INDIVIDUAL CHOICE

ECON 8001-2 **Instructor: Terry Hurley**

INTRODUCTION (MWG Chapter 1)

A key objective for microeconomic theory is to understand how individuals and firms allocate scarce resources. To accomplish this objective, economists have tended to emphasize two things: preferences and constraints. In this class, we will explore classical characterizations of consumer and firm preferences and constraints, and ask what are the implications of these characterizations in terms of behavior and individual wellbeing? We will also spend time talking about the prospects of being able to confirm or refute fundamental assertions of classical theory and the guidance the theory can provide in terms of applied analysis.

POSITIVE VS. NORMATIVE ECONOMIC ANALYSIS

Questions:

How do people make choices?
How do competitive markets affect people's choices?
How will people respond to continued rising healthcare costs?
Are people's choices good or bad?
Are the affects of competitive markets on people's choices good or bad?
Who will benefit most from continued increases in healthcare costs?

The first three questions above are positive in nature, while the last two are normative.

Definitions:
Positive Economic Analysis: Economic analysis that seeks to explain or predict how economic objects will behave.

Normative Economic Analysis: Economic analysis that seeks to explain how economic objects should behave.

Positive analysis is objective in nature and very much grounded in the tradition of science, while normative analysis is subjective in nature and very much grounded in the tradition of philosophy. While positive analysis seeks to ascertain the motivations and objectives of economic objects based on observing behavior, normative analysis posits motivations and objectives to prescribe behavior.

INDIVIDUAL CHOICE

Two Approaches to Modeling Individual Choice

a. Preference Based Approach
   - Unobservable preference relation is primitive.
   - Specify a preference relation while imposing rationality axioms and then derive the implications of those axioms in terms of individual choice.

b. Choice Based Approach
   - Observable choice behavior is primitive.

- Specify consistency assumptions regarding observed choice and then derive the implications of these assumptions in terms of observed choices.

PREFERENCE BASED APPROACH

Definitions:
Preference Relation (denoted by $\succsim$): If $x, y \in X$ and $x \succsim y$, then "x is at least as good as y."

Strict Preference Relation (denoted by $\succ$): If $x, y \in X$ and $x \succ y$, then "x is preferred to y" or $x \succ y$ if and only if $x \succsim y$ and not $y \succsim x$.

Indifference Relation (denoted by ~): If $x, y \in X$ and $x \sim y$, then "x is indifferent to y" or $x \sim y$ if and only if $x \succsim y$ and $y \succsim x$.

Complete Preference Relation: For all $x, y \in X$, $x \succsim y$ or $y \succsim x$.

Transitive Preference (*Strict Preference*, *Indifference*) *Relation*: For all $x, y, z \in X$, if $x \succsim (\succ, \sim)$ y and $y \succsim (\succ, \sim)$ z, then $x \succsim (\succ, \sim)$ z.

Irreflexive Preference (*Strict Preference*, *Indifference*) *Relation*: For all $x \in X$, not $x \succsim (\succ, \sim)$ x.

Reflexive Preference (*Strict Preference*, *Indifference*) *Relation*: For all $x \in X$, $x \succsim (\succ, \sim)$ x.

Symmetric Preference (*Strict Preference*, *Indifference*) *Relation*: For all $x, y \in X$, if $x \succsim (\succ, \sim)$ y and $y \succsim (\succ, \sim)$ x.

Rational Preference Relation: A preference relation is rational if it is (i) complete and (ii) transitive.

Utility Function (u: $X \rightarrow \Re$): Given the preference relation $\succsim$, a function such that for all $x, y \in X$, $u(x) \geq u(y)$ if and only if $x \succsim y$.

Often economists assume individual behavior is consistent with a rational preference relation. While the assumptions of transitivity and completeness are quite appealing for a lot of reasons, they certainly fall short of being worthy of the label "Universal Truths." Still, Propositions IC1 and particularly, IC2 give us some insight into why economists have been quite willing to accept the various shortcomings of these assumptions.

PROPOSITION IC1: For any rational preference relation,
(a) the strict preference relation is irreflexive and transitive,
(b) the indifference relation is reflexive, transitive, and symmetric, and
(c) for all $x, y, z \in X$, if $x \succ y$ and $y \succsim z$, then $x \succ z$.

Proof of (a): We know from the definition of a rational preference relation that for all $x, y \in X$, $x \succsim y$ or $y \succsim x$ by completeness and for all $x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$ by transitivity.

Let us start by showing that these assumptions imply the strict preference relation must be irreflexive. The proof will proceed by contradiction, assume for all $x \in X$, $x \succ x$. The definition of a strict preference relation implies that for all $x \in X$, if $x \succ x$ then $x \succsim x$ and not $x \succsim x$, which is a contradiction.

Now let us look at transitivity. Again, the proof proceeds by contradiction. Assume that for some $x, y, z \in X$, if $x \succ y$ and $y \succ z$, then not $x \succ z$. By definition of a strict preference relation, if $x \succ z$, then $x \succsim z$ and not $z \succsim x$. Not $x \succ z$ implies not $x \succsim z$ and $z \succsim x$, or $x \succsim z$ and $z \succsim x$. By the definition of a strict preference relation, $x \succ y$ implies $x \succsim y$ and not $y \succsim x$, and $y \succ z$ implies $y \succsim z$ and not $z \succsim y$. By the transitivity, $x \succsim y$ and $y \succsim z$ imply $x \succsim z$, which contradicts not $x \succsim z$ and $z \succsim x$. Now if $x \succsim z$ and $z \succsim x$ is true, $z \succsim x$ and $x \succsim y$ implies $z \succsim y$ by transitivity, but this contradicts not $z \succsim y$. **Q.E.D.**

Similar proofs can be developed for (b) and (c).

PROPOSITION IC2: A preference relation $\succsim$ can be represented by a utility function only if it is rational.

Proof: One way to prove PROPOSITION IC2, is to show that for all $x, y \in X$ there is no real value function $u(\cdot)$ such that $u(x) \geq u(y)$ if and only if $x \succsim y$ when $\succsim$ is not complete or not transitive.

Suppose $\succsim$ is not complete. Then for some $x, y \in X$, not $x \succsim y$ and not $y \succsim x$. By the definition of the utility function, $x \succsim y$ implies $u(x) \geq u(y)$. Not $x \succsim y$ then implies not $u(x) \geq u(y)$ or $u(y) > u(x)$. By the definition of the utility function, $y \succsim x$ implies $u(y) \geq u(x)$. Not $y \succsim x$ must then imply not $u(y) \geq u(x)$ or $u(x) > u(y)$. But $u(y) > u(x)$ and $u(y) > u(x)$ cannot both be true if $u(\cdot)$ is a real valued function.

Suppose $\succsim$ is not transitive. Then for some $x, y, z \in X$, $x \succsim y$ and $y \succsim z$ implies not $x \succsim z$. By the definition of the utility function, $x \succsim y$, $y \succsim z$, and $x \succsim z$ imply $u(x) \geq u(y)$, $u(y) \geq u(z)$, and $u(x) \geq u(z)$. Not $x \succsim z$, must then imply not $u(x) \geq u(z)$ or $u(z) > u(x)$. For real valued functions, $u(x) \geq u(y)$ and $u(y) \geq u(z)$ imply $u(x) \geq u(z)$, which contradicts $u(z) > u(x)$. **Q.E.D.**

PROPOSITION IC2 is nice because it says we can use real valued utility functions to represent rational preference relations. However, some care must be taken because it is possible to have rational preferences that we cannot represent with a real valued utility function. Later, we will explore what additional assumptions we need to ensure that we can find a real valued utility function to represent an individual's preferences in all possible cases.

CHOICE BASED APPROACH

Definitions:
Choice Set (denoted by b): A set of alternatives such that $b \subseteq X$ and $b \neq \emptyset$.

Feasible Choice Set (denoted by $\boldsymbol{B}$): The set of all feasible *Choice Sets* given physical, institutional, or any other possible constraints.

Choice Rule (denoted by $C(\cdot)$): For all $b \in \boldsymbol{B}$, $C(b) \subseteq b$ and $C(b) \neq \emptyset$.

Choice Structure: A feasible choice set and choice rule for that set: $(\boldsymbol{B}, C(\cdot))$.

Weak Axiom of Revealed Preference (*WARP*): A choice structure satisfies *WARP* if for some $b \in \boldsymbol{B}$ with $x, y \in b$ we have $x \in C(b)$ then for any $b' \in \boldsymbol{B}$ with $x, y \in b'$ and $y \in C(b')$, we must also have $x \in C(b')$.

Revealed Preference Relation (denoted by $\succsim^*$): Given $(\boldsymbol{B}, C(\cdot))$, "$x$ is revealed preferred to y" or $x \succsim^* y$, if and only if there is some $b \in \boldsymbol{B}$ such that $x, y \in b$ and $x \in C(b)$.

WORKING EXAMPLE: Consider two individuals: Spencer and Mason. Spencer has a Lebron James rookie basketball card (denoted by LJ) and Mason has an Adrian Peterson rookie football card (denoted by AP) and a Brett Favre MVP football card (denoted by BF).

For this example, one possible choice set is b = {{LJ, BF}, {AP, BF}}, while another is b' = {{LJ, BF}, {AP, BF}, {LJ, AP, BF}}.

Suppose Spencer and Mason's father tell them that they can only trade cards one for one. That is, Spencer could trade LJ for AP or for BF, but not for AP and BF. If this is the case, $b \in \boldsymbol{B}_M$ and $b' \notin \boldsymbol{B}_M$ where $\boldsymbol{B}_M$ is Mason's feasible choice set. Alternatively, $b, b' \notin \boldsymbol{B}_S$ where $\boldsymbol{B}_S$ is Spencer's feasible choice set.

Ignoring their father's wishes such that $b, b' \in \boldsymbol{B}_M$, a choice rule for Mason, $C_M(\cdot)$, might say {AP, BF} $\in C_M(b)$, which also tells us {AP, BF} $\succsim^*$ {LJ, BF}. Suppose also that {LJ, BF} $\notin C_M(b)$ such that not {LJ, BF} $\succsim^*$ {AP, BF}, which we can write as {AP, BF} $\succ^*$ {LJ, BF} where $\succ^*$ is simply the revealed preference version of $\succ$.

Question: If Mason's choice structure satisfies *WARP*, what can we say about $C_M(b')$?

Answer: If {AP, BF} $\in C_M(b)$ and {LJ, BF}$\in C_M(b')$, then {AP, BF} $\in C_M(b')$

A natural question to ask at this point is how do the two approaches compare? That is, are the two approaches just different ways of saying exactly the same thing? The answer to this question is "no." While it is possible to show that any rational preference relation will generate a choice structure that is consistent with *WARP*, if we have a choice structure that satisfies *WARP*, we cannot always find a rational preference relation that would generate it. Let us prove it.

Definition:

Preferred Alternatives Set (denoted by $C^*(\cdot, \succsim)$): If b is a nonempty set of alternatives and $\succsim$ is a preference relation defined on b, $C^*(b, \succsim) \equiv \{x \in b: x \succsim y \text{ for all } y \in b\}$.

Rationalizing Preference Relation: A preference relation $\succsim$ and choice structure ($\boldsymbol{B}$, $C(\cdot)$) such that $C(b) = C^*(b, \succsim)$ for all $b \in \boldsymbol{B}$.

PROPOSITION IC3: A choice structure, ($\boldsymbol{B}$, $C^*(\cdot, \succsim)$), generated by a rational preference relation $\succsim$ satisfies the *Weak Axiom of Revealed Preferences.*

Proof:

Suppose $x, y \in b$ for $b \in \boldsymbol{B}$ such that $x \in C^*(b, \succsim)$, then $x \succsim y$ by the definition of the preferred alternatives set. Now consider some $b' \in \boldsymbol{B}$ such that $x, y \in b'$ and $y \in C^*(b', \succsim)$. Then $y \succsim z$ for all $z \in b'$ also by the definition of the preferred alternatives set. By transitivity, $x \succsim z$ for all $z \in b'$ such that $x \in C^*(b', \succsim)$ as required by the *Weak Axiom of Revealed Preferences.* **Q.E.D.**

It is also easy to come up with an example to show that a choice structure that satisfies the *Weak Axiom of Revealed Preferences* may not be rationalizable by some rational preference structure.

Using our working example, let X = {{LJ}, {AP}, {BF}}, $\boldsymbol{B}$ = {{{LJ}, {AP}}, {{LJ}, {BF}}, {{AP}, {BF}}}, and

$$C(b) = \begin{cases} \{LJ\}, & for\ \{\{LJ\},\{AP\}\} \\ \{BF\}, & for\ \{\{LJ\},\{BF\}\} \\ \{AP\}, & for\ \{\{AP\},\{BF\}\} \end{cases}.$$

Questions: How can we show that this *Choice Structure* satisfies the *Weak Axiom of Revealed Preferences*?

A choice structure satisfies *WARP* if for some $b \in \boldsymbol{B}$ with $x, y \in b$ we have $x \in C(b)$ then for any $b' \in \boldsymbol{B}$ with $x, y \in b'$ and $y \in C(b')$, we must also have $x \in C(b')$.

Let $b = \{\{LJ\}, \{AP\}\}$ with $\{LJ\}, \{AP\} \in b$ and $\{LJ\} \in C(b)$, then the only $b' \in \boldsymbol{B}$ with $\{LJ\}$, $\{AP\} \in b'$ is b, but $\{AP\} \notin C(b')$, so *WARP* doesn't apply. Alternatively, we could have $b = \{\{LJ\}, \{AP\}\}$ with $\{LJ\}, \{LJ\} \in b$ and $\{LJ\} \in C(b)$, then the b' can be $\{\{LJ\}, \{AP\}\}$ or $\{\{LJ\}, \{BF\}\}$. For $b' = \{\{LJ\}, \{AP\}\}$, $\{LJ\} \in C(b')$, so *WARP* does apply and is satisfied. For $b' = \{\{LJ\}, \{BF\}\}$, $\{LJ\} \notin C(b')$, so *WARP* doesn't apply. Similar arguments can be made for $b = \{\{LJ\}, \{BF\}\}$ or $b = \{\{AP\}, \{BF\}\}$, so this choice structure does satisfy *WARP*.

Question: How can we show that this *Choice Structure* cannot be generated by a *Rational Preference Relation*?

Note that *C*({{*LJ*}, {*AP*}}) = {*LJ*} implies {*LJ*} **f** * {*AP*}, *C*({{*LJ*}, {*BF*}}) = {*BF*} implies {*BF*} **f** * {*LJ*}, and *C*({{*AP*}, {*BF*}}) = {*AP*} implies {*AP*} **f** * {*BF*}. Now if preferences were transitive as required by rationality, {*LJ*} **f** * {*AP*} and {*AP*} **f** * {*BF*} would imply {*LJ*} **f** * {*BF*}, which contradicts {*BF*} **f** * {*LJ*}. Therefore, this choice structure describes a preference relation that is not transitive.