PREFERENCE & UTILIY

ECON 8001-2 **Instructor: Terry Hurley**

INTRODUCTION (MWG Chapter 3 & Varian Chapter 7)

The first lectures for this class focused on individual choice in relation to people's preferences in very general terms. A key result is that it is may be possible to represent an individual with rational preferences using a real valued function that we referred to as the utility function. An important caveat was that while rational preferences were necessary to generate a utility function from an individual's preference relation, they were not sufficient. We will now expand on the prospects of using utility functions to describe people's preferences. In particular, we will be interested in obtaining a better understanding of how different classical assumptions on preference relations affect the existence and properties of the types of utility functions we might be able to generate from those relations. Over the next couple of lectures, these ideas will be developed in the context of the classical consumer choice problem.

CLASSICAL PREFERENCE ASSUMPTIONS & IMPLICATIONS

Notation:

L:	Total number of distinct goods and services, referred to as commodities, that are available to a consumer.
$x \in \Re^L$:	Vector of commodities. Note that these commodities can be very general representing different things at the same time and under the same general conditions of the world or the same thing at different times or under different conditions of the world.
$x_l \in \Re$:	Specific quantity of commodity l for $l = 1, 2, \ldots, L$. Note that positive values represent inflows of a commodity, while negative values represent outflows.
$X \subseteq \Re^L$:	The consumption set given institutional, physical, or any other possible constraints.

To keep our discussion more manageable, we will focus on $X = \Re^L_+ = \{x \in \Re^L: x_l \geq 0 \text{ for } l = 1, 2, \ldots, L\}$.

Definitions:

Strongly Monotone Preference Relation: A preference relation $\succsim$ on X such that for all $x, y \in X$, if $y \geq x$ ($y_l \geq x_l$ for $l = 1, 2, \ldots, L$) and $y \neq x$, then $y \succ x$.

Monotone Preference Relation: A preference relation $\succsim$ on X such that for all $x, y \in X$, if $y >> x$ ($y_l > x_l$ for $l = 1, 2, \ldots, L$), then $y \succ x$.

Locally Nonsatiated Preference Relation: A preference relation $\succsim$ on X such that for every $x \in X$ and every $\varepsilon > 0$, there is $y \in X$ such that $||y - x|| \leq \varepsilon$ and $y \succ x$.

These monotonicity and nonsatiation definitions characterize varying degrees of what is often referred to as the "more is better" axiom of classical utility theory. What I mean by varying degrees of "more is better" is that if preference relation is Strongly Monotone, then you can prove it is also Monotone, and if a preference relation is Monotone, then you can prove it is

Locally Nonsatiated. However, you cannot prove a Locally Nonsatiated preference relation is Monotone or that a Monotone preference relation is Strongly Monotone.

Convex Preference Relation: A preference relation $\succsim$ on X such that for any $a \in [0, 1]$ and all x, y, $z \in X$, if $y \succsim x$ and $z \succsim x$, then $ay + (1 - a)z \succsim x$.

Strictly Convex Preference Relation: A preference relation $\succsim$ on X such that for any $a \in (0, 1)$ and all x, y, $z \in X$, if $y \succsim x$, $z \succsim x$, and $y \neq z$, then $ay + (1 - a)z \succ x$.

These convexity definitions characterize varying degrees of the classical "preference for variety" axiom. A strictly convex preference relation is convex, but a convex preference relation need not be strictly convex.

Continuous Preference Relation: A preference relation $\succsim$ on X such that for any sequence of pairs $\{(x^n, y^n)\}_{n=1}^{\infty}$ with $x^n \succsim y^n$ for all n, $x = \lim_{n\to\infty} x^n$, and $y = \lim_{n\to\infty} y^n$, we have $x \succsim y$.

Upper Contour Set (denoted by $UCS(x)$): The set of all alternatives to x that are at least as good as x: $UCS(x) \equiv \{y \in X: y \succsim x\}$.

Lower Contour Set (denoted by $LCS(x)$): The set of all alternatives to x that x is at least as good as: $LCS(x) \equiv \{y \in X: x \succsim y\}$.

Indifference Contour Set (denoted by $ICS(x)$): The set of all alternatives to x that are at least as good as x and that x is at least as good as: $ICS(x) \equiv \{y \in X: x \succsim y \text{ and } y \succsim x\} = LCS_x \cap UCS_x$.

Now that we have a bunch of notation and definitions, we are ready to talk about how classical assumptions on preference relations impose structure on the utility correspondence or function that can be generated with these relations:

a) A rational preference relation (e.g. complete and transitive) is a necessary condition for having a utility function that represents this relation.
b) Assuming a monotone, strongly monotone, or locally nonsatiated rational preference relation lets us say that the utility function generated by the preference relation yields *Indifference Contour Sets* that are in a sense downward sloping (i.e. the utility function is increasing or non-decreasing).
c) Assuming a convex or strictly convex rational preference relation generates a utility function with *Indifference Contour Sets* that are convex to the origin and exhibit diminishing (non-increasing) marginal utility (i.e. the utility function is quasiconcave or strictly quasiconcave).
d) Finally, assuming a continuous rational preference relation generates utility functions and *Indifference Contour Sets* that are continuous. The assumption is important because it rules out cases like *Lexicographic preferences* which can be shown to be complete, transitive, strongly monotonic, and strictly convex, yet they cannot be represented by a utility function. Consider a two commodity world where $X \in \Re^2_+$. For $x, y \in X$, an example of *Lexicographic preferences* is $x \succsim y$ if either $x_1 > y_1$, or $x_1 = y_1$ and $x_2 > y_2$.

Two additional axioms that have found extensive use particularly in econometric applications are homothetic and quasi-linear preference relations.

Definitions:

Homothetic Preference Relation: A preference relation $\succsim$ on X such that for any $a \geq 0$ and all x, $y \in X$, if $x \sim y$, then $ax \sim ay$.

Figure PU1 illustrates the implications of a homothetic preference relation. Later, we will make extensive use of the Cobb-Douglas utility, which represents a homothetic preference relation.

Quasi-linear Preference Relation in x_1: A preference relation $\succsim$ on $X = (-\infty, \infty) \times \Re^{L-1}_{+}$ such that for $e = (1, 0,\ldots,0)$, any $a \in \Re$, any $l > 0$ and all $x, y \in X$

(i) $x + le \succ x$, and

(ii) if $x \sim y$, then $x + ae \sim y + ae$.

Figure PU2 illustrates the implications of a quasi-linear preference relation.