AGGREGATE DEMAND

ECON 8001-2 **Instructor: Terry Hurley**

INTRODUCTION (MWG Chapter 4 & Varian Chapter 9)

We have spent an enormous amount of time trying to better understand individual demand. What we need to do now is investigate how individual demands will coalesce in a market with many consumers, so we can begin talking about the functionality of markets in determining prices of commodities and the distribution of commodities across individuals. Of particular interest to us will be to what extent useful properties of individual demand carry forward into a market with many consumers. We will also be interested in understanding the limitations of aggregation in terms of empirical work and welfare evaluation.

PROPERTIES FOR INDIVIDUAL & AGGREGATE DEMAND

Obtaining aggregate demand from individual demand is straightforward. Suppose we have I individuals that are subscripted by i. The Marshallian demand for the ith individual with wealth w_i and facing prices $p >> 0$ is $x_i(p, w_i)$ (note that we are assuming everyone pays the same prices). Aggregate Demand is then just the sum of individual demand:

AD1 $$x(p, w_1, w_2,\dots, w_I) = \sum_{i=1}^{I} x_i(p,w_i).$$

Recall that if an individual's preference relation is rational, continuous, and locally nonsatiated, then the utility maximization problem has a solution and produces individual demands that are continuous, homogeneous of degree 0 in prices and wealth, and satisfy Walras Law. An interesting question is whether these properties carry forward to Aggregate Demand.

Continuity is the most straightforward. The sum of continuous functions is continuous, so if individual demands are continuous, aggregate demand will be continuous.

Homogeneity takes slightly more work. If Aggregate Demand is homogenous of degree 0 in prices and wealth then $x(ap, aw_1, aw_2,\dots, aw_I) = x(p, w_1, w_2,\dots, w_I)$ for any $a > 0$. Note that $x(ap, aw_1, aw_2,\dots, aw_I) = \sum_{i=1}^{I} x_i(ap,aw_i)$, but if individual Marshallian Demands are homogeneous of degree 0 in prices and wealth then $\sum_{i=1}^{I} x_i(ap,aw_i) = \sum_{i=1}^{I} x_i(p,w_i) = x(p, w_1, w_2,\dots, w_I)$.

Satisfaction of Walras Law is about the same amount of work. Aggregate demand satisfies Walras Law if $p \cdot x(p, w_1, w_2,\dots, w_I) = \sum_{i=1}^{I} w_i$. Note that $p \cdot x(p, w_1, w_2,\dots, w_I) = \sum_{i=1}^{I} p \cdot x_i(p,w_i)$. If individual demands satisfy Walras Law, then $p \cdot x_i(p, w_i) = w_i$ for all i such that $\sum_{i=1}^{I} p\cdot x_i(p,w_i) = \sum_{i=1}^{I} w_i$, which gives us the desired result.

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PROPOSITION AD1: If Marshallian Demand $x_i(p, w_i)$ is continuous, homogeneous of degree 0 in price and wealth, and satisfies Walras Law for all i = 1, …, I, then the aggregate demand $x(p, w_1, w_2, \ldots, w_I) = \sum_{i=1}^{I} x_i(p, w_i)$ is continuous, homogeneous of degree 0 in prices and wealth, and satisfies Walras Law.

It is important to note that while Aggregate Demand will have many of the same properties of individual Marshallian demand, there are some properties that may not be shared. One of these is the Weak Axiom of Revealed Preferences. If individual Marshallian Demand is homogeneous of degree 0 in prices and wealth; satisfies Walras Law; and satisfies the Weak Axiom of Revealed Preferences then for any $(p', w_i') = (p', p' \cdot x_i(p, w_i))$, $(p' - p) \cdot [x_i(p', w_i') - x_i(p, w_i)] \leq 0$. Again, this is analogous to saying Hicksian Demands have negative own price effects. Unfortunately, this property does not carry over to Aggregate Demand. MWG provide a variety of restrictions on the distribution of wealth and properties of individual Marshallian Demand that make it so the Aggregate Demand satisfies the Weak Axiom of Revealed Preferences. I will not spend time regurgitating them here because our limited time can be spent more productively elsewhere.

AGGREGATE DEMAND & AGGREGATE WEALTH

It is important to notice that Aggregate Demand depends on the distribution of wealth, which can create challenges for economists who often only observe total wealth (or average wealth), prices, and aggregate demand. Therefore, a question that has received some attention is: Under what restrictions on preferences is it reasonable to define aggregate demand in terms of aggregate, instead of individual wealth? That is, when does $x(p, w_1, w_2, \ldots, w_I) = x(p, W)$ where $W = \sum_{i=1}^{I} w_i$.

To gain insight into the answer to this question, we can totally differentiate with respect to individual and aggregate wealth:

AD2 $$\sum_{i=1}^{I} \frac{\partial x_{li}(p, w_i)}{\partial w_i} dw_i = \frac{\partial x_l(p, W)}{\partial W} dW \text{ for all } l.$$

Now suppose we hold aggregate wealth constant such that $dW = 0$, which also implies $\sum_{i=1}^{I} dw_i = 0$. Equation AD2 then implies

AD3 $$\sum_{i=1}^{I} \frac{\partial x_{li}(p, w_i)}{\partial w_i} dw_i = 0 \text{ for all } l.$$

Now if $\frac{\partial x_{li}(p,w_i)}{\partial w_i} = \frac{\partial x_{lj}(p,w_j)}{\partial w_j}$ for all *i*, *j*, and *l*, then equation AD3 will hold in general. But what does this mean? $\frac{\partial x_{li}(p,w_i)}{\partial w_i}$ is the slope of an Engel curve, so this restriction means that the slope of individual Engel curves are the same for all levels of wealth.

PROPOSITION AD2: A necessary and sufficient condition for $\frac{\partial x_{li}(p,w_i)}{\partial w_i} = \frac{\partial x_{lj}(p,w_j)}{\partial w_j}$ for all $i, j = 1, \ldots, I$ and $l = 1, \ldots, L$ at any price vector p is that preferences admit indirect utility functions of the Gorman form: $v_i(p, w_i) = a_i(p) + b(p)w_i$.

To see why this works, we can use Roy's Identity to recover an individual's Marshallian Demand and then we can use this demand to calculate the slope of the Engel Curves:

AD4 $$x_{li}(p, w_i) = -\frac{\frac{\partial v_i(p,w_i)}{\partial p_l}}{\frac{\partial v_i(p,w_i)}{\partial w_i}} = -\frac{\frac{\partial a_i(p)}{\partial p_l} + \frac{\partial b(p)}{\partial p_l} w_i}{b(p)} \text{ such that}$$

AD5 $$\frac{\partial x_{li}(p,w_i)}{\partial w_i} = -\frac{\frac{\partial b(p)}{\partial p_l}}{b(p)}.$$

Notice that equation AD5 (the slope of individual *i*'s Engel Curve for commodity *l*) depends only on $b(p)$, which is independent of w_i and even *i*. Therefore, it will be the same for all w_i and *i*. PROPOSITION AD2 however goes further than this because it says the Gorman form is both necessary and sufficient. Therefore, this is the only type of indirect utility function that will satisfy $\frac{\partial x_{li}(p,w_i)}{\partial w_i} = \frac{\partial x_{lj}(p,w_j)}{\partial w_j}$ for all *i*, *j*, and *l*.

This Gorman form for the indirect utility function is very restrictive, which has lead to attempts to find less restrictive relations that are still theoretically valid. For example, one might explore the conditions under which $x(p, F(w; \mu)) = x(p, w_1, w_2, \ldots, w_I)$ where $F(w; \mu)$ is the distribution of individual wealth with parameter vector μ (e.g. the mean and variance of wealth). Another approach that can be taken is to assume that individual wealth is known to be some function of aggregate wealth and prices such that $w_i = w_i(p, W)$ for all *i*, which implies $x(p, W) = \sum_{i=1}^{I} x_i(p, w_i(p,W))$. As you might suspect, you can find alternative sets of restrictions that will work under varying underlying assumptions and are not quite as unpalatable as assuming all individuals have Gorman preferences.

WELFARE ANALYSIS & AGGREGATE DEMAND

Previously, we showed how to evaluate the effect of changing circumstances on individual welfare using concepts like the Equivalent and Compensating Variations. The question of interest here is: To what extent can we use the Equivalent and Compensating Variations to evaluate the welfare of the masses (i.e. all consumers) using Aggregate Demand? To answer this question, we are going to need a little more structure. First, we are going to assume that there is some wealth distribution rule that depends only on prices and aggregate wealth: $w_i = w_i(p, W)$ for all i. We will assume this rule is continuous and homogeneous of degree one in prices and wealth, so that we preserve the continuity and homogeneity properties of Aggregate Demand that follow from individual demand. We will then define a *Positive Representative Consumer* whose preferences yield Aggregate Demand. Finally, we are going to have to make some assumptions regarding how we measure the welfare of the masses.

DEFINITIONS:

Positive Representative Consumer: A fictitious consumer with the preference relation $\succsim$ on $\Re^L_+$ such that the Aggregate Demand $x(p, W)$ satisfies $x(p, W) \succsim y$ for all $W \geq p \cdot y$.

Social Welfare Function (of the Bergson-Samuelson form): A real value function U: $\Re^I \to \Re$ that assigns a social utility level for every possible vector of individual utilities: $(u_1,\ldots, u_I) \in \Re^I$.

Now that we have more structure we can consider whether it is possible to draw welfare inferences for the masses based on the preferences of a *Positive Representative Consumer.*

Let us consider the problem of finding a wealth distribution rule that maximizes social welfare given prices and Aggregate Wealth. We can write this problem as

AD6 $$\max_{x_{li}\geq 0 \text{ for all } l \text{ and } i} U\left(u(x_1),\ldots,u(x_I)\right) \text{ subject to } p\cdot\sum_{i=1}^{I} x_i \leq W,$$

but this would not be taking advantage of what we already know. Recall that we can represent individual preferences using indirect utility functions, so we can simplify equation AD6 to

AD6' $$\max_{w_i\geq 0 \text{ for all } i} U\left(v_1(p,w_1),\ldots,v_I(p,w_I)\right) \text{ subject to } \sum_{i=1}^{I} w_i \leq W.$$

The Lagrangian for this problem is $L = U\left(v_1(p,w_1),\ldots,v_I(p,w_I)\right) + \lambda\left(W - \sum_{i=1}^{I} w_i\right)$, with the first order conditions

AD7 $$\frac{\partial L}{\partial w_i} = \frac{\partial U\left(v_1(p,w_1^*),\ldots,v_I(p,w_I^*)\right)}{\partial u_i}\frac{\partial v_i(p,w_i^*)}{\partial w_i} - \lambda^* \leq 0, \qquad \frac{\partial L}{\partial w_i}w_i^* = 0, \text{ and } w_i^* \geq 0 \text{ for}$$

all i, and

AD8 $\frac{\partial L}{\partial l} = W - \sum_{i=1}^{I} w_i * \geq 0$, $\frac{\partial L}{\partial l} l^* = 0$, and $l^* \geq 0$.

Now if this problem has a solution (we will not dwell on what conditions are required for a solution to exist because they should now be familiar for optimization problems like this), it will be of the form $w_i = w_i(p, W)$ for all i, which is the wealth distribution rule we are looking for, and we can write the value function as $V(p, W) = U(v_1(p,w_1(p,W)),\ldots, v_I(p,w_I(p,W)))$. Note that this value function looks an awful lot like an indirect utility function, which raises some interesting questions:

(i) Does $V(p, W)$ satisfy the standard properties of an indirect utility function?
(ii) Will Roy's Identity apply in terms of deriving Aggregate Demand?

If the answer to these questions is yes, then we can justify using our representative consumer's Aggregate Demand to make welfare inferences. Said another way, our Positive Representative Consumer will also be a *Normative Representative Consumer*:

DEFINITION:
Normative Representative Consumer: A Positive Representative Consumer with a Social Welfare Function U: $\Re^I \rightarrow \Re$ and income distribution rule $w_i = w_i(p, W)$ rule that is continuous and homogeneous of degree 1 in prices and wealth such that $U(v_1(p,w_1(p,W)),\ldots, v_I(p,w_I(p,W))) > U(v_1(p,w_1'),\ldots, v_I(p,w_I'))$ for all $(w_1',\ldots, w_I') \neq (w_1(p,W)),\ldots, w_I(p,W))$ and all (p, W).

Homogeneity of degree 0 is easy. $V(ap, aW) = V(p, W)$ for any $a > 0$ if the homogeneity property is valid. $V(ap, aW) = U(v_1(ap,w_1(ap, aW)),\ldots, v_I(ap,w_I(ap,Wa)))$. Since indirect utility functions are homogeneous of degree 0 in prices and wealth and the income distribution rule is assumed to be homogeneous of degree 1 in income and wealth, $U(v_1(ap,w_1(ap, aW)),\ldots, v_I(ap,w_I(ap,Wa)) = U(v_1(p,w_1(p, W)),\ldots, v_I(p,w_I(p,W)) = V(p, W)$ as desired. We should be a little careful here however because we have assumed the wealth distribution rule is homogeneous of degree 1 in prices and wealth, but not proven that it has to be.

To verify that the $V(p, W)$ is strictly increasing in W, we can differentiate:

AD9 $$\frac{\partial V(p,W)}{\partial W} = \sum_{i=1}^{I} \frac{\partial U}{\partial u_i}\frac{\partial v_i}{\partial w_i}\frac{\partial w_i}{\partial W}$$

where functional arguments are suppressed to ease notation. Note that for an interior solution equation AD7 implies

AD7' $$\frac{\partial U}{\partial u_i} = \frac{l^*}{\frac{\partial v_i}{\partial w_i}},$$

while equation AD8 tells us

AD8’ $W = \sum_{i=1}^{I} w_i(p,W)$.

Totally differentiating equation AD8’ with respect to *W* yields $dW = \sum_{i=1}^{I} \frac{\partial w_i(p,W)}{\partial W} dW$ or $\sum_{i=1}^{I} \frac{\partial w_i}{\partial W}$ = 1. Substituting all of these results back into equation AD9, yields

AD9’ $\frac{\partial V(p,W)}{\partial W} = l^*$,

which must be positive for an interior solution.

To verify that the $V(p, W)$ is non-increasing in prices, we can again differentiate

AD10 $\frac{\partial V(p,W)}{\partial p_l} = \sum_{i=1}^{I} \frac{\partial U}{\partial u_i} \left(\frac{\partial v_i}{\partial p_l} + \frac{\partial v_i}{\partial w_i} \frac{\partial w_i}{\partial p_l} \right)$.

Totally differentiating equation AD8’ with respect to p_l yields $0 = \sum_{i=1}^{I} \frac{\partial w_i(p,W)}{\partial p_l} dp_l$ or $\sum_{i=1}^{I} \frac{\partial w_i}{\partial p_l}$ = 0. Substituting this result and the result in equation AD7’ into equation AD10 yields

AD10’ $\frac{\partial V(p,W)}{\partial p_l} = l^* \sum_{i=1}^{I} \frac{\frac{\partial v_i}{\partial p_l}}{\frac{\partial v_i}{\partial w_i}}$.

Roy’s Identity for individual demands implies $\frac{\frac{\partial v_i}{\partial p_l}}{\frac{\partial v_i}{\partial w_i}}$ = $-x_{li}(p, w_i(p, W))$, which when combined with the fact that $l^* > 0$, means $\frac{\partial V(p,W)}{\partial p_l} < 0$ as it should for an indirect utility function.

It is also possible to show quasi-convexity in prices and continuity in prices and aggregate wealth (which is why the wealth distribution rule is assumed to be continuous), but I would rather turn to Roy’s Identity instead. If Roy’s Identity holds in the sense of permitting us to recover Aggregate Demand then

AD11 $$x_l(p,W) = -\frac{\frac{\partial V(p,W)}{\partial p_l}}{\frac{\partial V(p,W)}{\partial W}} = \sum_{i=1}^{I} x_{li}(p, w_i(p,W)) \text{ for all } l.$$

Stealing from our previous results gives us

AD12 $$-\frac{\frac{\partial V(p,W)}{\partial p_l}}{\frac{\partial V(p,W)}{\partial W}} = -\frac{l * \sum_{i=1}^{I} - x_{li}(p, w_i(p,W))}{l *} = \sum_{i=1}^{I} x_{li}(p, w_i(p,W))$$

as desired.

So it does indeed appear that we have an indirect utility function for a Normative Representative Consumer, which we can use for aggregate welfare analysis. To do this, we need to use Aggregate Demand to recover the expenditure function (i.e. the integrability problem): $E(p, U)$. We can then use this expenditure function to get an indirect utility function that we can use to estimate the Equivalent and Compensating Variations.

At this point, you all might be getting pretty excited about this seemingly powerful result. Don't get too excited. Remember that for all this to work, we had to assume that income was being distributed by some continuous and homogeneous of degree 1 wealth distribution rule that represented the optimal distribution of wealth given some Social Welfare Function, which are all pretty strict conditions.

I will conclude this section with a few final notes:

It is completely possible to have an Aggregate Demand relationship that is consistent with a Positive Representative Consumer, but not with a Normative one.

Individual preferences that are consistent with the Gorman form above will produce an aggregate indirect utility function of the Gorman form that is independent of our choice of the Social Welfare Function: $V(p, W) = \sum_{i=1}^{I} a_i(p) + b(p)W$. This means that if we are comfortable adopting the Gorman representation of the indirect utility function, we should be comfortable using this aggregate indirect utility function for our welfare analysis.

There has been an increasing interest in collecting micro level data to estimate individual specific demand relationships with microeconometric techniques. I suspect that an important motivation for this new emphasis is the restrictiveness of the environment necessary to develop theoretically valid positive and normative inferences from aggregate data. However, it is also motivated by the fact that computers are faster, can store more data, and it is easier and less costly than ever to collect micro level data.