Correction to Notes ECON 8001-02
Fall 2009

Date: 10-7-09

PREFERENCE & UTILITY
Page 1:
"These monotonicity and nonsatiation definitions characterize varying degrees of what is often referred to as the "more is better" axiom of classical utility theory. What I mean by varying degrees is that a monotone preference relation is strongly monotone and a strongly monotone preference relation is locally nonsatiated, but a strongly monotone preference relation need not be monotone and a locally nonsatiated preference relation need not be strongly monotone."

should read

"These monotonicity and nonsatiation definitions characterize varying degrees of what is often referred to as the "more is better" axiom of classical utility theory. What I mean by varying degrees of "more is better" is that if preference relation is Strongly Monotone, then you can prove it is also Monotone, and if a preference relation is Monotone, then you can prove it is Locally Nonsatiated. However, you cannot prove a Locally Nonsatiated preference relation is Monotone or that a Monotone preference relation is Strongly Monotone."

CLASSICAL CONSUMER PROBLEM:
Page 8:

"For $v(p, w)$ to be nonincreasing in p, $v(p', w) \geq v(p, w)$ meaning $u(x(p', w')) \geq u(x(p, w))$ for $p \geq p'$ and $p \neq p'$. By definition $B_{p',w} \equiv \{x \in \Re^L_+ : w \geq p' \cdot x\}$ and $B_{p,w} \equiv \{x \in \Re^L_+ : w \geq p \cdot x\}$, which implies $B_{p,w} \subset B_{p',w}$. By definition, $x(p', w) = \{x \in B_{p',w} : u(x) \geq u(y)$ for all $y \in B_{p',w}\}$ or $x(p', w) \succsim x(p, w)$ and $v(p', w) \geq v(p, w)$."

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EXPENDITURE MINIMIZATION
Page 1:

"**EM3'** $p - g^*\nabla u(h^*) \leq 0$, $h^*\cdot[\, p - g^*\nabla u(h^*)] = 0$, and $h^* \geq 0$."
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"**EM3'** $p - g^*\nabla u(h^*) \geq 0$, $h^*\cdot[\, p - g^*\nabla u(h^*)] = 0$, and $h^* \geq 0$."

Page 4:

"**EM6** $$\frac{\partial e(p,u)}{\partial p_l} = h_l(p,u) + \sum_{l=1}^{L} p_l \frac{\partial h_l(p,u)}{\partial p_l}$$

Equation EM3 for an interior solution implies $p_l = g^* \frac{\partial u(h^*)}{\partial h_l}$, which when substituted into equation EM6 yields

EM7 $$\frac{\partial e(p,u)}{\partial p_l} = h_l(p,u) + g^* \sum_{l=1}^{L} \frac{\partial u(h^*)}{\partial h_l} \frac{\partial h_l(p,u)}{\partial p_l}.$$

Now equation EM4 implies $u = u(h(p, u))$, which when we totally differentiate with respect to p_l yields

EM8 $$0 = \sum_{l=1}^{L} \frac{\partial u(h(p,u))}{\partial h_l} \frac{\partial h_l(p,u)}{\partial p_l}."$$

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Now equation EM4 implies $u = u(h(p, u))$, which when we totally differentiate with respect to p_l yields

EM8 $$0 = \sum_{k=1}^{L} \frac{\partial u(h(p,u))}{\partial h_k} \frac{\partial h_k(p,u)}{\partial p_l} \text{ .”}$$

DUALITY & INTEGRABILITY
Page 5:
“**PROPOSITION D4**: Suppose that $u(\cdot)$ is a continuous utility function representing a locally nonsatiated and strictly convex preference relation $\succsim$ on $X = \Re^L_+$. If $x_k(p, w)$ is normal (inferior) for all p and w, then $h_k(p, u'') >(<) h_k(p, u')$ if $u'' > u'$.

Proof: Suppose $u'' > u'$. Choose w' and w'' such that $w'' > w'$, $v(p, w'') = u''$, and $v(p, w') = u'$. By PROPOSITION D1, $x_k(p, w') = h_k(p, v(p, w'))$ and $x_k(p, w'') = h_k(p, v(p, w''))$. Since $x_k(p, w)$ is normal (inferior) for all p and w, $x_k(p, w'') >(<) x_k(p, w')$, such that $h_k(p, v(p, w''))$ $>(<) h_k(p, v(p, w'))$. Note that $h_k(p, v(p, w'')) = h_k(p, u'')$ for all p and w. Similarly, $h_k(p, v(p, w')) = h_k(p', u')$ for all p and w. Therefore, for any $u'' > u'$, $h_k(p, u'') >(<) h_k(p, u')$ if $x_k(p, w)$ is normal (inferior). **Q.E.D.”**

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“PROPOSITION D4: Suppose that $u(\cdot)$ is a continuous utility function representing a locally nonsatiated and strictly convex preference relation $\succsim$ on $X = \Re^L_+$. If $x_k(p, w)$ is normal (inferior) for all p and w, then $h_k(p, u'') >(<) h_k(p, u')$ if $u'' > u'$.

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Date: 11-3-09

EXCHANGE ECONOMY

Page 1:

"$x_{li}^{e} \geq 0$: Specific quantity of commodity l ($l = 1, 2, \ldots, L$) consumed by individual i."

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Page 3:

"Figure 1 illustrates the notion of market equilibrium embodied in equation EE5, while Figure 2 illustrates the notion of a change in market equilibrium due to a change in the price of commodity *l* embodied in equations EE6, EE7, and EE8. The top half of the Figure 2 reflects a scenario where the net effect of a change in the price of commodity *l* is a decrease in aggregate demand that results in an equilibrium price decrease for commodity *k*, while the top half reflects a scenario where the net effect of a change in the price of commodity *l* is an increase in aggregate demand that results in an equilibrium price increase for commodity *k*. These illustrations should be familiar."

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Page 4:

"**EE9** $CS(p_k^{*}, p_{\sim k}^{D}, p_k^{*} x_{k1}^{e} + p_{\sim k}^{S} \cdot x_{\sim k1}^{e}, \ldots, p_k^{*} x_{kI}^{e} + p_{\sim k}^{S} \cdot x_{\sim kI}^{e}) =$

$$\int_{p_k^{*}}^{\infty} x_k\left(p_k, p_{\sim k}^{D}, p_{\sim k}^{*} \cdot x_{\sim k1}^{e} + p_{\sim k}^{S} \cdot x_{\sim k1}^{e}, \ldots, p_{\sim k}^{*} \cdot x_{\sim kI}^{e} + p_{\sim k}^{S} \cdot x_{\sim kI}^{e}\right) dp_k \text{."}$$

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Page 4:

"EE10 $PS(p_k^*, x_{1k}^e,\ldots, x_{Ik}^e) = p_k^* \sum_{i=1}^{I} x_{ik}^e$.

Adding up consumer and producer surplus gives us a total measure of welfare which we will refer to as the *Total Surplus*:

EE11 $TS(p_k^*, x_{1k}^e,\ldots, x_{Ik}^e) =$
$CS(p_k^*, p_{\sim k}^D, p_k^* x_{1k}^e + p_{\sim k}^S \cdot x_{1\sim k}^e,\ldots, p_k^* x_{Ik}^e + p_{\sim k}^S \cdot x_{I\sim k}^e) +$
$PS(p_k^*, x_{1k}^e,\ldots, x_{Ik}^e)$."

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$PS(p_k^*, x_{k1}^e,\ldots, x_{kI}^e)$."

Page 5:

"A *Walrasian Equilibrium* is a vector or prices p^* and aggregate demand $x^D = x(p, p\cdot x_1^e, \ldots, p\cdot x_I^e)$ such that $x(p^*, p^*\cdot x_1^e, \ldots, p^*\cdot x_I^e) \leq \sum_{i=1}^{I} x_i^e$.

Aggregate Excess Demand ($z(p)$): The amount by which aggregate demand exceeds aggregate supply: $z(p) = x(p, p\cdot x_1^e, \ldots, p^S\cdot x_I^e) - \sum_{i=1}^{I} x_i^e$."

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Page 7:

"PROPOSITION EE4 is essentially saying that a Walrasian Equilibrium must exist provided our excess demand functions are continuous and Walras Law holds. There is however an important caveat that we need to recognize. Notice that PROPOSITION EE4 says $p^* \in S^{L-1}$, which means that this price vectors has $L - 1$ elements. But we said that there are L different commodities not $L - 1$. What is going on here? Recall that COROLLARY EE1 implied that our L markets are not independent, so if we know what the equilibrium is in $L - 1$ markets, we automatically know what equilibrium is in the final market. All this follows from the fact that demand is homogenous of degree 0 in prices and wealth, which we said implies only relative prices matter. Therefore, to solve for a Walrasian Equilibrium, we must normalize prices. One way to accomplish this normalization is to set the price of one commodity equal to 1. Another alternative used in the proof of PROPOSITION EE4 is to normalize such that $\sum_{l=1}^{L} p_l = 1$ and there are only $L - 1$ independent prices."

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Page 8:

"Note that we can do the same exercise for all consumers, but the question that may arise is why? To understand why, consider Figure 8 where I have sketched offer curves for Consumer *A* and *B*. Notice that these offer curves intersect at x_{1A}^* and x_{2A}^*, and x_{1B}^* and x_{2B}^* inside the region where Consumer *A*'s and *B*'s indifference curves overlap. Note that if they are ever going to intersect, it has to be in this region because the offer curves must lie on or above u_A^2 and u_B^1. Also notice the budget constraint tangent to u_A^2 at the endowment (labeled p^0); tangent to u_B^1 at the endowment (labeled p^4); and passing through the endowment and the intersection of these two offers curves. What do these budget constraints tell us? For p^0, x_{1A}^e and x_{2A}^e is optimal for Consumer A, while x_{1B}^0 and x_{2B}^0 is optimal for Consumer B. But $x_{1A}^e + x_{1B}^0 > x_1^e$ and $x_{2A}^e + x_{2B}^0 < x_2^e$, which means there is excess demand for commodity 1 and excess supply for commodity 2, which cannot be a Walrasian Equilibrium. For p^4, x_{1A}^4 and x_{2A}^4 is optimal for Consumer A, while x_{1B}^e and x_{2B}^e is optimal for Consumer B. But $x_{1A}^4 + x_{1B}^e < x_1^e$ and $x_{2A}^4 + x_{2B}^e > x_2^e$, which means there is excess supply for commodity 1 and excess demand for commodity 2, which cannot be a Walrasian Equilibrium. For p^*, x_{1A}^* and x_{2A}^* is optimal for Consumer A, while x_{1B}^* and x_{2B}^* is optimal for Consumer B. Also, $x_{1A}^* + x_{1B}^* = x_1^e$ and $x_{2A}^* + x_{2B}^* = x_2^*$, which means there is no excess supply and no excess demand for either commodity, which can be a Walrasian Equilibrium. Therefore, any point where the offer curves intersect will be a Walrasian Equilibrium.

With a better understanding of price determination in general equilibrium, it is now time to consider the relationship between exogenous factors and the equilibrium price. To do this, the first question we need to address is what is exogenous? In our simple model, the only things that are exogenous are endowments and preferences. We will focus on a change in endowments. From equation EE5 we know $x(p, p{\cdot}x_1^e, \ldots, p{\cdot}x_I^e) = \sum_{i=1}^{I} x_i^e$, but in a Walrasian Equilibrium these demands will not all be independent and we need to normalize price. Lets set $p_1^* = 1$ and totally differentiate the first $L - 1$ market clearing demand equations assuming our equilibrium price vector is unique and everything is differentiable:"

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Pages 8 and 9:
"With partial equilibrium analysis, we talked about welfare using consumer surplus. As mentioned earlier, consumer surplus has its problems in terms of trying to relate it directly back to individual preferences and welfare. In general equilibrium analysis, we will us the concept of *Pareto Efficiency* in order to develop more solid theoretical underpinnings for our welfare analysis."

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Page 9:

"**EE7** $$L = u_1(x_1) + \sum_{i=1}^{I} l_i\left(u_i(x_i) - u_i\right) + \sum_{i=1}^{I} g_l\left(\sum_{i=1}^{I}\left(x_{li}^e - x_{li}\right)\right)$$"

should read

"EE7 $$L = u_1(x_1) + \sum_{i=2}^{I} l_i\left(u_i(x_i) - u_i\right) + \sum_{l=1}^{L} g_l\left(\sum_{i=1}^{I}\left(x_{li}^e - x_{li}\right)\right)$$**"**

Page 11:
"Figure 10 illustrates how we can indentify all possible Pareto Efficient allocations given consumers' endowments. Note that many of these points do not represent Pareto improvements given initial endowments. They are nonetheless Pareto Efficient.

Taking the intersection of these Pareto improvements and Pareto Efficient set yields what is often referred to as the *Contract Curve* (see Figure 11). This Contract Curve shows all Pareto Efficient allocations that that consumers might be willing to trade to given their initial endowments."

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Date: 11-4-09

EXCHANGE ECONOMY

Page 8:
"Equation EE6 can be written in matrix form as $\Omega dp^{*} = \Phi dx^{e}$ where Ω has dimensions L -1 × L -1, dp^{*} has dimensions L -1 × 1, Φ has dimensions $I(L$ -1) × $I(L$ -1), and dx^{e} has dimensions $I(L$ -1) × 1. This system of equations can be solved using tools like Cramer's Rule. There is not much insightful to be gained by tackling this problem in generality (even if we just do a 2×2 Edgeworth box), so we will instead move on to Welfare analysis."

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Date: 11-19-09

HOMEWORK #6: ANSWERS

Problem 2, part d:
"From part (a), we know that ${x_1^a}^* = \frac{w^a}{4p_1^B}$ and ${x_1^b}^* = \frac{3w^b}{4p_2^B}$. Substituting these quantities in to $p_1^B\ {x_1^a}^* + p_2^B\ {x_2^a}^* = w^a$ and $p_1^B\ {x_1^b}^* + p_2^B\ {x_2^b}^* = w^b$ and rearranging yields ${x_2^a}^* = \frac{3w^a}{4p_2^B}$ and ${x_2^b}^* = \frac{w^b}{4p_2^B}$. Recall that $w^a = p_1^S {x_1^a}^e + p_2^S {x_2^a}^e + R^a$ and $w^b = p_1^S {x_1^b}^e + p_2^S {x_2^b}^e + R^b$. Given our assumption and Walrasian equilibrium price, $w^a = ({p_2}^* - 0.5t)15 + {p_2}^*15 = 30{p_2}^* - 7.5t$ and $w^b = ({p_2}^* - 0.5t)15 + {p_2}^*15 + 30t = 30{p_2}^* + 22.5t$. Substituting back into our demands then yields ${x_1^a}^* = \frac{30{p_2}^* - 7.5t}{4(({p_2}^* - 0.5t) + t)} = \frac{30{p_2}^* - 7.5t}{4{p_2}^* + 2t}$, ${x_2^a}^* = \frac{3(30{p_2}^* - 7.5t)}{4{p_2}^*} = \frac{90{p_2}^* - 22.5t}{4{p_2}^*}$, ${x_1^b}^* = \frac{3(30{p_1}^* + 22.5t)}{4(({p_2}^* - 0.5t) + t)} = \frac{3(30{p_1}^* + 22.5t)}{4{p_2}^* + 2t}$, and ${x_2^b}^* = \frac{30{p_1}^* + 22.5t}{4{p_2}^*}$.

For this solution to be Pareto Efficient, it must satisfy equation (1.1) – (1.7) (actually, we can forget about equation (1.5) because we are free to choose u^b so it is satisfied). Assuming the solution is interior, equations (1.1) – (1.4) imply $\frac{x_2^{a^*}}{3x_1^{a^*}} = \frac{3x_2^{b^*}}{x_1^{b^*}}$ (consumers equate their marginal rates of substitution) must be true. To verify this, note that $\frac{x_2^{a^*}}{3x_1^{a^*}} = \frac{\frac{3(30{p_2}^*-7.5t)}{4{p_2}^*}}{3\left(\frac{30{p_2}^*-7.5t}{4{p_2}^*+2t}\right)} = \frac{4{p_2}^*+2t}{4{p_2}^*}$ and $\frac{3x_2^{b^*}}{x_1^{b^*}} = \frac{3\left(\frac{30{p_1}^*+22.5t}{4{p_2}^*}\right)}{\frac{3(30{p_1}^*+22.5t)}{4{p_2}^*+2t}} = \frac{4{p_2}^*+2t}{4{p_2}^*}$. Therefore, in our Walrasian Equilibrium with a tax consumers will indeed equate their marginal rates of substitution. Equation (1.6) and (1.7) imply that total consumption will equal total endowments. Note that $x_1^{a^*} + x_1^{b^*} = \frac{30{p_2}^*-7.5t}{4{p_2}^*+2t} + \frac{3(30{p_1}^*+22.5t)}{4{p_2}^*+2t} = \frac{120{p_1}^*+60t}{4{p_2}^*+2t} = 30 = x_1^{a^e} + x_1^{b^e}$ and $x_2^{a^*} + x_2^{b^*} = \frac{90{p_2}^*-22.5t}{4{p_2}^*} + \frac{30{p_1}^*+22.5t}{4{p_2}^*} = \frac{120{p_2}^*}{4{p_2}^*} = 30 = x_2^{a^e} + x_2^{b^e}$ as required. Therefore, the Walrasian Equilibrium with a tax on consumers is Pareto Efficient."

should read

"From part (a), we know that $x_1^{a^*} = \frac{w^a}{4p_1^B}$ and $x_1^{b^*} = \frac{3w^b}{4p_2^B}$. Substituting these quantities in to $p_1^B\ x_1^{a^*} + p_2^B\ x_2^{a^*} = w^a$ and $p_1^B\ x_1^{b^*} + p_2^B\ x_2^{b^*} = w^b$ and rearranging yields $x_2^{a^*} = \frac{3w^a}{4p_2^B}$ and $x_2^{b^*} = \frac{w^b}{4p_2^B}$. Recall that $w^a = p_1^S x_1^{a^e} + p_2^S x_2^{a^e} + R^a$ and $w^b = p_1^S x_1^{b^e} + p_2^S x_2^{b^e} + R^b$. Given our assumption and Walrasian equilibrium price, $w^a = ({p_2}^* - 0.5t)15 + {p_2}^*15 + 30t = 30{p_2}^* + 22.5t$ and $w^b = ({p_2}^* - 0.5t)15 + {p_2}^*15 = 30{p_2}^* - 7.5t$. Substituting back into our demands then yields $x_1^{a^*} = \frac{30{p_2}^*+22.5t}{4(({p_2}^*-0.5t)+t)} = \frac{30{p_2}^*+22.5t}{4{p_2}^*+2t}$, $x_2^{a^*} = \frac{3(30{p_2}^*+22.5t)}{4{p_2}^*}$, $x_1^{b^*} = \frac{3(30{p_2}^*-7.5t)}{4(({p_2}^*-0.5t)+t)} = \frac{3(30{p_2}^*-7.5t)}{4{p_2}^*+2t}$, and $x_2^{b^*} = \frac{30{p_2}^*-7.5t}{4{p_2}^*}$.

For this solution to be Pareto Efficient, it must satisfy equation (1.1) – (1.7) (actually, we can forget about equation (1.5) because we are free to choose u^b so it is satisfied). Assuming the solution is interior, equations (1.1) – (1.4) imply $\frac{x_2^{a^*}}{3x_1^{a^*}} = \frac{3x_2^{b^*}}{x_1^{b^*}}$ (consumers equate their marginal rates of substitution) must be true. To verify this, note that $\frac{x_2^{a^*}}{3x_1^{a^*}} = \frac{\frac{3(30{p_2}^*+22.5t)}{4{p_2}^*}}{3\left(\frac{30{p_2}^*+22.5t}{4{p_2}^*+2t}\right)} = \frac{4{p_2}^*+2t}{4{p_2}^*}$ and $\frac{3x_2^{b^*}}{x_1^{b^*}} = \frac{3\left(\frac{30{p_2}^*-7.5t}{4{p_2}^*}\right)}{\frac{3(30{p_2}^*-7.5t)}{4{p_2}^*+2t}} = \frac{4{p_2}^*+2t}{4{p_2}^*}$. Therefore, in our Walrasian Equilibrium with a tax consumers will indeed equate their marginal rates of substitution. Equation (1.6) and (1.7) imply that total consumption will equal total endowments. Note that $x_1^{a^*} + x_1^{b^*} = \frac{3(30p_1{}^*+22.5t)}{4p_2{}^*+2t} + \frac{30p_2{}^*-7.5t}{4p_2{}^*+2t} = \frac{120p_1{}^*+60t}{4p_2{}^*+2t} = 30 = x_1^{a^e} + x_1^{b^e}$ and $x_2^{a^*} + x_2^{b^*} = \frac{30p_1{}^*+22.5t}{4p_2{}^*} + \frac{3(30p_2{}^*-7.5t)}{4p_2{}^*} = \frac{120p_2{}^*}{4p_2{}^*} = 30 = x_2^{a^e} + x_2^{b^e}$ as required. Therefore, the Walrasian Equilibrium with a tax on consumers is Pareto Efficient."

Date: 11-24-09

AGGREGATE SUPPLY

Page 6:
"Turning to cost functions, let $c_j(r, q_j)$ and $z_j(r, q_j)$ be the *j*th producer's cost function and conditional factor demands derived from a closed production set that also satisfies free disposal and convexity. Consider the problem $\min_{q_1 \geq 0, \ldots, q_J \geq 0} \sum_{j=1}^{j} c_j(r, q_j)$ subject to $q = \sum_{j=1}^{j} q_j$, which yields the distribution of output across producers that minimizes aggregate production: $q_j(r, q)$ for $j =$''

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Page 11:

"AS37 $$q = \begin{cases} \infty, & p > 75 \\ [0, 5, 10, \ldots, \infty], & p = 75 \\ 0, & p < 75 \end{cases}$$ "

Should read

"AS37 $$q = \begin{cases} \infty, & p > 25 \\ [0, 5, 10, \ldots, \infty], & p = 25 \\ 0, & p < 25 \end{cases}$$ "