DEMAND, WARP, & SARP

ECON 8001-2 **Instructor: Terry Hurley**

INTRODUCTION

When we first started talking about how people make choices, we discussed two different approaches: preference based and choice based. As we turned to exploring demand relations, we really focused in on the preference based approach and the assumptions required to make it work smoothly. Before moving away from individual demand, it is worth commenting on how all this ties back to the choice based approach because if you recall we could always construct a choice structure that satisfied the Weak Axiom of Revealed Preferences from a rational preference relation, but we could not always construct a rational preference relation starting from a choice structure that satisfied the Weak Axiom of Revealed Preferences.

WARP & MARSHALIAN DEMAND

First recall the definition for the Weak Axiom of Revealed Preferences:

Weak Axiom of Revealed Preference (*WARP*): A choice structure satisfies *WARP* if for some $b \in \boldsymbol{B}$ with $x, y \in b$ we have $x \in C(b)$ then for any $b' \in \boldsymbol{B}$ with $x, y \in b'$ and $y \in C(b')$, we must also have $x \in C(b')$.

This definition can be customized in terms of Marshallian Demand:

DEFINITION: The Mashallian Demand $x(p, w)$ satisfies the Weak Axiom of Revealed Preferences if for any two price wealth combinations (p, w) and (p', w'):

If $p \cdot x(p', w') \leq w$ and $x(p', w') \neq x(p, w)$, then $p' \cdot x(p, w) > w'$.

The definition simply says that if you can afford $x(p', w')$ given prices and wealth (p, w), but you do not choose it, then it must be the case that $x(p, w)$ is not affordable given prices and wealth (p', w'). If we combine this definition, with homogeneity of degree zero in prices and wealth, and satisfaction of Walras Law, then we can show something very interesting:

PROPOSITION WS1: Suppose the Marshallian Demand $x(p, w)$ is homogeneous of degree 0 in prices and wealth and satisfies Walras Law. Then $x(p, w)$ satisfies the Weak Axiom of Revealed Preferences if and only if for any compensated price change such that $(p', w') = (p', p' \cdot x(p, w))$,

$$(p' - p) \cdot [x(p', w') - x(p, w)] \leq 0,$$

with strict inequality whenever $x(p', w') \neq x(p, w)$.

Notice that this proposition looks an awful like PROPOSITION EM2, which is the *Compensated Law of Demand* for Hicksian Demand. It is also interesting to note that adding the Weak Axiom of Revealed Preferences to the assumptions of homogeneity of degree 0 in price and wealth, and Walras Law is enough to generate a negative semidefinite substitution matrix:

PROPOSITION WS2: Suppose the differentiable Marshallian Demand $x(p, w)$ is homogeneous of degree 0 in prices and wealth; satisfies Walras Law; and satisfies the Weak Axiom of Revealed Preferences. Then the Slutsky substitution matrix is negative semidefinite for any p and w: $v\, S(p, w)\, v \leq 0$ for any $v \in \Re^L$.

Now if you recall, the assumption we needed to get homogeneity of degree zero for the preference based approach was essentially optimization. The assumption we needed to get Walras to hold was local nonsatiation.

Still, an important property that the Weak Axiom of Revealed Preferences does not get us is symmetry. That is, $S(p, w)$ need not be symmetric. Therefore, the preference relation based approach and choice based approach do not yield equivalent demand relations if we appeal only to the Weak Axiom of Revealed Preferences. However, it is possible to strengthen the Weak Axiom of Revealed Preferences, in order to make the preference based and choice based approaches essentially equivalent.

DEFINITION: The Marshallian Demand $x(p, w)$ satisfies the ***Strong Axiom of Revealed Preferences*** if for any list $(p^1, w^1), \ldots, (p^N, w^N)$ with $x(p^{n+1}, w^{n+1}) \neq x(p^n, w^n)$ for all $n \leq N - 1$, we have $p^N \cdot x(p^1, w^1) > w^N$ whenever $p^n \cdot x(p^{n+1}, w^{n+1}) \leq w^n$ for all $n \leq N - 1$.

Essentially, the ***Strong Axiom of Revealed Preferences*** says that if $x(p^1, w^1)$ is revealed preferred to $x(p^N, w^N)$ (either directly or indirectly), then $x(p^N, w^N)$ cannot be revealed preferred to $x(p^1, w^1)$. If the Strong Axiom of Revealed Preferences holds, we can then find a rational preference relation that rationalizes it and the Marshallian Demand relation it implies:

PROPOSITION WS3: If the Marshallian Demand $x(p, w)$ satisfies the Strong Axiom of Revealed Preferences then there is a rational preference relation $\succsim$ on $X = \Re^L_+$ that rationalizes $x(p, w)$ such that for all p and w, $x(p, w) \succ y$ for every $y \neq x(p, w)$ where $y \in B_{p,w}$.

Therefore, if we are only willing to appeal to the Weak Axiom of Revealed Preferences, we cannot guarantee many of the results we developed for Marshallian Demand using the preference based approach. However, we can get these results back if we are willing to appeal to the Strong Axiom of Revealed Preferences.