosphere follows the following process

\[
S_{t+1} = (1 - \gamma) S_t + k_t - k_{t+1} - z_t
\]  

(3)

where \( \gamma \in [0, 1) \) is the rate of natural reabsorption of carbon and \( S_0 \) is given. The presence of carbon in the atmosphere generates a negative externality which is assumed to take the form of a per period disutility cost. An individual’s utility in period \( t \) is given by

\[
U(c_t, z_t, S_{t+1}) = u(c_t) - v(z_t) - x(S_{t+1})
\]

where \( u \) is assumed to be increasing, concave and twice differentiable with \( \lim_{c \to 0} u'(c) = \infty \). The disutility cost function \( v \) is increasing, convex, twice differentiable and satisfies \( \lim_{z \to 0} v'(z) = 0 \). The function \( x \) is assumed to be increasing, convex, twice differentiable and satisfies \( \lim_{S \to 0} x'(S) = 0 \). Over time, individuals care about their utility and that of their children. Thus, the utility of an individual born in period \( t \) is given by

\[
v_t = U(c_t, z_t, S_t) + \beta v_{t+1}
\]

(4)

where \( \beta \in (0, 1) \) is the altruistic weight over the child’s utility, \( v_{t+1} \). This demographic specification is consistent with one in which households consists of a single infinitely lived individual who care about the value

\[
\sum_{t=0}^{\infty} \beta^t U(c_t, z_t, S_{t+1})
\]

(5)

**Social Welfare.** When a representative infinitely lived individual inhabits the economy, it is natural to consider a social welfare function that coincides with (5). However, when the economy is composed of an infinite sequence of altruistically linked generations, a more general social welfare function involves weighting the utility of each generation separately. In particular, this paper considers a utilitarian criterion that weighs current as well as future unborn generations according to the following function

\[
\sum_{s=0}^{\infty} \alpha_s \left[ \sum_{t=s}^{\infty} \beta^{t-s} U(c_t, z_t, S_{t+1}) \right]
\]

(6)
where \( \{\alpha_s\}_{s=0}^{\infty} \) is any arbitrary weighting scheme across generations. In particular, note that a special weighting scheme is the one that places all weight on the current generation. In this special case, equation (6) collapses to (5).

We can further simplify the welfare function to get
\[
\sum_{t=0}^{\infty} \hat{\beta}_t U(c_t, z_t, S_{t+1})
\]  
(7)

where \( \hat{\beta}_t \equiv \sum_{\tau=0}^{t} \alpha_{\tau} \beta^{t-\tau} \) represents the social discount function. Note that when future generations enter in the calculation of social welfare, even though each individual at any given point in time discounts the future by \( \beta \), society as a whole does it at a different rate given by
\[
\frac{\hat{\beta}_{t+1}}{\hat{\beta}_t} = \beta + \frac{\alpha_{t+1}}{\hat{\beta}_t} \geq \beta
\]  
(8)

It becomes clear that the social discount factor is higher than the private one if weights are strictly positive for all generations. This approach to discounting is the one in Farhi and Werning (2007). The next section characterizes the social optimal allocation as the solution to a planning problem under some restrictions on the weighting scheme.

3 Social Planning Problem

Given welfare weights \( \{\alpha_t\}_{t=0}^{\infty} \), the social planning problem is to choose \( \{c_t^*, k_t^*, z_t^*, S_t^*\}_{t=0}^{\infty} \) in order to maximize the social welfare function
\[
\sum_{t=0}^{\infty} \hat{\beta}_t U(c_t, z_t, S_{t+1})
\]
subject to the carbon cycle (3), the resource constraint (2) together with (1) and initial conditions \( \{k_0, S_0\} \), as well as the social discount function, \( \hat{\beta}_t \equiv \sum_{\tau=0}^{t} \alpha_{\tau} \beta^{t-\tau} \).

Social Discounting. Expression (8) highlights two important implications of the social welfare function considered in this paper. First, the difference between the social and private discount rates depends on the choice of the Pareto weights. Second, unless restrictive assumptions are made on the weighting scheme, social preferences display a time
varying discount function and therefore are subject to time inconsistency problems of the sort studied by Strotz (1955).

We say that the social planning problem is time consistent if the initial solution at \( t = 0 \) also solves the problem at any future period \( t \). That is, if future generations are allowed to reevaluate the climate policy, they will find optimal to follow the same path \( \{c_t^*, k_t^*, z_t^*, S_t^*\}_{t=0}^\infty \) that was prescribed by previous ones. For the rest of the paper, I will assume that the welfare weights satisfy Assumption 1. This assumption ensures that the social discount function takes the standard geometric form and, hence, the social planning problem is time consistent. I will remove this assumption in section 7.

**Assumption 1** The welfare weights \( \{\alpha_t\}_{t=0}^\infty \) in the social welfare function 6 satisfy one of the following conditions:

(i) \( \alpha_0 = 1 \) and \( \alpha_t = 0 \ \forall \ t \neq 0 \)

(ii) \( \alpha_0 = \frac{1}{\beta - \bar{\beta}} \) and \( \alpha_{t+1} = \hat{\beta}^t \) for some constant \( \hat{\beta} > \beta \)

Condition (i) describes a social welfare criterion that places direct weight on the current generation while future unborn generations are only valued indirectly through the altruism of the current one. This is a special case in which the social discount function becomes \( \hat{\beta}_t = \beta^t \) so that social and private discounting coincide.

Condition (ii) is a result derived by Bernheim (1989). For the social planning problem to be time consistent, it is required that weights on future generations decrease geometrically from \( t = 1 \) on. The current generation receives extra weight. Furthermore, welfare weights that satisfy condition (ii) induce a social discount rate that is lower than that of the private individuals in the society.

### 3.1 Optimal Allocation

Given welfare weights \( \{\alpha_t\}_{t=0}^\infty \) that satisfy the conditions (i) or (ii), define the **socially optimal allocation** as the path for consumption, fossil fuel, sequestration and carbon level, \( \{c_t^*, k_t^*, z_t^*, S_t^*\}_{t=0}^\infty \), that solves the social planning problem.
It is useful to define $\hat{\beta}' q_t^*$ and $\hat{\beta}' p_t^*$ as the social (shadow) cost of carbon and the social value of fossil fuel, respectively. Formally, the first one corresponds to the Lagrange multiplier on the carbon cycle constraint (3) and the second to the multiplier on the resource constraint (2).

The social cost of carbon is at the center of all models of climate change so it is important to understand the basic economics behind this concept. At the optimal allocation, the first order condition with respect to $S_{t+1}^*$ gives

$$q_t^* = \hat{\beta} (1 - \gamma) q_{t+1}^* + x'(S_{t+1}^*)$$

(9)

The social cost of carbon is derived from solving this equation recursively and it is equal to

$$q_t^* = \sum_{j=1}^{\infty} [\hat{\beta} (1 - \gamma)]^{j-1} x'(S_{t+j}^*)$$

(10)

Essentially, it measures the marginal damage of increasing carbon emission in an extra unit. The fact that it is given by the discounted sum of damages originated from that extra unit over time highlights the dynamic nature of the externality. Moreover, it also highlights the reason why discounting is a controversial issue in economics of climate change. The discount factor directly affects the social value of the externality and, hence, the desirability of any policy aimed at controlling it. Since it will be useful to derive the results, I will denote $\mu_t^*$ the social cost of carbon expressed in terms of utils. That is

$$\mu_t^* \equiv \frac{q_t^*}{u'(c_t^*)}$$

(11)

The other optimality conditions are standard. The marginal cost of removing carbon emissions through sequestration must be equalized to its social value which is the accumulated benefits from having one less unit of carbon in the atmosphere forever after

$$v'(z_t^*) = q_t^*$$

(12)

The social value of fossil fuels is driven by a version of the Hotelling rule (Hotelling (1931)) reflecting the exhaustible nature of fossil fuels

$$\hat{\beta} (p_{t+1}^* - q_{t+1}^*) = p_t^* - q_t^*$$

(13)
The Hotelling rule prescribes that the price of an exhaustible resource must increase at the rate of discount. This ensures that the benefit from extracting the resource is equalized at all dates at which a positive amount of the good is extracted. Optimality implies that the benefits from extracting must be net from the associated climate damages. Equation (13) combined with the first order condition with respect to consumption and sequestration delivers the Euler equation for this economy

$$\hat{\beta}[u'(c^*_t) - v'(z^*_t)] = u'(c^*_t) - v'(z^*_t)$$

(14)

The Euler equation simply states that it is impossible to increase total welfare by moving consumption across generations. Using (10) and (12), the Euler equation can be written in terms of environmental damages in the following way

$$\hat{\beta}[u'(c^*_t) - \sum_{j=1}^{\infty} [\hat{\beta}(1-\gamma))]^{j-1}x'(S^*_{t+j})] = u'(c^*_t) - \sum_{j=1}^{\infty} [\hat{\beta}(1-\gamma))]^{j-1}x'(S^*_{t+j})$$

(15)

It becomes clear that leaving fossil fuels for future generations not only gives them the possibility of enjoying higher consumption but also endow them with extra power to pollute. Hence, delaying consumption also means delaying environmental damages. The fact that damages are cumulative, in the sense that they derive from the stock (and not the flow) of carbon, implies that the gains from delaying consumption are typically higher than the costs. This is because the damages, if occur later, are spread over less periods of time. Note however, that discounting directly counteracts this force: the lower the discount factor the less important these gains from delaying carbon emissions become.

The social cost of carbon (10), the first order condition with respect to sequestration (12) and the Euler equation (14) together with the feasibility constraint (2) and the carbon cycle (3) fully characterize the socially optimal allocation.

The two key decisions society face are how quickly oil reserves are used up and how much effort each generation should devote to mitigate the associated externality. Since individuals are small to be able to affect the aggregate level of carbon, they will typically get both decisions wrong: consume too fast and fail to make the efficient level of effort on mitigation. A role for policy intervention arises.
4 Market Economy

The social planner chooses allocations taking into account that both consumption of fossil fuels and mitigation efforts affect the level of carbon in the atmosphere. Instead, households and firms in a decentralized economy are small and can not individually influence the level of carbon. For this reason, they fail to take into account the externality and the equilibrium allocation that arises in a market economy will typically not coincide with the planner’s one. Nevertheless, we say that the social planner’s allocation is implementable if we can find policies and equilibrium prices such that the allocation and prices are a competitive equilibrium given these policies.

There are two instruments designed to control carbon emissions that deserve special attention: carbon taxes and a cap and trade system. The first is a price-based policy while the second is a quantity control. From a theoretical point of view, both instruments are equivalent in the sense that they are equally capable of implementing the desired allocation. Motivated by this well known result, this section proposes two alternative market economies: one competitive economy with carbon taxes and a second economy with a cap and trade system. In the next section, I use these two decentralized environments to derive the main results of the paper.

4.1 Carbon Taxes

Consider the following market economy. A continuum of mass one firms, or a representative firm, operates a linear technology \( f(y_t) = y_t \) to produce the consumption good. Firms own the economy’s stock of fossil fuel and use it as an input for production. Extraction of fossil fuels increases emissions on a one to one basis. However, emissions can be offset if firms buy sequestration services from households at a market price \( w_t \). Firms face two forms of taxation: a carbon tax, \( \tau^k_t \), on emissions and a tax credit, \( \tau^z_t \), on sequestration. Taxes are defined in units of the consumption good. Hence, per period profits of the firm are given by

\[
\pi_t = p_t(k_t - k_{t+1}) - w_t z_t^d - p_t \tau^k_t(k_t - k_{t+1}) + p_t \tau^z_t z_t^d
\]
The problem of the firm is to choose a sequence \( \{k_t, z_t^d\}_{t=0}^{\infty} \) in order to maximize discounted profits given by

\[ \Pi_0(k_0) = \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t} \frac{1}{R_s} \right) \pi_t \]  

with the initial stock of fossil fuel \( k_0 \) given. All profits are rebated to households as dividends.

There is a continuum of mass one households who derive utility from consumption of the single good in the economy and incur in a disutility cost when they provide sequestration services \( z_t \). Households dislike carbon in the atmosphere but they are small to individually control its level. They save in a risk free asset, \( b_{t+1} \geq 0 \) which bears a gross one-period rate of return \( R_t > 1 \). The asset can be thought of as bequests from individuals in generation \( t \) to the ones in the next generation. Households consume, provide sequestration services and save subject to the following set of budget constraints for \( t = 0, 1, 2, ... \)

\[ p_t c_t + b_{t+1} = w_t z_t + R_t b_t + T_t + \pi_t \]  

where \( p_t \) is the price of the consumption good at date \( t \) in units of a numeraire, \( T_t \) represents a lump sum rebate from the government, \( \pi_t \) are the profits received from the firm and initially \( b_0 = 0 \). The problem of the households is to choose a sequence \( \{c_t, z_t, b_t\}_{t=0}^{\infty} \) in order to maximize (5) subject to (17), taking prices and taxes as given.

A government collects carbon taxes and pays subsidies. Any surplus (or deficit) is rebated in a lump-sum transfer to households. The sequence of government’s budget constraint for \( t = 0, 1, ... \) is given by

\[ p_t \tau_t^h (k_t - k_{t+1}) - p_t \tau_t^z z_t^d = T_t \]  

Finally, market clearing for this economy requires

\[ c_t = k_t - k_{t+1} \]  

\[ z_t^d = z_t \]  

\[ b_t = 0 \]  

for every period \( t \).
A competitive equilibrium with taxes \( \{ \tau^k_t, \tau^z_t, T_t \}_{t=0}^{\infty} \) is a sequence of prices \( \{ p_t, w_t, R_t \}_{t=0}^{\infty} \) and allocations \( \{ c_t, z_t, z^d_t, k_t, b_t \}_{t=0}^{\infty} \) such that: (i) given taxes and prices, the allocation solves the consumer’s problem maximizing (5) subject to (17), and the firm’s problem maximizing (16), (ii) given the allocation, transfers are such that the government budget constraints (18) is satisfied and (iii) prices clear the markets (19)-(21).

I characterize next a competitive equilibrium with taxes. Firms optimizing behavior delivers the following condition on prices

\[
\frac{p_{t+1}(1 - \tau^k_{t+1})}{p_t(1 - \tau^k_t)} = R_{t+1}
\]

which is the market version of the Hotelling rule: the price of an exhaustible resource must increase at the rate of interest. In this economy, the Hotelling rule applies for prices net of tax payments. This rule ensures that firms are indifferent between extracting the resource at any date as long as they make the same profits in present value terms. In addition, firms are indifferent between buying sequestration services from households or not if the price they pay for these equals the tax credit they then receive from the government, \( \tau^z_t = \frac{w_t}{p_t} \).

The household’s problem is characterized by two relevant margins: the intratemporal decision between consumption and sequestration and the intertemporal choice regarding how much to bequeath to the next generation. Combining the first order conditions with respect to sequestration and consumption delivers the following necessary equilibrium condition for an interior solution

\[
\frac{v'(z_t)}{u'(c_t)} = \frac{w_t}{p_t}
\]

This condition is standard and says that consumers will decide on consumption and sequestration in order to equalize marginal utilities to the ratio of prices which, from the firm’s problem, is equal to the sequestration tax credit. It is clear that, absent any tax policy, neither firms nor households have incentives to do sequestration. In fact, as will become clear in the section No Policy Intervention, sequestration would be zero in this economy. Once the government sets a price for this missing market, households are willing to offer these services since it is a source of income that allows them to consume. And firms are indifferent between paying for these services or not as long as they are compensated through a tax return.
There is another way of thinking about this condition which is: sequestration subsidies allow the government to control how much of the reduction in carbon emissions comes from ‘abatement’ as opposed to ‘mitigation’. In general, abatement refers to a direct reduction in emissions, which in this economy occurs through a decrease in consumption (or equivalently, a reduction in the extraction of fossil fuels). Instead, mitigation corresponds to actions that offset the emissions that have already been released (sequestration in this economy).

Combining the first order condition with respect to consumption for two subsequent periods delivers the following Euler equation for this economy

\[
\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1 - \tau_k^t}{1 - \tau_k^{t+1}}
\]

(24)

where I have used the Hotelling rule from the firm to substitute away the interest rate. The Euler equation rules the rate at which fossil fuels are eaten up over time. Note that in an economy without carbon taxes, consumers would be willing to equalize the marginal utility of consumption in present value terms. That is, the marginal utility of consumption would grow at the discount rate. This reflects the exhaustibility nature of the resource and it is a version of the Hotelling rule from the perspective of the consumer.

The intratemporal condition (23) and the Euler equation (24), together with the condition for prices in the firm’s problem, the budget constraint of the government (18) and the market clearing conditions (19)-(20) for every period \( t \) fully characterize a competitive equilibrium with taxes.

The next subsection proposes a cap and trade system as an alternative competitive environment.

### 4.2 Cap and Trade System

Consider the following alternative market economy. The demographics, preferences and technologies are identical to the previous setup. Only the policy instruments differ. Instead of carbon taxes, the government introduces a cap and trade system which sets a cap on net emissions of carbon \( \{\theta_t\}_{t=0}^\infty \) together with a cap on sequestration \( \{\phi_t\}_{t=0}^\infty \). The reason why the introduction of two caps is necessary, as opposed to only one cap on the net emissions
of carbon, is at the heart of this paper and will become clear in the section Optimal Policy. The government endows households with both carbon permits and sequestration licenses. Sequestration licenses are authorizations to sell sequestration services which firms can count as offsets of their emissions. If a firm wants to use sequestration as a reduction in its emissions, then it must buy sequestration from the authorized households. Households and firms are then allowed to trade carbon permits and sequestration rights at market prices $\hat{\tau}_t$ and $\hat{\tau}_t^z$, respectively.

In order to produce one unit of output, a firm must purchase one carbon permit. Alternatively, the firm can buy sequestration from authorized households and offset the emission that releases through production so that net emissions are zero. The possibility of offsetting emissions is available as long as sequestration licenses have not been exhausted. Hence, firms face the following constraints for every period $t$

$$\theta_t^d \geq k_t - k_{t+1} - z_t^d$$

(25)

$$\phi_t^d \geq z_t^d$$

(26)

where $\theta_t^d$ and $\phi_t^d$ corresponds to firm’s demand of carbon permits and sequestration rights, respectively. Per period profits of the firm are given by

$$\pi_t = p_t(k_t - k_{t+1}) - w_t z_t^d - \hat{\tau}_t \theta^d_t - \hat{\tau}_t^z \phi^d_t$$

(27)

The problem of the firm is to choose a sequence $\{k_t, z_t^d, \theta_t^d, \phi_t^d\}_{t=0}^{\infty}$ in order to maximize discounted profits $\sum_{t=0}^{\infty} (\prod_{s=0}^{t-1} \frac{1}{R_s}) \pi_t$ subject to the constraints (25) and (26). All profits are rebated to households as dividends.

Households provide sequestration services at a competitive price $w_t$. Carbon permits and licenses can not be stored. Households consume, provide sequestration services and bequeath wealth subject to the following set of budget constraints for $t = 0, 1, 2, ...$

$$p_t c_t + b_{t+1} = w_t z_t + R_t b_t + \hat{\tau}_t \theta_t + \hat{\tau}_t^z \phi_t + \pi_t$$

(28)

where prices and taxes are defined in units of a numeraire and $\pi_t$ are the profits received from the firm. The problem of the households is to choose a sequence $\{c_t, z_t, b_t\}_{t=0}^{\infty}$ in order to maximize (5) subject to (28).
Finally, market clearing for this economy requires that the following conditions must be satisfied for every period $t$

$$c_t = k_t - k_{t+1} \quad (29)$$
$$b_t = 0 \quad (30)$$
$$z_t = z_t^d \quad (31)$$
$$\theta_t = \theta_t^d \quad (32)$$
$$\phi_t = \phi_t^d \quad (33)$$

The last three equations represent the markets for carbon emission and sequestration permits.

A competitive equilibrium with a cap and trade system $\{\theta_t, \phi_t\}_{t=0}^\infty$ is a sequence of prices $\{p_t, w_t, R_t, \hat{\tau}_t, \hat{\tau}^z_t\}_{t=0}^\infty$ and allocations $\{c_t, z_t, k_t, b_t, z_t^d, \theta_t^d, \phi_t^d\}_{t=0}^\infty$ such that: (i) given prices and the caps, the allocation solves the consumer’s problem maximizing (5) subject to (28), and the firm’s problem maximizing (43) subject to (25), (ii) given the allocation, prices clear the markets (29)-(33).

I characterize next a competitive equilibrium with a cap and trade system. It is easy to see that the equilibrium conditions display prices $\hat{\tau}_t$ and $\hat{\tau}^z_t$ where taxes were before. Firms optimizing behavior delivers a version of the Hotelling rule for this economy:

$$\frac{p_{t+1} - \hat{\tau}_{t+1}}{p_t - \hat{\tau}_t} = R_{t+1} \quad (34)$$

which states that in order for firms to be indifferent between extracting at any date, the price of the exhaustible resource (net of payments for carbon permits) must grow at the rate of interest. Further, the equilibrium price on sequestration services is given by $w_t = \hat{\tau}_t - \hat{\tau}^z_t$. This equilibrium condition highlights the essence of pricing carbon emissions. Once individuals face a price on an otherwise unpriced good (carbon emissions), markets that were missing arise. Without the introduction of a cap and trade system, firms would not pay for sequestration in this economy. With the carbon trading scheme, firms pay for sequestration because it is a way to ‘save’ on carbon permits. A firm that wants to produce a unit of output can avoid paying for a carbon permit by paying for sequestration of the resulting emission.
And as long as the price of the sequestration permit is lower than the price of a carbon permit, the firm is indifferent between paying for any positive amount of sequestration.

Household’s intratemporal equilibrium condition is given by

$$
\frac{v'(z_t)}{w'(c_t)} = \frac{w_t}{p_t}
$$

Again, consumers choose between consumption and sequestration in order to equalize the marginal utilities to the ratio of prices. The optimality condition of the firm can be used to substitute away the wage. Finally, the intertemporal choice regarding how much fossil fuel bequeath to the next generation is ruled by the following Euler condition

$$
\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1 - \tau_t/p_t}{1 - \tau_{t+1}/p_{t+1}}
$$

which is the same as in the previous setup. The price of carbon permits in real terms plays here the role that the tax on carbon emissions had in the tax economy. The intratemporal condition (35) and the Euler equation (36), together with the conditions for prices in the firm’s problem and the market clearing conditions (29)-(32) for every period $t$ fully characterize a competitive equilibrium with a cap and trade system.

5 Optimal Policy

This section presents the main results of the paper: when the welfare of future generations is explicitly considered in climate change analysis, standard policies are insufficient to control carbon emissions. More sophisticated ones are required to achieve optimality. I build towards the main results in the following way: First, the subsection No Policy Intervention studies the business as usual (or laissez faire) economy as a benchmark case. Next, the subsection Standard Policies shows that in the special case in which only the first generation receives positive weight in the social welfare, so that private and social discounting coincide, standard policies such as carbon taxes and caps on net emissions are optimal. Finally, the subsection Sophisticated Policies characterizes optimal policies when the social welfare function assigns direct weights on future unborn generations, inducing a lower social discount rate. It shows
that the optimal policy involves carbon taxes on emissions which are different from carbon subsidies on sequestration. It also shows that an optimal cap and trade system does not work unless it specifies a cap on sequestration licenses.

5.1 No Policy Intervention

Consider the laissez-faire (or business-as-usual) equilibrium in the economies described in the previous section. The next proposition is straightforward.

Proposition 2 Let $\tau^k_t = \tau^z_t = 0 \forall t$ in the tax economy. Then the competitive equilibrium allocation is not optimal. Alternatively, suppose there is no cap and trade system. Then the competitive equilibrium allocation is not optimal.

Without policy intervention, the economy collapses to a simple version of a cake-eating problem. Importantly, $z_t = 0, \forall t$. No mitigation of carbon emissions takes place in the economy since households derive disutility from it and get no payments in exchange. The market for sequestration is missing because it does not exist a price on carbon emissions.

The only relevant decision households face is how much fossil fuel to hand on to the next generation. The intertemporal necessary condition for an interior solution is given by

$$\beta u'(c_{t+1}) = u'(c_t)$$

which is a version of the Hotelling rule from the perspective of the consumers: The marginal utility of consumption of an exhaustible good grows over time at the discount rate $1/\beta$. It simply means that the marginal utility cost for generation $t$ of reducing consumption by $\epsilon$ equals the marginal utility gain the subsequent generation gets from consuming the extra $\epsilon$ of fossil fuels (which weights $\beta$ on the current’s generation utility). If the Euler equation holds, then it is impossible to increase utility by moving consumption across generations.

By comparing (37) with (14), it follows that the rate of depletion of fossil fuels is not optimal in the laissez faire economy. There are two reasons for this. First, individuals do not take into account that, associated to the utility gains and costs derived from moving consumption across generations, there are gains and costs from the environmental damages that
consumption generates. Second, when the social welfare criterion induces a social discount factor that differs from the private one, then discounting becomes an extra reason why the rate of depletion in the laissez faire economy differs from the optimal one. This is because the next generation’s utility gain from inheriting an extra unit of fossil fuels weights $\beta$ on the current’s generation utility but it is worth $\hat{\beta}$ from a social point of view.

The remaining of this section studies optimal environmental policies which are designed to achieve the socially optimal allocation as the equilibrium outcome of a decentralized market. In order to do so, optimal policies need to correct for the environmental externality. In this economy, optimal policies operate through two channels: correcting the rate of depletion of fossil fuels (abatement) and inducing the efficient level of sequestration (mitigation).

5.2 Standard Policies

Consider a social welfare criterion which values the welfare of future generations only through the altruism of the current ones. The welfare weights satisfy condition ($i$) in section 3 and the social discount factor coincides then with that of private individuals: $\hat{\beta} = \beta$. The social planning problem corresponds to that of a representative infinitely lived individual which is the problem solved in many positive studies, as in Nordhaus and Boyer (2003).

Characterizing optimal policies consists on finding a set of instruments that makes the equilibrium conditions in the market economy coincide with the optimality conditions in the social planning problem. The following proposition states one of the results of the paper: when social and private discounting coincide, it is sufficient to control the net emissions of carbon. The proof is relegated to the Appendix.

**Proposition 3 (Standard Policies)** Suppose that the social welfare weights are defined by condition ($i$) so that $\beta = \hat{\beta}$. If $\{c^*_t, z^*_t, k^*_t, S^*_t\}_{t=0}^{\infty}$ solves the social planning problem, then it solves the competitive equilibrium with taxes $\{\tau^k_t, \tau^z_t, T_t\}_{t=0}^{\infty}$ defined by

$$\tau^k_t = \tau^z_t = \mu^*_t$$

for every period $t$ and all proceeds from taxation rebated/financed lump-sum through $T_t$. Alternatively, it solves the competitive equilibrium with a cap and trade system with a cap on
net emissions \( \{ \theta_t \}_{t=0}^{\infty} \) defined by

\[
\theta_t = k^*_t - k^*_{t+1} - z^*_t
\]

for every period \( t \) and no cap on sequestration \( \{ \phi_t \}_{t=0}^{\infty} \).

The proposition highlights some basic principles of pricing carbon emissions. Optimal carbon taxes and subsidies in a decentralized economy must be set equal to the social cost of carbon in the planning problem, \( \mu^*_t \). This result is often used in the literature as a shortcut to characterize optimal taxes. In particular, it is enough to solve for the path of carbon emissions in a social planner’s problem and then argue that the carbon tax equals the shadow value of carbon in that problem. There is no special need to work out the details of a market economy. The next subsection shows that this shortcut is no longer available when social discounting differs from the private one.

By condition (11), the social cost of carbon is equal to

\[
\mu^*_t = \sum_{j=1}^{\infty} \beta (1 - \gamma)^{j-1} x'(S^*_t + j) u'[c^*_t]
\]

At any point in time, taxes and subsidies are equal to the discounted sum of future marginal damages from carbon emissions, expressed in units of fossil fuel consumption. This tax formula highlights the dynamic structure of the externality: carbon emissions are cumulative and add to a stock that only depreciates at a rate lower than one. Set in this way, taxes induce the current generation to pay for emissions an amount equal to the present discounted value of marginal costs imposed on the future ones. In the same way, the current generation is rewarded for removing emissions an amount equal to the present discounted value of benefits created on the future generations. It follows that taxes are sensitive to the discount factor and, for that reason, this last one has been the subject of much controversy.

In an economy with a cap and trade system, if the government sets carbon emissions cap to those which correspond to the carbon emitted in the optimal allocation, the social value of carbon \( \mu^*_t \) arises as the equilibrium price of carbon permits. To see this, note that equation (24) evaluated at the star allocation together with the equilibrium condition for
prices \( \hat{\tau}_t - \hat{\tau}^z_t = \hat{w}_t \) implies that the equilibrium price of carbon permits satisfies

\[
\hat{\tau}_t = \frac{v'(z^*_t)}{u'(c^*_t)} = \mu_t^*
\]

(39)

where the second equality comes from (12). Since there is no cap on sequestration, \( \tau^z_t \) is zero. In order to burn fossil fuels, firms must either buy permits to pollute or pay households for sequestration. The equilibrium price of fossil fuels reflects this cost and hence consumers internalize it. As a consequence, the optimal path of sequestration and fossil fuel consumption is implemented.

Hence, when social and private discount rates coincide, the social cost of carbon is the outcome of either a carbon tax policy or a cap and trade system. The optimal allocation can be implemented by either one of the two systems. This is a well-known equivalence result. It implies in this economy that it is enough to control the net emissions of carbon (emissions net of sequestration). The next subsection shows that when future generations are directly valued in the societal welfare criterion, controlling the net emissions of carbon is not sufficient to achieve optimality.

It is important to notice that the (after-tax) price of fossil fuels in the decentralized economy follows the Hotelling rule. When social and private discounting coincide, this is optimal since it is consistent with condition (13) in the planning problem. It means that, once corrected for climate damages, individual agents deplete fossil fuels at the optimal rate. This will no longer be the case in the next subsection. The Hotelling rule is at the heart of the results.

### 5.3 Sophisticated Policies

This section presents the main results of the paper in Propositions 5 and 7, and their corollaries. The results share the same basic intuition: divergence in discounting introduces an extra reason for policy intervention. Even in the absence of the climate externality, the social optimal allocation implies a path of extraction of fossil fuels, sequestration and carbon in the atmosphere that differs from the one that would be chosen by the individuals acting in the economy. A more patient planner wants to implement a slower rate of extraction of fossil
fuels which adds to the already existing need to slowing down extraction in order to reduce emissions.

I first show in the next proposition that the type of policies that typically work to control carbon emissions, as the ones described in the previous subsection, are not optimal when social and private discounting differ. In particular, it is not sufficient to control the net emissions of carbon. A social planner that directly cares about future generations wants to treat differently the gross emissions of carbon (which increase carbon in the atmosphere) from the sequestration of emissions (which decrease it).

**Proposition 4 (Insufficiency of Standard Policies)** Suppose the social welfare weights are defined by condition (ii) so that \( \beta < \hat{\beta} \). Assume further that the policy instruments are restricted to have the form \( \tau^k_t = \tau^z_t \) \( \forall t \) in the tax economy or to have no cap on sequestration in the cap and trade system. Then the competitive equilibrium allocation is not optimal.

There are two ways of delivering the optimal level of carbon to the next generation. One requires the current generation to work more; the other to consume less. Even though both are equivalent in terms of the amount of the externality the future generation inherits (i.e. the level of carbon), there is a higher social return in doing so through less consumption. It implies future generations will not only inherit less carbon but also more reserves of fossil fuels, which they value. Moreover, society cares that they value them in a way that each individual parent in the current generation does not internalize through their altruism. In this sense, a social discount factor which is higher than the private one due to intergenerational equity reasons acts as a new externality which optimal policies need to correct for. A basic principle in public finance is that optimal policies require to have as many instruments as margins are to be corrected. Standard policies provide only one instrument. An extra lever is missing if climate policies are meant to deal not only with the environmental externality itself but also with intergenerational equity.

The following proposition presents one of the main results of this paper: when social and private discounting differ, an optimal tax scheme requires subsidizing the mitigation technology with a carbon tax credit but taxing emissions at a rate different than that implied
by simply imposing a carbon tax. The proof is relegated to the Appendix.

**Proposition 5 (Optimal Taxes)** Suppose that the social welfare weights are defined by condition (ii) so that \( \beta < \hat{\beta} \). If \( \{c^*_t, z^*_t, k^*_t, S^*_t\}_{t=0}^{\infty} \) solves the social planning problem, then it solves the competitive equilibrium with taxes \( \{\tau^k_t, \tau^z_t, T_t\}_{t=0}^{\infty} \) defined as follows

\[
1 - \tau^k_t = \left( \frac{\hat{\beta}}{\beta} \right)^t (1 - \mu^*_t) \quad ; \quad \tau^z_t = \mu^*_t
\]

for every period \( t \) and all proceeds from taxation rebated/financed lump-sum through \( T_t \).

The optimal tax on emissions is not equal to the value of the externality those emissions generate. Divergence in discounting shows up as an extra term in the tax formula. Optimal taxes are now a function not only of the climate externality, \( \mu^*_t \), but also of the ratio of the discount factors, \( \frac{\hat{\beta}}{\beta} \). Moreover, note that a tax on emissions is optimal even absent a climate externality. If \( \mu^*_t \) is equal to zero in the tax formula, the optimal tax is equal to \( \left( \frac{\hat{\beta}}{\beta} \right)^t \). There is a subtle but important point that I want to make with this proposition. The optimal policy distinguishes between the private and the social discount factor, which contains the weighting scheme, and depends on both. Introducing a high social discount factor in climate change analysis is not a relabeling of the social planning problem which is just solved with a higher discount factor. In order to be able to characterize optimal carbon policies, it is necessary to specify the social weighting scheme attached to future generations and differentiate it from the private discount factor. Only then optimal policies can be specified.

The social planning problem where future generations receive direct weight is essentially different from the one in which they do not. Hence, it requires different policies. In particular, the social value of carbon is no longer a sufficient indicator of the price that should be charged for carbon emissions. Therefore, there is no short cut to setting a carbon tax. In particular, it is not enough to solve for the path of carbon emissions that arise in a social planner’s problem and argue that the carbon tax equals the shadow cost of carbon in that problem. When social and private discounting differ due to intergenerational equity considerations, particular attention need to be posed on the decentralized environment in which climate policies are meant to implement the optimal path of carbon.
One drawback of the tax scheme in Proposition 5 is that the tax rate on emissions diverge. However, note that it is only the ratio of taxes which enters into the intertemporal condition (24) and this ratio does not diverge. Nevertheless, a basic principle in public finance is that there are typically many instruments that can decentralize an optimal allocation. The next result presents, as a Corollary to Proposition (5), an alternative decentralization which involve the combination of standard policies with a subsidy on fossil fuel reserves. Moreover, the tax rate does not diverge.

**Corollary 6 (A Subsidy on Oil Reserves)** The optimal allocation \( \{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty} \) can also be decentralized with standard policies and a subsidy \( \tau_t^s \) on oil reserves defined by

\[
\tau_t^k = \tau_t^z = \mu_t^*; \quad \tau_t^s = (\frac{\beta}{\beta} - 1)(1 - \mu_t^*)
\]

If the tax on emissions is set as a carbon tax (i.e. a tax equal to the value of the externality), then the optimal tax scheme requires subsidizing firms so that they keep more fossil fuels underground. Per period profits of the firm are given by

\[
\pi_t = p_t(k_t - k_{t+1}) - w_t z_t^d - p_t \tau_t^k (k_t - k_{t+1}) + p_t \tau_t^z z_t^d + p_t \tau_t^s k_t
\]

The tax credit on fossil fuels, \( \tau_t^s \), is composed of two terms: a time-varying one that reflects the value of the externality and a constant one that recovers the divergence in discounting. The intuition for this result is similar to the one before. When social and private discounting differ, there is an extra return on achieving a given level of carbon through less extraction, as opposed to more sequestration. Firms in the competitive equilibrium are indifferent between extracting fossil fuels at any date as long as they get the same return per unit of extraction. With a linear technology and a carbon tax on emissions, the return of keeping an extra unit of oil underground is equal to one minus the tax, which equals \( \mu_t^* \). That is, the marginal productivity of oil net of taxes. The path of extraction is then corrected to reflect the climate externality. On top of that, a subsidy \( \frac{\beta}{\beta} - 1 > 1 \) adds the extra return on oil underground that is missing. Note that, absent a climate externality, a constant subsidy on fossil fuels reserves would still be optimal.
The need of introducing a subsidy on oil reserves was also proposed in Sinn (2008), although for different reasons. In that paper, firms deplete fossil fuels too quickly due to weak property rights. As in this paper, a subsidy on the stock of oil is then required to provide incentives for firms to postpone extraction. This result also resembles that in Farhi and Werning (2010), although they work in a different environment. They find that, when the welfare of future generations is directly valued in social welfare, it is optimal to subsidize bequeaths.

The following proposition presents the second main result of the paper: if sequestration can be used to meet compliance obligations, then an optimal cap and trade system must include a cap on sequestration.

**Proposition 7 (Optimal Cap and Trade System)** Suppose that the social welfare weights are defined by condition (ii) so that $\beta < \hat{\beta}$. If $\{c^*_t, z^*_t, k^*_t, S^*_t\}_{t=0}^\infty$ solves the social planning problem, then it solves the competitive equilibrium with a cap and trade system that includes a cap on net emissions and on sequestration defined by

$$\theta_t = k^*_t - k^*_{t+1} - z^*_t$$

$$\phi_t = z^*_t$$

The intuition for this result comes from the fact that, absent a cap on sequestration, firms will be burning oil too fast compared to the social optimum. Furthermore, since this implies too much emissions, firms will rely on sequestration to meet their compliance obligations. The market outcome of a standard cap and trade system with no limit on sequestration would have too much sequestration and too much extraction. Therefore, future generations would still inherit the optimal level of carbon (since this is controlled by the cap on net emissions) but would get too little oil. By setting a cap on the amount of offsets that firms can use, the government is indirectly setting a cap on fossil fuel extraction. The cap on sequestration is then the tool that is required to induce private agents to deliver not only the optimal level of carbon to the next generation but also the optimal stock of fossil fuels.

This is a policy that on the face of it seems a little perplexing. Removing carbon from the atmosphere is a social good and it is not straightforward that an optimal policy would set a
cap on it. However, this result resembles some of the policies that are currently in place. The European Union Emissions Trading Scheme (EU ETS) allows firms the use of compliance carbon credits up to a limit, which varies across member countries. California’s greenhouse gas (GHG) cap-and-trade program allows the use of offset credits to meet up to 8 percent of the firms’ triennial compliance obligation. The Regional Greenhouse Gas Initiative (RGGI) let regulated firms to use offsets to meet up to 3.3 percent of their compliance obligations. In all cases, carbon offsets must be authorized and meet some regulatory criteria. Another way to read the results in this paper is that they provide a rationale of why these types of caps on sequestration are optimal.

Finally, the next result presents an alternative cap and trade system.

**Corollary 8 (A floor on Sequestration)** Alternatively, the optimal allocation can be decentralized with a cap and trade system which sets a floor on sequestration and does not allow to use offsets credits as a compliance instrument. The optimal cap and floor are defined as

\[ \theta_t = k_t^* - k_{t+1}^* \]

\[ \zeta_t = z_t^* \]

With this alternative cap and trade system, firms face the following constraints for every period \( t \)

\[ \theta_t^d \geq k_t - k_{t+1} \]  
\[ z_t^d \geq \zeta_t^* \]

(41)  
\[ 42 \]

where \( \theta_t^d \) corresponds to the firm’s demand of carbon permits on gross emissions (“gross” since offsets are not allowed as a compliance tool) and \( \zeta_t^* \) is the floor on sequestration set by the government. Per period profits of the firm are given by

\[ \pi_t = p_t(k_t - k_{t+1}) - w_t z_t^d - \hat{\gamma}_t \theta_t^d \]

(43)

In this economy, firms undertake sequestration of carbon emissions only if they obtain some policy-related benefit. Either they receive a tax credit for it or they can use it as source of
compliance in a cap and trade system. If none of these benefits are in place, then firms have no incentives to undertake sequestration. For this reason, a floor on sequestration is required in order to achieve the optimal level of emissions offsets in the market economy.

The main take-away of all the above results is that the standard way of designing price or quantity controls to correct an externality fails to implement the social optimum when this is computed assuming social preferences differ from private preferences. In this paper, the difference arises from a social welfare function that places direct Pareto weight to future unborn generations but the main idea goes through whatever the motivation for having a social discount factor different from the private one. The characterization of the optimal instruments is specific to the model I have setup, which is parsimonious enough to allow me to derive closed form expressions for the optimal policies. However, the main substantive conclusion of the paper, which is that policies have to be designed taking into account future generations, is likely to remain in a more complex model in which additional more realistic features are added.

6 Numerical Example

This section presents an example economy. The objective of this numerical exercise is to illustrate the main features of the optimal policies as well as of the path for sequestration, consumption and the stocks of carbon and fossil fuels that solve the social planning problem. It was computed assuming the following functional forms and parameters: $\hat{\beta} = 0.98$, $\beta = 0.96$, $U(c, z, S) = \log(c) - \phi_z z^2 - \phi_s S^2$, $\phi_z = 500$, $\phi_s = 1$, $\gamma = 0.001$, $k \in [0, 1]$, $S \in [0, 1]$, $k_0 = 1$, $S_0 = .3$.

The picture below shows the time path for consumption and sequestration and for the stocks of carbon and fossil fuels. The red line corresponds to the laissez faire solution and the blue and green lines to the social optimal allocation for different assumptions on discounting. The picture shows that fossil fuels are depleted in all three cases. Depletion occurs at a faster rate in the laissez faire economy. Since fossil fuels are an exhaustible resource, a faster depletion implies that without policy intervention the consumption of later generations is too
low. In addition, the laissez faire economy exhibits no sequestration and a too high level of carbon.

Figure 1: Social Planner’s allocation vs Laissez Faire

The picture also shows that a lower discount rate (high discount factor) requires a slower rate of depletion of fossil fuels (the green line in the upper right panel is flatter than the blue one). When future generations are directly valued in social welfare then, they inherit higher reserves of fossil fuels and enjoy a higher consumption. This feature of the optimal allocation is critical and explain the main results of the paper: When society values future generations welfare directly, there is an extra value in delivering a low level of carbon through less consumption rather than through less sequestration (even though both options are technologically equally efficient for doing so). This is reflected in the optimal allocation in a
flatter path of consumption and a lower sequestration, compared to the solution when future generations are only value through the altruism of the current one. The path of sequestration in this last case (the blue line in the bottom right panel) has a flavor of the “ramp-up” solution found by Nordhaus and Boyer (2003). They prescribe a climate-policy ramp with mitigation efforts back-loaded on latter generations, which is what the optimal allocation with equal social and private discounting displays in my model.

The next figure illustrates one of the main results of the paper. The upper panel shows the optimal tax on emissions and the tax credit on sequestration that correspond to Proposition 5. The lower panel shows the optimal tax on emissions and the tax credit on sequestration and on fossil fuel reserves that correspond to Corollary 6.

In the upper right panel, the tax credit on sequestration is a carbon tax credit: It is equal
to the shadow cost of carbon. For this example, I backed out taxes by plugging the optimal path of \(\{c_t^*, z_t^*\}_{t=0}^{\infty}\) into the households’ intratemporal condition in the market economy with taxes. However, since the tax is equal to the shadow cost of carbon in the social planning problem, an alternative procedure is to compute it as the ratio of minus the derivative of the value function with respect to carbon to the derivative with respect to fossil fuel in the social planner’s problem. That is

\[
\mu_t^* = -\frac{V_s(k, S)}{V_k(k, S)}
\]

Both procedures deliver the same tax credit for sequestration. This is a shortcut often used in the literature which allows to characterize carbon taxes without specifying the decentralized environment.

However, one of the points of this paper is to show that this shortcut is no longer available when the planning problem is applying a social discount factor different from that of the individuals in the society. The tax on emissions in this case is not equal to the shadow cost of carbon. It is not simply a carbon tax. The optimal design of policies requires then to model the market economy explicitly. In the simple market economy I propose in this paper, the tax on emissions is equal to the blue line in the left upper panel, which is different from the carbon tax credit on sequestration. The lower panels show an alternative tax scheme in which the tax rate on emissions does not diverge (Corollary 6). In that decentralization, the tax on emissions and on sequestration is set equal to the shadow cost of carbon (lower right) and coupled with a tax credit on the stock of fossil fuels, as shown by the red line (lower left).

Finally, Figure 5 shows these taxes for different values of the social discount factor. When future generations are valued only through the altruism of the current one, then social and private discounting coincide and taxes correspond to the blue line in the picture. Note that the tax credit on the reserves of oil is zero. This means that standard carbon taxes are sufficient to implement the social optimum in this case. The blue line corresponds to a discount rate of approximately 4% (\(\hat{\beta}_1 = 0.96\)). Alternatively, when future generations receive direct weight in the social welfare, the social discount factor is higher than the private one
and taxes reflect this difference. As the welfare weights on future generations vary from zero to some positive numbers, the optimal taxes vary from the ones in blue to the ones in light blue. The light blue line corresponds to a discount rate of approximately 1% ($\hat{\beta}_4 = 0.99$).

Figure 3: Discounting and Optimal Taxes

The picture shows that a low discount rate actually implies lower taxes on emissions and tax credits on sequestration. The reason is that the optimal allocation with a low discount rate displays a lower level of carbon and taxes are equal to the shadow cost of carbon, which is then lower for a lower carbon. However, in order to achieve this low level of carbon, an optimal tax scheme has to include also a tax credit on the reserves of oil, as the ones displayed in the right panel. This subsidy is higher the lower it is the level of carbon that is desirable to implement. Or equivalently, the lower the discount rate that is applied to compute the social optimum.
7 A Time-varying Social Discount Rate

The theoretical results derived in the previous section are restricted to welfare weights that satisfy conditions (i) or (ii) and hence entail a constant social discount rate. When any other weighting scheme is considered, a time varying social discount rate arises. In the spirit of Strotz (1955), a time varying social discount rate means that the current planner values the welfare of two subsequent generations in the future different from how future ones will value them. For example, one can imagine a planner who cares about the current generation, their children and grandchildren, but relatively less about generations coming after them.

The solution to the social planning problem with a time-varying social discount rate is more complicated since the problem is subject to time inconsistency: Each planner can not count on future ones to follow a specific climate policy since it will not be optimal when evaluated from their point of view. Therefore, the social planning problem becomes a dynamic game among planners, where each planner chooses the climate policy today taking as given the policies that future ones will undertake.

Standard control theory is not appropriate to solve this problem and hence the rest of this section is necessarily more technical. However, the main results of the paper follow solely from considering a social welfare function that places direct Pareto weights on future generations and do not depend on these weights creating time varying social discounting or dynamic inconsistency. Some readers less interested in the technical methods required to deal with dynamic inconsistency may wish to skim over the rest of this section.

To keep the recursive structure of the problem, I restrict attention to a particular type of time-varying social discounting: Hyperbolic discounting. Hyperbolic social discounting implies that the social discount rate in the short run, $(\hat{\delta} \hat{\beta})^{-1}$ with $\hat{\delta} < 1$, is higher than in the long run, $(\hat{\beta})^{-1}$. In the Appendix A.2, I show that this social discount function arises from a weighting scheme which places direct weights on the current generation and their children, and weights start to decrease geometrically after them. Provided that the welfare weights are restricted to be positive for every generation, the social discount rate is still lower than the private one in every period. This is not straightforward to see but it is a result that comes
out clearly from the proof of the proposition in the appendix.

Hyperbolic discounting have been discussed in the literature, although not based on intergenerational equity reasons. Weitzman (2001) argues that the social discount rate used to discount intergenerational disutility damages from climate change should decline over time to reflect the overall divergence in opinions about its proper value (See also Hepburn et al. (2010) for a review of hyperbolic discounting with an application to environmental issues). As long as the social discount rate is different from the private one, whatever the motivation or the specification of the discount function, the results in this paper are likely to go through.

With a time inconsistent social planning problem, the notion of optimality becomes problematic in itself. I proceed as follows: I first discuss the solution to the social planning problem under the assumption that society counts with a commitment technology to overcome the time inconsistency problems. I then characterize the Markov Perfect (time consistent) solution to the planning problem when there is no commitment.

**Optimal Allocation with Commitment.** One way to deal with time inconsistency is for society to develop ways to commit itself to future actions (such as through laws or a constitution). I will not model these commitment technologies explicitly, but will simply assume for now that they exist. This assumption allows to solve for the optimal allocation with commitment. The characterization of the allocation with commitment at $t = 0$ coincides with the one without commitment, which I explain next. From $t = 1$ on, the characterization it coincides with the one discussed in section 3.1 for a planner with a constant discount rate. The optimal plan that arises from solving the dynamic game with commitment is represented by the red line in the pictures below.

**Constrained-Optimal Allocation.** If the political system is such that no commitment technology is available then, the social planning problem becomes one in which each planner decides how much fossil fuel and carbon to hand on to the next generation under the constraint that policies from tomorrow on must be taken as given. I refer to this allocation as the ‘Constrained optimal’ since it is the best outcome society can seek constrained by the fact that current generations can not commit future ones to continue with a specific climate policy.
For recursive structures the solution to this game can be characterized as a policy fixed point using standard recursive methods. I restrict attention to time consistent Markov perfect equilibrium. The current planner takes as given the policies of future planners, \( \varphi = \{ \varphi^k (k, S), \varphi^S (k, S) \} \), and chooses a pair of functions \( \{ k' = g^k (k, S; \varphi); S' = g^S (k, S; \varphi) \} \) in order to solve

\[
\max_{k', S'} \ u(c) - v(z) - x(S') + \delta \beta V(k', S')
\]

\( st \ c + k' = k \) \hspace{1cm} (44)

\[
S' = (1 - \gamma) S + k - k' - z \hspace{1cm} (45)
\]

\[
V(k, S) = u(\varphi^c (k, S)) - v(\varphi^z (k, S)) - x(\varphi^S (k, S)) + \delta \beta V(\varphi^k (k, S), \varphi^S (k, S)) \hspace{1cm} (46)
\]

where \( \varphi^c (k, S) = k - \varphi^k (k, S) \) and \( \varphi^z (k, S) = (1 - \gamma) S + \varphi^c (k, C) - \varphi^S (k, S) \). \( V \) is an indirect utility function that satisfies the functional equation (46). Note that this equation reflects that future planner’s decisions follow the policy rule \( \varphi \). It also incorporates that the current planner has a long run discount rate \( (\hat{\beta})^{-1} \) that differs from the short run one, \( (\delta \hat{\beta})^{-1} \).

The Markovian assumption is reflected in the policy functions being time independent and only a function of the current stock of fuel and carbon. The maximization problem can be thought of as a mapping function from the space of policy functions into itself. One can feed a policy function \( \varphi \) and the problem delivers a policy function \( g \). Solving for the equilibrium in this context involves looking for a policy fixed point of this mapping function.

The functions \( \{ g, \varphi \} \) correspond to a *time-consistent Markov Perfect Equilibrium* if

\[
g^k (k, S) = \varphi^k (k, S) \quad ; \quad g^S (k, S) = \varphi^S (k, S) \quad \forall (k, S)
\]

The *time-consistent allocation* is the path for consumption, fossil fuel, sequestration and carbon level, \( \{ c^*_t, k^*_t, z^*_t, S^*_t \}_{t=0}^{\infty} \), that satisfies \( k^*_t = g^k (k^*_{t-1}, S^*_{t-1}) \) and \( S^*_t = g^S (k^*_{t-1}, S^*_{t-1}) \) for every period \( t \), with initial \( k^*_0 \) and \( S^*_0 \) given and where \( c^*_t \) and \( z^*_t \) are recovered from (44) and (45).

The characterization of the time consistent allocation supposes the derivation of the fol-
ollowing two generalized Euler equations:

\[
\begin{align*}
  u'(c_t) - v'(z_t) &= \delta \beta [u'(c_{t+1}) - v'(z_{t+1})][1 + g^k_1(k_{t+1}, S_{t+1})(\frac{1}{\delta} - 1)] \\
  &\quad - \delta \beta [v'(z_{t+1}) - x'(S_{t+2})]g^k_1(k_{t+1}, S_{t+1})(\frac{1}{\delta} - 1) \\
  x'(S_{t+1}) - v'(z_t) &= -\delta \beta v'(z_{t+1})(1 - \gamma) + \delta \beta [u'(c_{t+1}) - v'(z_{t+1})]g^k_2(k_{t+1}, S_{t+1})(\frac{1}{\delta} - 1) \\
  &\quad - \delta \beta [v'(z_{t+1}) - x'(S_{t+2})]g^k_2(k_{t+1}, S_{t+1})(\frac{1}{\delta} - 1)
\end{align*}
\] (47) (48)

I have included the derivation of these generalized-Euler equations is in the appendix. These are the main behavioral equations in the planner’s problem. They constitute a system of functional equations in the policy rules \( g^k(k, S) \) and \( g^s(k, S) \). Note the presence of the derivatives of the policy functions. When \( \hat{\delta} = 1 \), the social discount rate is lower than the private one but constant, and then these derivatives disappear by the envelope theorem. They show up here to reflect that the current planner decides how much fossil fuel and carbon to pass on to the next one based on what he expects future planners will do, given that they will be evaluating the future using a different discount rate. In turn, what future planners do depends on how much fuel and carbon they inherit. This is the essence of the dynamic game between planners.

Hyperbolic discounting is often associated with the presence of impatience. In fact, for \( \delta < 1 \), there is a sense in which there is in this economy too much consumption and not enough sequestration. The current planner favors the present generation so they will be allowed to eat more and sequester less. But the next planner favors the next generation so they will be allowed to eat more and sequester less, and so on. This is the standard interpretation for hyperbolic preferences with a bias towards the present. However, since the current planner places direct weights also on future generations, the social discount rate is always lower than the private one (both in the short run and in the long run). Therefore, also with hyperbolic social discounting, all generations are required to consume less and sequester more, compared to the optimal allocation where future generations are only valued through the altruism of the current one (i.e. a planning problem with \( \hat{\beta} = \beta \)).
Numerical methods are needed to further characterize the constrained optimal allocation. **Numerical Solution.** The parametrization of this example economy is the same as in section 6, with \( \hat{\delta} = 0.98 \). It implies that the social discount rate is closer to the private one in the short run and decreases in the long run. The blue line corresponds to the allocation with commitment and the green line to the constrained optimal one. The two paths coincide at period zero. After that, the commitment solution displays lower extraction and relatively more sequestration for the about a century, which is a expected result since the current generation can commit future ones to do so. The current generation extracts relatively more than generations to come (there is a jump downwards, which is hardly seeable from the picture). Carbon in the atmosphere is overall lower when there is commitment given that

![Graphs showing allocation with and without commitment]

**Figure 4: Social Planner’s allocation with and w/o Commitment**

37
the emissions from consumption are lower and sequestration slightly higher in early periods.

The following picture delivers the main result from this section. It shows the optimal (blue line) and constrained optimal (green line) policies when the social discount rate is hyperbolic. The main take away from the picture is that the results, as stated in Corollary 6, go through: If the tax on emissions is set equal to the tax credit on sequestration, then the optimal tax scheme requires subsidizing firms so that they keep more fossil fuels underground.

Figure 5: Taxes and Subsidies

8 Conclusions

If we believe that, as advocated by Stern (2007), the problem of climate change involves not only the environmental damage itself but also a concern about intergenerational equity, then we need to start thinking about policy instruments designed to approach both sides of the same problem. This paper develops a simple model of climate change and shows that when future generations are directly valued in the social welfare, so that the social discount rate is lower than the private one, standard price or quantity controls are insufficient to implement
the social optimum.

A Appendix

A.1 Main Proofs

Proof of Proposition 2. The proof consists on showing that the equilibrium conditions do not coincide with the optimality conditions in the planning problem. Let $\tau^k_t = \tau^z_t = 0 \forall t$. Then, firms’ optimization implies that $w_t = 0$ and the equilibrium condition (23) becomes $v'(z_t) = 0$, while optimal sequestration satisfies (12). In addition, the equilibrium intertemporal condition is (37) which does not coincide with (14). In the cap and trade economy, if there is no caps then $\hat{\tau}_t = \hat{\tau}^z_t = 0$. The equilibrium condition (35) becomes $v'(z_t) = 0$, while optimal sequestration satisfies (12). In addition, the equilibrium intertemporal condition is (37) which does not coincide with (14). ■

Proof of Proposition 3. The proof consists on showing that all conditions for an equilibrium are satisfied by the efficient allocation $\{c^*_t, z^*_t, k^*_t, S^*_t\}_{t=0}^\infty$, when policy instruments are set optimally. I will refer to the optimal allocation and the star allocation indistinctly. The first part of the proof shows that all conditions for a competitive equilibrium with taxes are satisfied. It is useful to rewrite taxes in terms of allocations (instead of shadow prices) as follows

$$\tau^k_t = \tau^z_t = \frac{v'(z^*_t)}{u'(c^*_t)}$$

The intertemporal condition (24), evaluated at the star allocation, is

$$\frac{\beta u'(c^*_{t+1})}{u'(c^*_t)} = 1 - \frac{\tau^k_t}{1 - \tau^k_{t+1}}$$

Plug the optimal tax rate to get

$$\frac{\beta u'(c^*_{t+1})}{u'(c^*_t)} = 1 - \frac{v'(z^*_t)}{u'(c^*_t)} \frac{v'(z^*_{t+1})}{u'(c^*_{t+1})}$$
Rearranging terms we get

\[ \beta [u'(c_{t+1}^*) - v'(z_{t+1}^*)] = u'(c_t^*) - v'(z_t^*) \]

which is satisfied by the optimal allocation since it coincides with (14), given that \( \hat{\beta} = \beta \).

Consider now the equilibrium condition (23), together with the optimality condition for firms, evaluated at the star allocation

\[ \frac{u'(c_t^*)}{v'(z_t^*)} = \frac{1}{\tau_t^z} \]

This condition holds by definition of the optimal taxes. The market clearing condition for fossil fuel (19) is already satisfied by the efficient allocation. Given the sequence of taxes \( \{\tau_t^k, \tau_t^z\}_{t=0}^\infty \) transfers \( \{T_t\}_{t=0}^\infty \) are defined so that the budget constraint of the government (18) is satisfied. Plugging transfers and profits into the budget constraint of the consumer delivers \( b_{t+1} = R_t b_t \) for all \( t \), which together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial \( b_0 = 0 \).

Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm, condition (22). This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied by the optimal allocation.

The second part of the proof shows that, when caps on carbon emissions are set optimally, all conditions for a competitive equilibrium with a cap and trade system are satisfied by the optimal allocation. Since there is no cap on sequestration, \( \hat{\tau}^z_t = 0 \). The intratemporal condition (35) together with the condition on prices from the firm’s problem implies that the equilibrium price of permits on net emissions is given by

\[ \frac{v'(z_t)}{u'(c_t)} = \frac{\hat{\tau}_t}{p_t} \]

Plug it into the intertemporal condition (36) to get

\[ \frac{\beta u'(c_{t+1})}{u'(c_t)} = 1 - \frac{v'(z_t)}{u'(c_t)} \frac{1}{1 - \frac{v'(z_{t+1})}{u'(c_{t+1})}} \]

Rearranging terms and using we get

\[ \beta [u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t) \quad (49) \]
Given \( \{\theta_t\}_{t=0}^{\infty} \), the equilibrium sequence \( \{c_t, z_t\}_{t=0}^{\infty} \) must also satisfy

\[
\theta_t = c_t - z_t
\]  

(50)

Conditions (53) and (50) become a system of two equations with two unknowns: \( c_t, z_t \). Using (14), the definition of caps given in the Proposition and the condition \( \hat{\beta} = \beta \), the system (53) and (50) is satisfied by the optimal allocation. The market clearing condition for fossil fuel (19) is already satisfied by the efficient allocation. The budget constraint of the consumer delivers \( b_{t+1} = R_t b_t \) for all \( t \), which together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial \( b_0 = 0 \). Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm, condition (34). This completes the proof that all conditions for a competitive equilibrium with a cap and trade system are satisfied by the optimal allocation. ■

**Proof of Proposition 4.** It is sufficient to show that at least one of the conditions for an equilibrium is violated by the optimal allocation. In the tax economy, the intratemporal condition (23), together with the condition on prices from the firm’s problem and the restriction on taxes, can be rewritten as

\[
\frac{v'(z_t)}{u'(c_t)} = \tau_t^k
\]

Plug it into the intertemporal condition (24) to get

\[
\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1 - \frac{v'(z_t)}{u'(c_t)}}{1 - \frac{v'(z_{t+1})}{u'(c_{t+1})}}
\]

Rearranging terms we get that the competitive allocation satisfies

\[
\beta[u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t)
\]

(51)

But this condition is violated by the star allocation. The optimal allocation satisfies

\[
\hat{\beta}[u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t)
\]

(52)
for $\hat{\beta} > \beta$. It follows that the optimal allocation violates condition (24) for an equilibrium. Hence, the optimal allocation does not solve the competitive equilibrium with taxes. Standard carbon taxes alone are not optimal. In the cap and trade economy, the intratemporal condition (35) together with the condition on prices from the firms’ problem imply that the equilibrium price of permits is given by

$$\frac{v'(z_t)}{u'(c_t)} = \hat{\tau}_t$$

Plug it into the intertemporal condition (36) to get

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1 - \frac{v'(z_t)}{u'(c_t)}}{1 - \frac{v'(z_{t+1})}{u'(c_{t+1})}}$$

Rearranging terms we get that the competitive allocation satisfies

$$\beta [u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t)$$

But this condition is violated by the star allocation. The optimal allocation satisfies

$$\hat{\beta} [u'(c_{t+1}) - v'(z_{t+1})] = u'(c_t) - v'(z_t)$$

for $\hat{\beta} > \beta$. It follows that the optimal allocation violates condition (36) for an equilibrium. Hence, the optimal allocation does not solve the competitive equilibrium with a standard cap and trade. A cap and trade system with caps on the net emissions on carbon alone is not optimal. ■

**Proof of Proposition 5.** The proof consists on showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c^*_t, z^*_t, k^*_t, S_t^*_t\}_{t=0}^\infty$, when taxes and subsidies are set optimally. It is useful to rewrite taxes in terms of allocations (instead of shadow prices) in the following way

$$1 - \tau^k_t = \left(\frac{\hat{\beta}}{\beta}\right)^t \left[1 - \frac{v'(z^*_t)}{u'(c^*_t)}\right]$$

The intertemporal condition (24), evaluated at the star allocation, is

$$\frac{\beta u'(c^*_{t+1})}{u'(c^*_t)} = \frac{1 - \tau^k_t}{1 - \tau^k_{t+1}}$$
Plug the optimal tax rate to get
\[
\beta u' \left( c^*_{t+1} \right) u' \left( c^*_t \right) = \frac{\left( \frac{\hat{\beta}}{\beta} \right)^t \left[ 1 - \frac{v' (z^*_{t+1})}{w (c^*_{t+1})} \right]}{\left( \frac{\hat{\beta}}{\beta} \right)^{t+1} \left[ 1 - \frac{v' (z^*_{t+1})}{w (c^*_{t+1})} \right]}
\]
Rearranging terms we get
\[
\hat{\beta} [u' (c^*_{t+1}) - v' (z^*_{t+1})] = u' (c^*_t) - v' (z^*_t)
\]
which is satisfied by the efficient allocation since it coincides with (14). Consider now the equilibrium condition (23) evaluated at the star allocation
\[
\frac{u' (c^*_t)}{v' (z^*_t)} = \frac{1}{\tau^*_t}
\]
This condition holds by definition of the optimal taxes. The market clearing condition for fossil fuel (19) is already satisfied by the efficient allocation. Given the sequence of taxes \( \{ \tau^k_t, \tau^z_t \}_{t=0}^\infty \), transfers \( \{ T_t \}_{t=0}^\infty \) are defined so that the budget constraint of the government (18) is satisfied. Plugging transfers and profits into the budget constraint of the consumer delivers \( b_{t+1} = R_t b_t \) for all \( t \), which together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial \( b_0 = 0 \).
Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm, condition (22). This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied.  

**Proof of Proposition 6.** The proof consists on showing that all conditions for an equilibrium are satisfied by the optimal allocation \( \{ c^*_t, z^*_t, k^*_t, S^*_t \}_{t=0}^\infty \), when taxes and subsidies are set optimally. The intertemporal condition for this tax economy, evaluated at the star allocation, is
\[
\beta u' \left( c^*_{t+1} \right) u' \left( c^*_t \right) = \frac{1 - \tau^k_t}{1 - \tau^k_t + \tau^s_{t+1}}
\]
Plug the optimal tax rate to get
\[
\frac{\beta u' \left( c^*_{t+1} \right) u' \left( c^*_t \right)}{1 - \tau^k_t} = \frac{1 - \frac{v' (z^*_t)}{w (c^*_t)}}{1 - \frac{v' (z^*_{t+1})}{w (c^*_{t+1})} + \left( \frac{\hat{\beta}}{\beta} - 1 \right) \left[ 1 - \frac{v' (z^*_{t+1})}{w (c^*_{t+1})} \right]}
\]
Rearranging terms we get
\[ \hat{\beta}[u'(c^*_t) - v'(z^*_t)] = u'(c^*_t) - v'(z^*_t) \]
which is satisfied by the efficient allocation since it coincides with \((14)\). Consider now the equilibrium condition \((23)\) evaluated at the star allocation
\[ \frac{u'(c^*_t)}{v'(z^*_t)} = \frac{1}{\tau^*_t} \]
This condition holds by definition of the optimal taxes. The market clearing condition for fossil fuel \((19)\) is already satisfied by the efficient allocation. Given the sequence of taxes \(\{\tau^*_t, \tau^*_z, \tau^*_s\}_{t=0}^\infty\), transfers \(\{T_t\}_{t=0}^\infty\) are defined so that the budget constraint of the government
\[ p_t \tau^*_t (k_t - k_{t+1}) - p_t \tau^*_z z^*_t - p_t \tau^*_s k_t = T_t \]
is satisfied. Plugging transfers and profits into the budget constraint of the consumer delivers \(b_{t+1} = R_t b_t\) for all \(t\), which together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial \(b_0 = 0\). Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm, condition \((22)\). This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied. ■

**Proof of Proposition 7.** The proof is by construction. The constraints \((25)\) and \((26)\), together with the market clearing condition for the caps, eq. \((32)\) and \((33)\), imply that the competitive allocation coincides with the optimal. Equilibrium permit prices are then recovered from the equilibrium conditions \((35)\) and \((36)\). ■

**Proof of Proposition 8.** The proof is by construction. The constraints \((41)\) and \((42)\), together with the market clearing condition for the cap, eq. \((32)\) and for sequestration \((31)\), imply that the competitive allocation coincides with the optimal. Equilibrium prices for carbon permits and for sequestration are then recovered from the equilibrium conditions \((35)\) and \((36)\). ■
A.2 Intergenerational Accounting

This section elaborates on the social welfare function used in this paper and provides the proof for Proposition 9.

The paper considers a utilitarian criterion that weighs current as well as future unborn generations according to the following function

$$\sum_{s=0}^{\infty} \alpha_s v_s$$

where $v_s = \sum_{t=s}^{\infty} \beta^{t-s} U(c_t, z_t, S_{t+1})$ is the welfare of generation $t$ and $\{\alpha_s\}_{s=0}^{\infty}$ is an arbitrary weighting scheme across generations. This welfare function can be further simplified to

$$\sum_{t=0}^{\infty} \hat{\beta}_t U(c_t, z_t, S_{t+1})$$

where $\hat{\beta}_t \equiv \sum_{t=0}^{t} \alpha_t \beta^{t-\tau}$ represents the social discount function.

This welfare function is defined for any arbitrary weighting scheme. This paper focus on three special cases: (1) no direct weights on future generations, (2) geometric weights, (3) quasi geometric weights. Cases (1) and (2) imply that the social planning problem is time consistent while case (3) introduces time inconsistency.

We say that the social planning problem is time consistent if the initial solution at $t = 0$ also solves the problem at any future period $t$. Following Strotz (1955), a necessary and sufficient condition for time consistency is that the discount function takes the special form

$$\hat{\beta}_t = \gamma^t$$

for all $t$ and $\gamma > 0$.

A.2.1 No direct weight on future generations

This case corresponds to condition (i) in the paper. When the social criterion places positive weight only on the current generation, $\alpha_0 = 1$ and future unborn generations are only valued through the altruism of their ancestors so that $\alpha_t = 0 \forall t > 0$, it is easy to see that (56) delivers

$$\sum_{t=0}^{\infty} \beta^t U(c_t, z_t, S_{t+1})$$
That is, the social welfare function coincides with the private objective function (5). Social and private discounting are equal and take the standard geometric form. Applying Strotz (1955) result, the social planning problem is time consistent.

A.2.2 Geometric-weights

This case corresponds to condition (ii) in the paper. When the social criterion places strictly positive geometric weights, $\alpha_s = \hat{\beta}^s$ to generations $s = 1, 2, ...$, for some constant $\hat{\beta} > \beta$, then the social welfare function (56) can be rewritten as

$$\sum_{t=0}^{\infty} \hat{\beta}^t U(c_t, z_t, S_{t+1})$$ (58)

That is, the social objective function differs from the private (5) in that it applies a social discount factor which is higher than the private one, $\hat{\beta} > \beta$. That is, social discounting takes the standard geometric form and the social planning problem is time consistent. This result was derived by Bernheim (1989). The interested reader can refer to Theorem 2 in that paper.

A.2.3 Quasi-Geometric weights

This case corresponds to the one used in section 7 in the paper. The weighing scheme represents a one period deviation from the previous one. Geometric weights are applied to generations $t + 2$ onwards, while the current generation and their children receive extra weight.

**Proposition 9** Suppose, at any given point in time, welfare weights are given by $\alpha_t = \frac{1}{\beta - \hat{\beta}}$, $\alpha_{t+1} = \frac{\delta \hat{\beta} - \beta}{\beta - \hat{\beta}}$ and $\alpha_{t+1+j} = \delta^j \hat{\beta}$ for $j \geq 1$, for constants $\hat{\beta} > \beta$ and $\delta \hat{\beta} > \beta$. Then, the social discount function $\hat{\beta}_t$ takes the following values

$$1, \delta \hat{\beta}, \delta^2 \hat{\beta}, \delta^3 \hat{\beta}, ....$$ (59)

for $t, t+1, t+2, ...$

Further, the social welfare function (7) turns into the following social objective

$$U(c_t, z_t, S_{t+1}) + \delta \sum_{j=1}^{\infty} \hat{\beta}^j U(c_{t+j}, z_{t+j}, S_{t+1+j})$$ (60)
Proof of Proposition 9. The proof is by construction. I construct welfare weights such that, at any point in time $t$, the discount factor between $t$ and $t + 1$ equals

$$\frac{\hat{\beta}_{t+1}}{\hat{\beta}_t} = \delta \hat{\beta}$$

and between any other two periods $j > t$ equals

$$\frac{\hat{\beta}_{j+1}}{\hat{\beta}_j} = \hat{\beta}$$

(62)

Without loss of generality, I take the current period $t$ to be $t = 0$. I need to find constants $\kappa, \hat{\beta}, \delta$ such that

$$\sum_{\tau=0}^{t} \alpha_\tau \beta^{t-\tau} = \kappa \hat{\beta}^t \ \forall t \geq 1$$

(63)

with the additional constraint that

$$\frac{\beta \alpha_0 + \alpha_1}{\alpha_0} = \delta \hat{\beta}$$

(64)

for $t = 0$. Note that if welfare weights satisfy equations (63)-(64), then they also satisfy (61)-(62). Express (63) in two subsequent periods and subtract one from the other to obtain

$$\alpha_{t+1} = \kappa \hat{\beta}^t (\hat{\beta} - \beta) \ \forall t \geq 1$$

The restriction that weights must be strictly positive implies that $\hat{\beta} > \beta$. Further, note that (63) holds for all $t \geq 1$ so, at $t = 1$, it delivers

$$\beta \alpha_0 + \alpha_1 = \kappa \hat{\beta}^t$$

(65)

Equations (65) and (64) constitute a system of two equations in two unknowns. Solving for $\alpha_0$ and $\alpha_1$, we obtain

$$\alpha_0 = \frac{\kappa}{\delta} ; \alpha_1 = \frac{\kappa}{\delta} \left( \delta \hat{\beta} - \beta \right)$$

The requirement that weights must be strictly positive implies that $\delta \hat{\beta} > \beta$. Normalizing weights so that they add up to a convenient constant we can solve for $\kappa$ to get $\kappa = \frac{\delta}{\beta - \hat{\beta}}$. Hence, weights are given by

$$\alpha_0 = \frac{1}{\hat{\beta} - \beta} ; \alpha_1 = \frac{\delta \hat{\beta} - \beta}{\hat{\beta} - \beta}$$

Add up to $\frac{1}{\beta - \hat{\beta}} [1 + \delta \hat{\beta} - \beta + \frac{\delta^2 \hat{\beta} (\hat{\beta} - \beta)}{1 - \beta}]$
for the current and next generation and decrease geometrically from period \( t = 1 \) on according to
\[
\alpha_{t+1} = \delta \hat{\beta}^t \quad \forall \ t \geq 1
\]

Note that social preferences are still more patient than private, since \( \hat{\beta} > \beta \) and \( \delta \hat{\beta} > \beta \). More patient social preferences arise not from the specific weights that the planner chooses but from the restriction that those weights must be strictly positive for all generations. This weighting scheme represents a one-period deviation from the previous one.

### A.3 Time-Consistent Markov Allocation

#### A.3.1 Generalized Euler Equations

Assuming differentiability of the value function and the policy functions, if a solution exists and is interior, the first order conditions for the planner’s problem are
\[
\begin{align*}
u'(g^c (k, S)) - v'(g^z (k, S)) &= \delta \hat{\beta} V_1 (g^k (k, S), g^S (k, S)) \quad (66) \\
v' (g^z (k, S)) &= x' (g^s (k, S)) - \delta \hat{\beta} V_2 (g^k (k, S), g^S (k, S)) \quad (67)
\end{align*}
\]

where \( g^c (k, S) = k - g^k (k, S) \) and \( g^z (k, S) = (1 - \gamma) S - g^s (k, S) + g^c (k, S) \). For ease of exposition, I suppress the arguments of the policy functions in what follows so that, for example, \( g^S \) is the shortcut for \( g^S (k, S) \). Using (46), the corresponding derivatives of the function \( V \) are given by
\[
\begin{align*}
V_1 (k, S) &= u'(g^c )[1 - g^k_1] - v'(g^z)[-g^*_{1} + 1 - g^k_1] - x'(g^s)g^s_1 \quad (68) \\
&\quad + \hat{\beta}[V_1 (g^k, g^*)g^k_1 + V_2 (g^k, g^*)g^s_1] \\
V_2 (k, S) &= -u'(g^c)g^k_2 - v'(g^z)[1 - \gamma - g^s_2 - g^k_2] - x'(g^s)g^s_2 \quad (69) \\
&\quad + \hat{\beta}[V_1 (g^*, g^*)g^k_2 + V_2 (g^k, g^*)g^s_2]
\end{align*}
\]
Further, the derivatives of the value function can be substituted away using (66)-(67).

\[
V_1(k, S) = \left[ u'(k - g^k) - v'(g^z) \right] [1 - g_k^k(1 - \frac{1}{\delta})] + \left[ v'(g^z) - x'(g^S) \right] g_k^S(1 - \frac{1}{\delta})
\]

\[
V_2(k, S) = \left[ u'(k - g^k) - v'(k - g^k) \right] g_z^S(1 - \frac{1}{\delta}) - v'(g^z)(1 - \gamma) + \left[ v'(g^z) - x'(g^S) \right] g_z^S(1 - \frac{1}{\delta})
\]

Finally, update these equations one period ahead and plug them back into the first order conditions to get the following Generalized Euler Equations, rewritten in sequential form for ease of reading

\[
u'(c_t) - v'(z_t) = \delta \hat{\beta} [u'(c_{t+1}) - v'(z_{t+1})][1 + g_1^k(k_{t+1}, S_{t+1})(\frac{1}{\delta} - 1)]
\]

\[- \delta \hat{\beta} [v'(z_{t+1}) - x'(S_{t+2})] g_z^k(k_{t+1}, S_{t+1})(\frac{1}{\delta} - 1)
\]

\[
x'(S_{t+1}) - v'(z_t) = - \delta \hat{\beta} v'(z_{t+1})(1 - \gamma) + \delta \hat{\beta} [u'(c_{t+1}) - v'(z_{t+1})] g_z^k(k_{t+1}, S_{t+1})(\frac{1}{\delta} - 1)
\]

\[- \delta \hat{\beta} [v'(z_{t+1}) - x'(S_{t+2})] g_z^k(k_{t+1}, S_{t+1})(\frac{1}{\delta} - 1)
\]

A.3.2 Computational Procedure

The algorithm is an application of endogenous grid methods to a problem with two state variables and two control variables. I assume continuity and differentiability of the policy functions. The solution to the planner’s problem involves solving equations (70) and (71), which contain four partial derivatives of the policy functions. This implies that there are two Euler equations but six unknowns: the two policy functions \(g^k(k, S)\) and \(g^s(k, S)\) and the four partial derivatives \(g_k^1(k, S), g_k^2(k, S), g_S^1(k, S), g_S^2(k, S)\). Although I have not proved that the equilibrium is unique, the algorithm converges always to the same solution. The steps involved in the algorithm are as follows:

1. Define a grid on current fossil fuel \(k \equiv \{k_1, k_2, k_3, \ldots\}\) and on tomorrow’s carbon level \(S \equiv \{S'_1, S'_2, S'_3, \ldots\}\).

A similar algorithm can be found in Hintermaier and Koeniger (2010). Convergence depends on the value of \(\delta\) and on the number of grid points. For some low values of \(\delta\) and for fine grids, the algorithm fails to converge. This problem is also found in Maliar and Maliar (2005). They do not apply endogenous grid methods but their algorithm also exploits the Euler equation.
2. Use the first order condition with respect to fossil fuel to get the value $k'(k, S')$ that satisfies the optimality condition with respect to $k$

$$u'(k, k') = \beta[V_1(k', S') - V_2(k', S')] - x'(S')$$  \hfill (72)

Use the envelope conditions to compute the partial derivatives of the value function.

3. The optimal values for $k'(k, S')$ are given by

$$\min\{k'(k, S'), k\}$$

The non negativity constraint is satisfied by assuming Inada conditions.

4. The values for sequestration $z(k, S')$ are recovered from the first order condition with respect to carbon

$$v'(z(k, S')) = x'(S') - \beta V_2(k', S')$$

5. Use the equation for the carbon cycle to recover the current state $S$

$$S = S' - (k - k'(k, S')) + z(k, S')$$

6. Use interpolation to retrieve the new policy functions $\{k'(k, S), S'(k, S)\}$ on the endogenous grid obtained for $S$.

7. Iterate until convergence of the policy functions

References


