

WELFARE EVALUATION

ECON 8001-2

Instructor: Terry Hurley

INTRODUCTION

Up to this point, we have focused primarily on positive economic analysis. That is, we have worked to construct a framework that will be useful for interpreting why people make the choices they do and for understanding how these choices might change when individual circumstances change. We have also paid some attention to identifying ways to test whether this framework offers an adequate representation of how people make choices. What we want to do now is explore how this framework can be used in a normative sense. That is, to what extent can we use this framework to determine whether someone is better or worse off if circumstances change?

COMPENSATING & EQUIVALENT VARIATION

In the last section we saw how to derive an expenditure function from observed Marshallian Demand and how this expenditure function tells us something about someone's preference relation. We will now link this idea to the indirect utility function and show how we can use the two concepts to develop inferences regarding an individual's wellbeing (i.e. utility) in response to changing circumstances.

Recall that utility functions and, therefore, indirect utility functions are not unique. That is, any strictly increasing transformation will yield a new utility or indirect utility function that represents the same preference relation as the original one. Also recall that the expenditure function is strictly increasing in u . Therefore, if we choose arbitrary prices $p^b \gg 0$, the function $e(p^b, v(p, w))$ will give us a valid indirect utility function that represents the same preference relation as $v(p, w)$. The interesting thing about this indirect utility function is that it yields a measure of individual wellbeing with the same money metric as wealth (e.g. dollars). We can use this money metric to evaluate individual wellbeing under varying circumstances. For example, if a new government policy changes current prices, p^0 , to a new set of prices, p^1 , we can evaluate whether $e(p^b, v(p^0, w))$ is greater than or less than $e(p^b, v(p^1, w))$. If it is greater, then the individual's wellbeing is higher with the current prices. If it is less, then the individual's wellbeing is lower with the current prices.

The price vector one chooses for p^b is arbitrary. Two possible choices that have attracted the most attention from economists are current prices ($p^b = p^0$) and new prices ($p^b = p^1$). Using current prices produces the welfare measure referred to as the *Equivalent Variation*:

$$\mathbf{WE1} \quad EV(p^0, p^1, w) = e(p^0, v(p^1, w)) - e(p^0, v(p^0, w)) = e(p^0, v(p^1, w)) - w.$$

Using new prices produces the welfare measure referred to as the *Compensating Variation*:

$$\mathbf{WE2} \quad CV(p^0, p^1, w) = e(p^1, v(p^1, w)) - e(p^1, v(p^0, w)) = w - e(p^1, v(p^0, w)).$$

There are several other ways to write these relationships to help gain intuition. For example,

$$\mathbf{WE3} \quad v(p^0, w + EV(p^0, p^1, w)) = v(p^1, w) \text{ and}$$

WELFARE EVALUATION

ECON 8001-2

Instructor: Terry Hurley

WE4
$$v(p^0, w) = v(p^1, w - CV(p^0, p^1, w)).$$

Equation WE3 says that the Equivalent Variation is the amount of money an individual must be paid to be as well off with current prices as it is with new prices. Equation WE4 says that the Compensating Variation is the amount of money the individual would be willing to give up with new prices to be just as well off as with the current prices. Note that in either case, the Equivalent and Compensating Variation can be positive or negative.

Now let us assume that the price of the k th commodity decreases: $p_l^0 = p_l^1$ for all $l = 1, \dots, L$ and $l \neq k$ and $p_k^1 < p_k^0$. Let $p_{\sim k} = (p_1^0, \dots, p_{k-1}^0, p_{k+1}^0, \dots, p_L^0) = (p_1^1, \dots, p_{k-1}^1, p_{k+1}^1, \dots, p_L^1)$. Since $e(p^1, v(p^1, w)) = w$ and $\frac{\partial e(p, u)}{\partial p_k} = h_k(p, u)$,

WE5
$$\begin{aligned} EV(p^0, p^1, w) &= e(p^0, v(p^1, w)) - w \\ &= e(p^0, v(p^1, w)) - e(p^1, v(p^1, w)) \\ &= \int_{p_k^1}^{p_k^0} h_k((p_k, p_{\sim k}), v(p^1, w)) dp_k, \end{aligned}$$

which says the Equivalent Variation for a change in the price of commodity k is equal to the area to the left of the Hicksian Demand evaluated from p_k^1 to p_k^0 at the new level of wellbeing. Also, since $e(p^0, v(p^0, w)) = w$,

WE6
$$\begin{aligned} CV(p^0, p^1, w) &= w - e(p^1, v(p^0, w)) \\ &= e(p^0, v(p^0, w)) - e(p^1, v(p^0, w)) \\ &= \int_{p_k^1}^{p_k^0} h_k((p_k, p_{\sim k}), v(p^0, w)) dp_k, \end{aligned}$$

which says the Compensating Variation for a change in the price of commodity k is equal to the area to the left of the Hicksian Demand from p_k^1 to p_k^0 evaluated at the current level of wellbeing.

We have been careful to distinguish between the Equivalent and Compensating Variations because in general they are different and as such will give different quantitative answers. Qualitatively, the answers will be the same when comparing two alternative price vectors. That is, if the Equivalent Variation says an individual is better off, the Compensating Variation will also say the individual is better off. The two differ however in terms of saying how much better off because the utility function is an ordinal rather than cardinal concept.

To better understand the relationship between the two note that we can write the difference as

WE7
$$EV(p^0, p^1, w) - CV(p^0, p^1, w) = \int_{p_k^1}^{p_k^0} [h_k((p_k, p_{\sim k}), v(p^1, w)) - h_k((p_k, p_{\sim k}), v(p^0, w))] dp_k.$$

WELFARE EVALUATION

ECON 8001-2

Instructor: Terry Hurley

Recall that PROPOSITION D4 tells us that if the Marshallian Demand $x_k(p, w)$ is normal (inferior), then $h_k(p, u'') >(<) h_k(p, u')$ if $u'' > u'$. Also, recall that PROPOSITION CP9 tells us that $v(p^1, w) \geq v(p^0, w)$. Therefore, $h_k(p, v(p^1, w)) \geq(\leq) h_k(p, v(p^0, w))$ for all p if $x_k(p, w)$ is normal (inferior), which implies $EV(p^0, p^1, w) \geq(\leq) CV(p^0, p^1, w)$ if $x_k(p, w)$ is normal (inferior). Figure WE1 illustrates this result. The point being that the Equivalent Variation exceeds the Compensating Variation for a price decrease when the commodity of interest is Normal. When the commodity of interest is inferior, the Compensating Variation will exceed the Equivalent Variation for a price decrease. Finally, if Marshallian Demand is independent of wealth, COROLLARY D4 implies that the Equivalent and Compensating Variation will be equal.

The bottom line is that we can use either one to compare and rank a change in the price of a commodity. Depending on whether the commodity is normal or inferior and whether price increases or decreases, we know which one will give a larger value of the welfare change, but this should not concern us because we have only required preferences to be ordinal (not cardinal).

All this said, one must be more careful when comparing multiple price changes. To understand why, suppose current prices are p^0 and that the government is considering taxing either gasoline or cars to build new roads. The tax on gasoline results in the new price vector p^g , while the tax on cars results in the new price vector p^c . Let us use the Equivalent and Compensating Variations to evaluate these price changes. The Equivalent Variation for a change in prices from p^0 to p^g and from p^0 to p^c are

$$\text{WE8} \quad EV(p^0, p^g, w) = e(p^0, v(p^g, w)) - w \text{ and}$$

$$\text{WE9} \quad EV(p^0, p^c, w) = e(p^0, v(p^c, w)) - w.$$

The Compensating Variation for a change in prices from p^0 to p^g and from p^0 to p^c are

$$\text{WE10} \quad CV(p^0, p^g, w) = w - e(p^g, v(p^0, w)) \text{ and}$$

$$\text{WE11} \quad CV(p^0, p^c, w) = w - e(p^c, v(p^0, w)).$$

If we compare the Equivalent Variations for these two alternative tax policies, the Equivalent Variation for the gasoline (car) tax will be larger if

$$\text{WE12} \quad EV(p^0, p^g, w) >(<) EV(p^0, p^c, w) \text{ or}$$

$$\text{WE13} \quad e(p^0, v(p^g, w)) >(<) e(p^0, v(p^c, w)).$$

Since the expenditure function is increasing in u , equation WE13 says that when the indirect utility for p^g and w is greater (less) than the indirect utility for p^c and w , the Equivalent Variation for p^g and w will be greater (less) than the Equivalent Variation p^c and w . This is precisely the type of ranking we want.

WELFARE EVALUATION

ECON 8001-2

Instructor: Terry Hurley

Now, if we compare the Compensating Variations for these two alternative tax policies, the Compensating Variation for the gasoline (car) tax will be larger if

$$\text{WE14} \quad CV(p^0, p^g, w) >(<) CV(p^0, p^c, w) \text{ or}$$

$$\text{WE15} \quad e(p^c, v(p^0, w)) >(<) e(p^g, v(p^0, w)).$$

If the indirect utility function for p^g and w is greater (less) than the indirect utility function for p^c and w can we say for sure that the Compensating Variation for p^g and w is greater (less) than the Compensating Variation for p^c and w ? The answer to this question is NO! We know the expenditure function is non-decreasing in p_l for any l , but this really does nothing definitive for us in equation WE15.

What is going on here? The comparison between (p^g, w) and (p^c, w) in equation WE13 uses the indirect utility function $e(p^0, u)$, while the comparison in equation WE15 uses two distinct indirect utility functions: $e(p^c, u)$ and $e(p^g, u)$. But we have already talked about how we cannot compare different indirect utility functions because the concept is ordinal and not cardinal. The bottom line is that care needs to be taken when constructing multiple welfare comparisons using Equivalent and Compensating Variations because they can lead to different conclusions. In the case above, the Equivalent Variation will give us a valid assessment of welfare changes, while the Compensating Variation may not.

CONSUMER SURPLUS

Using areas defined by demand curves to measure welfare changes to price changes should be familiar. It is often a topic introduced in Microeconomic Principles and certainly introduced in Intermediate Microeconomics. However, you may have noticed that we did things somewhat differently. When welfare analysis is first introduced, it is usually introduced in the context of Marshallian Demand and called *Consumer Surplus*. *Consumer Surplus* for a commodity is defined as the area under the Marshallian Demand above the price of the commodity, which we can write as

$$\text{WE16} \quad CS(p^0, w) = \int_{p_k^0}^{\infty} x_k(p_k, p_{\sim k}, w) dp_k.$$

As with our earlier example, considering a decrease in the price of the k th commodity resulting in the new price vector p^1 , the change in consumer surplus can be written as

WELFARE EVALUATION

ECON 8001-2

Instructor: Terry Hurley

$$\begin{aligned}
 \Delta CS(p^0, p^1, w) &= CS(p^1, w) - CS(p^0, w) \\
 \text{WE17} \quad &= \int_{p_k^1}^{\infty} x_k(p_k, p_{-k}, w) dp_k - \int_{p_k^0}^{\infty} x_k(p_k, p_{-k}, w) dp_k, \\
 &= \int_{p_k^1}^{p_k^0} x_k(p_k, p_{-k}, w) dp_k
 \end{aligned}$$

which looks an awful lot like equations WE5 and WE6. We know WE5 and WE6 produce different quantitative results except when Marshallian Demand is independent of wealth, so the question is does equation WE17 typically correspond to equation WE5, equation WE6, or is it something different entirely. The answer to this question is that it is something different. Indeed, what we can show is that consumer surplus must be bounded by the Equivalent and Compensating Variations.

Recall that PROPOSITION D1 implies $h_k(p^0, v(p^0, w)) = x_k(p^0, w)$ and $h_k(p^1, v(p^1, w)) = x_k(p^1, w)$, while PROPOSITION D3 implies $\frac{\partial h_k(p, v(p, w))}{\partial p_k} = \frac{\partial x_k(p, w)}{\partial p_k} + \frac{\partial x_k(p, w)}{\partial w} x_k(p, w)$. Together,

these results say that if Marshallian Demand is normal $h_k(p, v(p^0, w)) < x_k(p, w)$ for all $p_k < p_k^0$ and $h_k(p, v(p^1, w)) > x_k(p, w)$ for all $p_k > p_k^1$, which implies $h_k(p, v(p^1, w)) > x_k(p, w) > h_k(p, v(p^0, w))$ for $p_k^0 > p_k > p_k^1$ or $EV(p^0, p^1, w) > \Delta CS(p^0, p^1, w) > CV(p^0, p^1, w)$. The top half of Figure WE2 illustrates. Alternatively, if Marshallian Demand is inferior $h_k(p, v(p^0, w)) > x_k(p, w)$ for all $p_k < p_k^0$ and $h_k(p, v(p^1, w)) < x_k(p, w)$ for all $p_k > p_k^1$, which implies $h_k(p, v(p^0, w)) > x_k(p, w) > h_k(p, v(p^1, w))$ for $p_k^0 > p_k > p_k^1$ or $CV(p^0, p^1, w) > \Delta CS(p^0, p^1, w) > EV(p^0, p^1, w)$, as illustrated in the bottom half of Figure WE2.

What is going on here? What is going on is that price changes affect Marshallian Demand in two ways: a substitution effect and a wealth effect. They affect Hicksian Demand in only one way: a substitution effect. Substitution effects do not reflect an increase in welfare, while wealth effects do. By using Hicksian Demands to measure welfare effects, we control for changes in consumption attributable to substitution away from more expensive commodities.

WELFARE ANALYSIS WITH LIMITED INFORMATION

The Equivalent and Compensating Variations provide useful tools for welfare analysis. Some may prefer to use Consumer Surplus even though it is not an exact measure of a change in someone's wellbeing because it is immediately observable. Still, if we know Marshallian Demand, integrability tells us that we can derive the corresponding expenditure function and Hicksian Demands (with sometimes significantly more work) needed to evaluate the Equivalent or Compensating Variation. But what if we do not know everything there is to know about Marshallian Demand? Can we still make useful welfare statements? In some instances, we can.

Suppose we observe that an individual is consuming x^0 at price p^0 . Would this person be worse off if prices changed to p^1 where $(p^1 - p^0) \cdot x^0 < 0$? The answer to this question is NO! Indeed,

WELFARE EVALUATION

ECON 8001-2

Instructor: Terry Hurley

the person must be better off if preferences are locally nonsatiated. Note that $(p^1 - p^0) \cdot x^0 < 0$ implies $p^1 \cdot x^0 < p^0 \cdot x^0 = w$ (the last equality by Walras Law). Therefore, x^0 is affordable under the new prices, but it does not exhaust all income, which implies there must be some x^1 that is strictly preferred to x^0 given the new prices.

But what if we look at p^1 where $(p^1 - p^0) \cdot x^0 > 0$? Under these circumstances, it is possible to show (using local nonsatiation) that there is some $\bar{a} \in (0, 1)$ such that for all $a < \bar{a}$ $(p^0, w = p^0 \cdot x^0)$ is strictly better for an individual than $((1 - a)p^0 + ap^1, w)$, which is really not quite as useful.

Figure WE1: Equivalent and Compensating Variation comparison for and decrease in the price of commodity k .

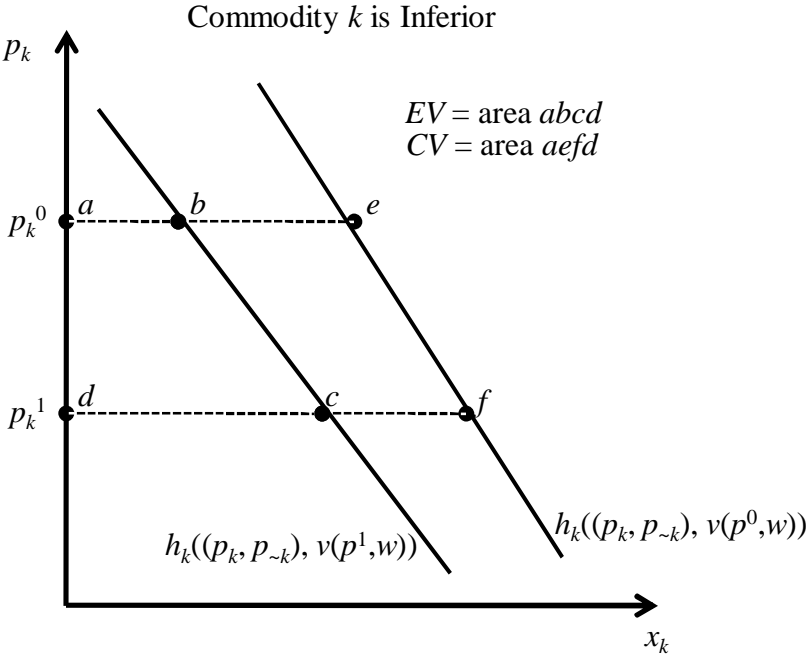
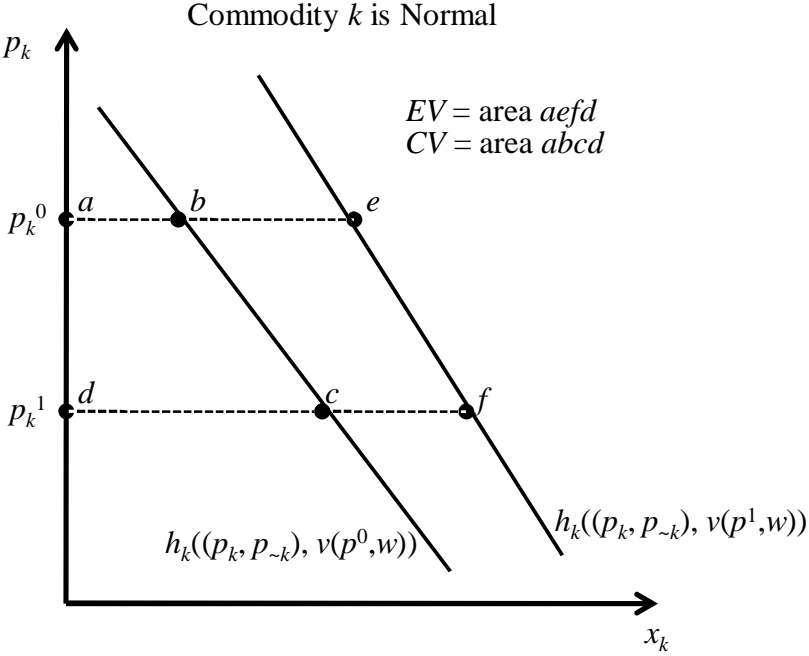


Figure WE2: Equivalent Variation, Compensating Variation, and change in Consumer Surplus for and decrease in the price of commodity k .

