

MICROECONOMIC ANALYSIS

ECON 8001-2

Fall 2009

Instructor: Terry Hurley

Due: 10-13-09

TA: Giovanni Alarcon

HOMEWORK #5

Note: When writing up your answers, carefully define all new notation and terms that you introduce, and write in complete sentences and paragraphs.

1. Suppose we have only two consumers labeled a and b , and two goods labeled 1 and 2. Individual a 's wealth is w^a , while individual b 's wealth is w^b . Individual a 's consumption of good 1 and 2 is denoted by x_1^a and x_2^a , while individual b 's consumption of good 1 and 2 is denoted by x_1^b and x_2^b . Individual a 's preferences are represented by the indirect utility

function $v^a(p_1, p_2, w^a) = \frac{w^a}{p_1^{\frac{1}{3}} p_2^{\frac{2}{3}}}$, while individual b 's are represented by the indirect utility

function $v^b(p_1, p_2, w^b) = \frac{w^b}{p_1^{\frac{2}{3}} p_2^{\frac{1}{3}}}$. Both of these indirect utility function are consistent with

utility functions for continuous, locally nonsatiated, and strictly convex preference relations.

- (a) Use these indirect utility functions to derive aggregate demand for both goods.
(b) Is it possible to write this aggregate demand as just a function of aggregate wealth (i.e. $W = w^a + w^b$) and prices? Explain.

2. Given the consumers in question 1, consider the social welfare function $U(u^a, u^b) = u^a u^b$.

- (a) Set up the social welfare maximization problem for distributing aggregate wealth W between a and b and derive the first order conditions.
(b) Use these first order conditions to find the wealth distribution rule that maximizes social welfare (note that the solution will be interior because the objective is concave and the constraint is linear if you set up the problem right). Is this wealth distribution rule homogeneous of degree 1 in prices and wealth?
(c) Derive the value function for this social welfare optimization problem by substituting the wealth distribution rule back into the objective function.

3. Treating the value function you derived in question 2 (c) as an indirect utility function:
- (a) Under what conditions is using this value function for welfare analysis theoretically justified? Show that this value function satisfies these conditions.
 - (b) Use the change in consumer surplus to calculate the welfare effect of a unit tax of $t > 0$ on good 1 such that the price of good after the tax is $p_1' = p_1 + t$. Assume that none of the tax revenues are returned to the consumers.
 - (c) Derive the expenditure function for this indirect utility function. How will the change in consumer surplus compare to the equivalent and compensating variations calculated using this expenditure function? You do not need to calculate the equivalent and compensating variations, but you do need to otherwise justify your answer.