MICROECONOMIC ANALYSIS
ECON 8001-2
Fall 2009

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HOMEWORK #6

Note: When writing up your answers, carefully define all new notation and terms that you introduce, and write in complete sentences and paragraphs.

1. Consider a world with only two consumers labeled ***a*** and ***b***, and two commodities labeled 1 and 2. Consumer ***a***'s continuous, strictly convex, and locally nonsatiated preference relation **f̰** ^{a} on $\hat{A}_+^2$ can be represented by the utility function $u^a(x_1^a, x_2^a) = x_1^a {x_2^a}^3$, while consumer ***b***'s continuous, strictly convex, and locally nonsatiated preference relation **f̰** ^{b} on $\hat{A}_+^2$ can be represented by the utility function $u^b\left(x_1^b, x_2^b\right) = {x_1^b}^3 x_2^b$. There is no production. Instead, consumer ***a*** is blessed with the endowment $\left({x_1^a}^e, {x_2^a}^e\right)$ = (15, 15), while consumer ***b*** is blessed with the endowment $\left({x_1^b}^e, {x_2^b}^e\right)$ = (15, 15).

   (a) Set up the Pareto Efficient optimization problem for maximizing consumer ***a***'s utility subject to consumer ***b***'s utility being at least u^b (assuming u^b is feasible) and total consumption not exceeding total endowments. Derive the first order conditions (***Note that the second order conditions will be satisfied***).
   (b) Characterize the set of Pareto Efficient allocations in terms of x_2^a assuming and interior solution (i.e. $x_1^a > 0$, $x_2^a > 0$, $x_1^b > 0$ and $x_2^b > 0$).
   (c) Is there a Pareto efficient allocation where $x_1^a > 0$, $x_2^a = 0$, $x_1^b > 0$ and $x_2^b = 30$? Explain.
   (d) Is $x_1^a = 0$, $x_2^a = 0$, $x_1^b = 30$ and $x_2^b = 30$ a Pareto Efficient allocation? Explain.
   (e) Using an Edgeworth box, illustrate (i) representative indifference curves for each consumer (be sure to carefully label your axes, the direction of increased consumption for each consumer, and the direction of increased utility for each consumer), (ii) the conditions satisfied by Pareto Efficient allocations, and (iii) the entire set of Pareto efficient allocations.

2. For the consumers in problem 1, assume they can sell commodities 1 and 2 at prices $p_1^S > 0$ and $p_2^S > 0$ and buy commodities 1 and 2 at prices $p_1^B > 0$ and $p_2^B > 0$. Define consumer ***a***'s wealth as $w^a = p_1^S x_1^{a^e} + p_2^S x_2^{a^e} + R^a$ and consumer *b*'s wealth as $w^b = p_1^S x_1^{b^e} + p_2^S x_2^{b^e} + R^b$ where R^a and R^b will be defined later.

   (a) Derive aggregate demand for commodity 1 in terms of p_1^B, p_2^B, w^a, and w^b.
   (b) Assume there is a tax on commodity 1 such that $p_1^B = p_1 + t$ where $t \geq 0$. Find the partial equilibrium price for commodity 1 (i.e. p_1) in terms p_2^B, t, w^a, and w^b. How does a change in the tax affect this partial equilibrium price? What is the equilibrium price paid by the buyer? How does a change in the tax affect this equilibrium price? What is the economic intuition of this result?
   (c) Again, assume there is a tax on commodity 1 such that $p_1^B = p_1 + t$ where $t \geq 0$. Also assume that $p_1^S = p_1$ (i.e. the selling price for commodity 1 is less than the buying price due to the tax) and $p_2^S = p_2^B = p_2$ (i.e. the buying and selling price for commodity 2 is the same). Find the Walrasian equilibrium price and quantity for commodity 1 assuming $R^a = 30t$ and $R^b = 0$ (that is, the total revenue generated by the tax is distributed back to consumer ***a***). How does a change in the tax affect this Walrasian equilibrium price? What is the equilibrium price paid by the buyer? How does a change in the tax affect this equilibrium price? How does this result compare to your result in part (b)? What is the economic intuition of this result?
   (d) Derive the Walrasian equilibrium allocation of commodities for each consumer assuming the solution is interior. Is this allocation Pareto Efficient? Justify your answer (***Hint: Take advantage of the first order conditions you derived in problem 1***). What is the economic intuition of your result?

3. Consider an exchange economy with *I* individuals and *L* commodities. Assume that for all *i* $\succsim^i$ on $\mathbb{R}_+^L$ is continuous, strictly convex, and strongly monotone preference relation that can be represented by the utility function $u^i(x^i)$. Let $U(u^1,\ldots, u^I)$ be a strictly increasing social welfare function such that $\dfrac{\partial U(u^1,\ldots,u^I)}{\partial u_i} > 0$ for all *i*. Finally, let $X^e \in \mathbb{R}_{++}^L$ be the aggregate endowments of commodities available to the economy.
   (a) Setup the social welfare maximization problem with resource constraints using individual utility functions (not indirect utility functions). Write down the Largangian for this problem and derive the first order conditions.
   (b) Assuming we have an interior solution, how do the implications of these first order conditions compare to the implications of the first order conditions we derived in class for a Pareto Efficient allocation? What do the results of this comparison tell us about the efficiency (in a Pareto sense) of this social welfare optimum?