

MICROECONOMIC ANALYSIS

ECON 8001-2

Fall 2009

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Due: 11-10-09

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HOMEWORK #7: ANSWERS

Note: When writing up your answers, carefully define all new notation and terms that you introduce, and write in complete sentences and paragraphs.

1. Consider the production function $q = f(z) \in \mathfrak{R}_+$ where $z \in \mathfrak{R}_+^N$. Assume that this production function is differentiable in z for all z .
 - (a) Show that the elasticity of scale is equal to the sum of output elasticities: $e(z) = \sum_{n=1}^N e_n(z)$ where $e(z)$ is the elasticity of scale and $e_n(z)$ is the output elasticity for factor n .
 - (b) If $f(z)$ is homothetic and $f(z^0) = f(z^1)$ for $z^0, z^1 \in \mathfrak{R}_+^N$, show that $f(az^0) = f(az^1)$ for $a > 0$.
 - (c) If $f(z)$ is homothetic, show that the marginal rate of technical substitution for any two factors l and k must be equal when evaluated at $z \in \mathfrak{R}_+^N$ and az for $a > 0$.
2. Consider the function $g(r_1, r_2, q) = (r_1 + r_1^{a_1} r_2^{a_2} + r_2)q^b$ where $r_1 > 0$ and $r_2 > 0$ are factor prices; $q \in \mathfrak{R}_+$ is output; and a_1, a_2 , and b are constant parameters.
 - (a) Assume this function is a valid cost function for a firm that produces output q using two factors ($z_1 \geq 0$ and $z_2 \geq 0$ corresponding to r_1 and r_2) with an input requirement set that is regular, monotonic, and strictly convex. Derive the firm's conditional factor demands.
 - (b) What are four other conditions this cost function must satisfy? Determine what restrictions on a_1, a_2 , and b (if any) are needed to satisfy these properties.
 - (c) Derive a transformation function that is consistent with this cost function assuming $a_1 = a_2 = 1/2$.
 - (d) What additional restrictions on b (if any) are needed for the transformation function you derived in part (c) to exhibit constant returns to scale?
3. Using the transformation function you derived in question 2 part (c):
 - (a) assuming z_2 is fixed at z_2^0 in the short-run, derive the conditional factor demand for z_1 .
 - (b) given this short-run conditional factor demand, find the short-run total, variable, and fixed costs. Also find the short-run average total, variable, and fixed costs.
 - (c) assuming $a_1 = a_2 = 1/2$, substitute the long-run conditional factor demand you derived for z_2 in problem 2 part (a) into the total cost function you found in part (b) for z_2^0 . How does this cost function compare to the cost function in problem 2 assuming $a_1 = a_2 = 1/2$? What is the economic intuition of your result?