MICROECONOMIC ANALYSIS
ECON 8001-2
Fall 2009

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HOMEWORK #4: ANSWERS

Note: When writing up your answers, carefully define all new notation and terms that you introduce, and write in complete sentences and paragraphs.

1. Consider the function

$$f(p_1, p_2, u) = p_1^{\frac{a_1}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} u^{\frac{1}{a_1+a_2}}$$

where $p_1 > 0$, $p_2 > 0$, and $u > 0$ are the price of good 1, the price of good 2, and utility; and a_1 and a_2 are constant parameters.

a) Derive the Hicksian demands for good 1 and 2 assuming this function is a valid expenditure function for an individual with a strictly convex, locally nonsatiated, continuous, and rational preference relation $\succsim$ on $X = \Re_+^2$.
b) List four properties that an expenditure function must satisfy. What restrictions on a_1 and a_2, if any, are required to satisfy these properties?
c) Assuming a_1 and a_2 satisfy the conditions necessary for $f(p_1, p_2, m)$ to be a valid expenditure function, derive the corresponding indirect utility function and Marshallian demands for x_1 and x_2.
d) Derive the effect of a change in the price of x_1 on the Marshallian demands for x_1 and x_2. How much of the change in Marshallian demand is due to an income effect and how much is due to a substitution effect? Illustrate these effects in a figure and discuss their economic implications.
e) Suppose the government is considering a unit tax of t on either x_1 or x_2. Derive conditions under which the consumer would be better off with a tax on good 1 instead of a tax on good 2 (Note: For these alternative tax policies, only consider the partial equilibrium effects and assume that the revenues from the tax are not used to benefit the consumer in any way). Assuming $a_1 = a_2$, what is the economic intuition of your result?

ANSWER:

a) To get the Hicksian demands, we can differentiate the $f(p_1, p_2, u)$ with respect to p_1 and p_2:

$$h_1(p_1, p_2, u) = \frac{\partial f(p_1, p_2, u)}{\partial p_1} = \frac{a_1}{a_1 + a_2} u^{\frac{1}{a_1+a_2}} \left(\frac{p_2}{p_1}\right)^{\frac{a_2}{a_1+a_2}} \text{ and}$$

$$h_2(p_1, p_2, u) = \frac{\partial e(p_1, p_2, U)}{\partial p_2} = \frac{a_2}{a_1 + a_2} u^{\frac{1}{a_1+a_2}} \left(\frac{p_1}{p_2}\right)^{\frac{a_1}{a_1+a_2}}.$$

b) To be a valid expenditure function, $f(p_1, p_2, u)$ must be (i) homogeneous of degree one in prices, (ii) strictly increasing in utility and non-decreasing in prices; (iii) concave in prices, and (iv) continuous in prices and utility.

For (i), $f(p_1, p_2, u)$ will be homogeneous of degree one if $f(p_1, p_2, u) = tf(tp_1, tp_2, u)$ for any $t > 0$. Note that $f(tp_1, tp_2, u) = (tp_1)^{\frac{a_1}{a_1+a_2}} (tp_2)^{\frac{a_2}{a_1+a_2}} u^{\frac{1}{a_1+a_2}} =$ $t^{\frac{a_1}{a_1+a_2}} t^{\frac{a_2}{a_1+a_2}} p_1^{\frac{a_1}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} u^{\frac{1}{a_1+a_2}} = tp_1^{\frac{a_1}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} u^{\frac{1}{a_1+a_2}} = tf(tp_1, tp_2, u)$ for any $a_1 + a_2 \neq 0$, so $f(p_1, p_2, u)$ is indeed homogeneous of degree 1 in prices for $a_1 + a_2 \neq 0$.

For (ii),

$$\frac{\partial f(p_1, p_2, u)}{\partial u} = \frac{1}{a_1 + a_2} p_1^{\frac{a_1}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} u^{\frac{1}{a_1+a_2}-1} > 0,$$

$$\frac{\partial f(p_1, p_2, u)}{\partial p_1} = \frac{a_1}{a_1 + a_2} u^{\frac{1}{a_1+a_2}} \left(\frac{p_2}{p_1}\right)^{\frac{a_2}{a_1+a_2}} \geq 0, \text{ and}$$

$$\frac{\partial f(p_1, p_2, u)}{\partial p_2} = \frac{a_2}{a_1 + a_2} u^{\frac{1}{a_1+a_2}} \left(\frac{p_1}{p_2}\right)^{\frac{a_1}{a_1+a_2}} \geq 0$$

only if $a_1 \geq 0$, $a_2 \geq 0$, and $a_1 + a_2 > 0$.

For (iii), note that $f(p_1, p_2, u)$ will be concave in prices if the Hessian

$$H = \begin{bmatrix} \frac{\partial^2 f(p_1, p_2, u)}{\partial p_1^2} & \frac{\partial^2 f(p_1, p_2, u)}{\partial p_1 \partial p_2} \\ \frac{\partial^2 f(p_1, p_2, u)}{\partial p_1 \partial p_2} & \frac{\partial^2 f(p_1, p_2, u)}{\partial p_2^2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{a_1 a_2}{(a_1 + a_2)^2} u^{\frac{1}{a_1+a_2}} p_1^{-\frac{a_1+2a_2}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} & \frac{a_1 a_2}{(a_1 + a_2)^2} u^{\frac{1}{a_1+a_2}} p_1^{-\frac{a_2}{a_1+a_2}} p_2^{\frac{-a_1}{a_1+a_2}} \\ \frac{a_1 a_2}{(a_1 + a_2)^2} u^{\frac{1}{a_1+a_2}} p_1^{-\frac{a_2}{a_1+a_2}} p_2^{\frac{-a_1}{a_1+a_2}} & -\frac{a_1 a_2}{(a_1 + a_2)^2} u^{\frac{1}{a_1+a_2}} p_1^{\frac{a_1}{a_1+a_2}} p_2^{-\frac{2a_1+a_2}{a_1+a_2}} \end{bmatrix}$$

is negative semidefinite. H is negative semidefinite if its eigenvalues are all nonpositive. The eigenvalues for H can be found by solving

$$\begin{bmatrix} -\frac{a_1a_2}{(a_1+a_2)^2}u^{\frac{1}{a_1+a_2}}p_1^{-\frac{a_1+2a_2}{a_1+a_2}}p_2^{\frac{a_2}{a_1+a_2}}-e & \frac{a_1a_2}{(a_1+a_2)^2}u^{\frac{1}{a_1+a_2}}p_1^{-\frac{a_2}{a_1+a_2}}p_2^{\frac{-a_1}{a_1+a_2}} \\ \frac{a_1a_2}{(a_1+a_2)^2}u^{\frac{1}{a_1+a_2}}p_1^{-\frac{a_2}{a_1+a_2}}p_2^{\frac{-a_1}{a_1+a_2}} & -\frac{a_1a_2}{(a_1+a_2)^2}u^{\frac{1}{a_1+a_2}}p_1^{\frac{a_1}{a_1+a_2}}p_2^{-\frac{2a_1+a_2}{a_1+a_2}}-e \end{bmatrix} = 0$$

for e:

$$\left(\frac{a_1a_2}{(a_1+a_2)^2}u^{\frac{1}{a_1+a_2}}p_1^{-\frac{a_1+2a_2}{a_1+a_2}}p_2^{\frac{a_2}{a_1+a_2}}+e\right)\left(\frac{a_1a_2}{(a_1+a_2)^2}u^{\frac{1}{a_1+a_2}}p_1^{\frac{a_1}{a_1+a_2}}p_2^{-\frac{2a_1+a_2}{a_1+a_2}}+e\right) - \left(\frac{a_1a_2}{(a_1+a_2)^2}u^{\frac{1}{a_1+a_2}}p_1^{-\frac{a_2}{a_1+a_2}}p_2^{\frac{-a_1}{a_1+a_2}}\right)^2 = 0 \Rightarrow$$

$$e\frac{a_1a_2}{(a_1+a_2)^2}u^{\frac{1}{a_1+a_2}}\left(p_1^{-\frac{a_1+2a_2}{a_1+a_2}}p_2^{\frac{a_2}{a_1+a_2}}+p_1^{\frac{a_1}{a_1+a_2}}p_2^{-\frac{2a_1+a_2}{a_1+a_2}}\right)+e^2=0,$$

which implies $e = 0$ and $e=-\frac{a_1a_2}{(a_1+a_2)^2}u^{\frac{1}{a_1+a_2}}\left(p_1^{-\frac{a_1+2a_2}{a_1+a_2}}p_2^{\frac{a_2}{a_1+a_2}}+p_1^{\frac{a_1}{a_1+a_2}}p_2^{-\frac{2a_1+a_2}{a_1+a_2}}\right)\le 0$ provided $a_1 \ge 0$, $a_2 \ge 0$, and $a_1 + a_2 > 0$.

c) Duality implies that the expenditure function satisfies the property that $w = e(p_1, p_2, v(p_1, p_2, w))$ or for our expenditure function $w = p_1^{\frac{a_1}{a_1+a_2}}p_2^{\frac{a_2}{a_1+a_2}}v(p_1,p_2,w)^{\frac{1}{a_1+a_2}}$. Solving for $v(p_1, p_2, w)$, yields $v(p_1,p_2,w)=\frac{w^{a_1+a_2}}{p_1^{a_1}p_2^{a_2}}$. To get the Marshallian demand, we can use Roy's Identity:

$$x_i(p_1,p_2,w) = -\frac{\frac{\partial v(p_1,p_2,w)}{\partial p_i}}{\frac{\partial v(p_1,p_2,w)}{\partial w}}$$ such that

$$x_1(p_1,p_2,w) = -\frac{-a_1\frac{w^{a_1+a_2}}{p_1^{a_1+1}p_2^{a_2}}}{\frac{(a_1+a_2)w^{a_1+a_2-1}}{p_1^{a_1}p_2^{a_2}}} = \frac{a_1w}{p_1(a_1+a_2)}$$ and

$$x_2(p_1,p_2,w) = -\frac{-a_2\frac{w^{a_1+a_2}}{p_1^{a_1}p_2^{a_2+1}}}{\frac{(a_1+a_2)w^{a_1+a_2-1}}{p_1^{a_1}p_2^{a_2}}} = \frac{a_2w}{p_2(a_1+a_2)}.$$

d) The effect of a change in p_1 on the Marshallian demand for x_1 and x_2 is

$$\frac{\partial x_1(p_1,p_2,w)}{\partial p_1} = -\frac{a_1 w}{p_1^2(a_1+a_2)} \text{ and } \frac{\partial x_2(p_1,p_2,w)}{\partial p_1} = 0 .$$

Slutskys Equation implies that $\frac{\partial x_l(p,w^*)}{\partial p_k} = \frac{\partial h_l(p,u^*)}{\partial p_k} - \frac{\partial x_l(p,w^*)}{\partial w} x_k(p,w^*)$ or

$$\frac{\partial x_1(p_1,p_2,w)}{\partial p_1} = -\frac{a_1 a_2}{(a_1+a_2)^2} u^{\frac{1}{a_1+a_2}} p_1^{-\frac{a_1+2a_2}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} - \frac{a_1^2 w}{p_1^2(a_1+a_2)^2} \text{ and}$$

$$\frac{\partial x_2(p_1,p_2,w)}{\partial p_1} = \frac{a_1 a_2}{(a_1+a_2)^2} u^{\frac{1}{a_1+a_2}} p_1^{-\frac{a_2}{a_1+a_2}} p_2^{\frac{-a_1}{a_1+a_2}} - \frac{a_1 a_2 w}{p_1 p_2 (a_1+a_2)^2}$$

Where the first terms on the right-hand-side of these equations is the substitution effect and the second terms are the income effects. Note that if we substitute the indirect utility function into these equations we get

$$\frac{\partial x_1(p_1,p_2,w)}{\partial p_1} = -\frac{a_1 a_2}{(a_1+a_2)^2}\left(\frac{w^{a_1+a_2}}{p_1^{a_1} p_2^{a_2}}\right)^{\frac{1}{a_1+a_2}} p_1^{-\frac{a_1+2a_2}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} - \frac{a_1^2 w}{p_1^2(a_1+a_2)^2} \text{ and}$$
$$= -\frac{a_1 a_2}{(a_1+a_2)^2}\frac{w}{p_1^2} - \frac{a_1^2 w}{p_1^2(a_1+a_2)^2} = -\frac{a_1 w}{p_1^2(a_1+a_2)}$$

$$\frac{\partial x_2(p_1,p_2,w)}{\partial p_1} = \frac{a_1 a_2}{(a_1+a_2)^2}\left(\frac{w^{a_1+a_2}}{p_1^{a_1} p_2^{a_2}}\right)^{\frac{1}{a_1+a_2}} p_1^{-\frac{a_2}{a_1+a_2}} p_2^{\frac{-a_1}{a_1+a_2}} - \frac{a_1 a_2 w}{p_1 p_2 (a_1+a_2)^2}$$
$$= \frac{a_1 a_2}{(a_1+a_2)^2}\frac{w}{p_1^{\frac{a_1}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}}} p_1^{-\frac{a_2}{a_1+a_2}} p_2^{\frac{-a_1}{a_1+a_2}} - \frac{a_1 a_2 w}{p_1 p_2 (a_1+a_2)^2} = 0$$

as expected.

For good 1, the substitution effect is negative as required, the income effect is also negative which implies good 1 is normal. Therefore, the negative substitution and income effects reinforce each other. For good 2, the substitution effect is positive while the income effect is negative. Therefore, good 2 can be classified as of Hicksian substitute and normal good. It is also important to note that the substitution effect is equal in magnitude to the income effect so the net effect is 0. Therefore, we cannot classify good 2 as a Marshallian substitute or complement. The Figure below illustrates. What is important for this figure to be accurate is that the income effect $x_1(p_1, p_2, w)$ is negative and the net effect of the change on $x_2(p_1, p_2, w)$ is zero.

e) In this problem, we want to compare the partial equilibrium consequences of two alternative unit taxes. Two possible tools we can use are the equivalent and compensating variations. Given starting prices p_1^S and p_2^S and ending prices p_1^E and p_2^E, the equivalent and compensating variations are defined as

$$EV(p_1^S, p_2^S, p_1^E, p_2^E, w) = e(p_1^S, p_2^S, v(p_1^E, p_2^E, w)) - w \text{ and}$$

$$CV(p_1^S, p_2^S, p_1^E, p_2^E, w) = w - e(p_1^E, p_2^E, v(p_1^S, p_2^S, w)).$$

In the problem at hand, there is one set of starting prices, $p_1^S = p_1$ and $p_2^S = p_2$, and two sets of ending prices, $p_1^{E1} = p_1 + t$ and $p_2^{E1} = p_2$ and $p_1^{E2} = p_1$ and $p_2^{E2} = p_2 + t$. Therefore, EV will give us a theoretically valid comparison, while CV will not. For a tax on good 1, the equivalent variation is

$$EV^1 = p_1^{\frac{a_1}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} \left(\frac{w^{a_1+a_2}}{(p_1+t)^{a_1} p_2^{a_2}} \right)^{\frac{1}{a_1+a_2}} - w = \left(\frac{p_1^{\frac{a_1}{a_1+a_2}}}{(p_1+t)^{\frac{a_1}{a_1+a_2}}} - 1 \right) w,$$

while for a tax on good 2, the equivalent variation is

$$EV^2 = p_1^{\frac{a_1}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} \left(\frac{w^{a_1+a_2}}{p_1^{a_1} (p_2+t)^{a_2}} \right)^{\frac{1}{a_1+a_2}} - w = \left(\frac{p_2^{\frac{a_2}{a_1+a_2}}}{(p_2+t)^{\frac{a_2}{a_1+a_2}}} - 1 \right) w.$$

Before comparing these results, we need to remember what EV^1 and EV^2 will represent. Since we are effectively evaluating an increase in prices, EV^1 and EV^2 will measure a loss in the individual's welfare. Therefore, we are interested in whether a loss in the individual's welfare is less when good 1 is taxed. Comparing these results, $EV^1 > EV^2$ if $\left(\frac{p_1^{\frac{a_1}{a_1+a_2}}}{(p_1+t)^{\frac{a_1}{a_1+a_2}}} - 1 \right) w > \left(\frac{p_2^{\frac{a_2}{a_1+a_2}}}{(p_2+t)^{\frac{a_2}{a_1+a_2}}} - 1 \right) w$ or $\left(\frac{p_1}{(p_1+t)} \right)^{a_1} > \left(\frac{p_2}{(p_2+t)} \right)^{a_2}$. For $a_1 = a_2$, this condition boils down to $p_1 > p_2$. That is, the individual will prefer to have good 1 taxed if it is the higher priced good. To understand the intuition of this result note that the tax revenue collected by the government for a tax on good 1 is $R_1 = tx_1(p_1+t, p_2, w) = \frac{a_1 w}{(p_1+t)(a_1+a_2)}$, while the tax revenue for a tax on good 2 is $R_2 = tx_2(p_1, p_2+t, w) = \frac{a_2 w}{(p_2+t)(a_1+a_2)}$. Note that $R_2 > R_1$ when $\frac{a_2 w}{(p_2+t)(a_1+a_2)} > \frac{a_1 w}{(p_1+t)(a_1+a_2)}$ or $p_1 > p_2$ when $a_1 = a_2$. Therefore, the individual will pay less of his income in taxes if good 1 is taxed when the price of good 1 is greater than the price of good 2. Of course, this intuition works because $a_1 = a_2$ implies that the consumer's preferences for good 1 and 2 are in a sense symmetrical. If they were not, then more would come into play than just minimizing the tax burden.

2. Consider a two commodity world with the general expenditure and indirect utility functions $e(p_1, p_2, u)$ and $v(p_1, p_2, w)$. In class, we showed how you can integrate the Hicksian Demand for a commodity to evaluate the Equivalent and Compensating Variation when the price of that commodity changed.

   a) Show how the Hicksian Demands can be integrated to calculate the Equivalent and Compensating Variations when the prices of both commodities decrease.
   b) Use the results in part (a) to decompose the total effect of a change in the price of both goods on the Equivalent and Compensating Variations into the proportion attributable to a change in the price of commodity 1 and the proportion attributable to a change in the price of commodity 2.
   c) Show that your answer for part (a) and (b) is not unique.
   d) What are the implications of this non-uniqueness for welfare analysis?

ANSWER:
(a) and (c)

The Equivalent Variation is defined as

$$EV((p_1^0, p_2^0), (p_1^1, p_2^1), w) = e((p_1^0, p_2^0), v((p_1^1, p_2^1), w)) - w$$

which can also be written as

$$EV((p_1^0, p_2^0), (p_1^1, p_2^1), w) = e((p_1^0, p_2^0), v((p_1^1, p_2^1), w)) - e((p_1^1, p_2^1), v((p_1^1, p_2^1), w)).$$

If we add and subtract $e((p_1^0, p_2^1), v((p_1^1, p_2^1), w))$,

$$\begin{aligned} EV((p_1^0, p_2^0), (p_1^1, p_2^1), w) = {} & e((p_1^0, p_2^0), v((p_1^1, p_2^1), w)) - e((p_1^0, p_2^1), v((p_1^1, p_2^1), w)) \\ & + e((p_1^0, p_2^1), v((p_1^1, p_2^1), w)) - e((p_1^1, p_2^1), v((p_1^1, p_2^1), w)) \\ = {} & \int_{p_2^1}^{p_2^0} h_2\left(p_1^0, p_2, v\left(p_1^1, p_2^1, w\right)\right) dp_2 + \int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^1, v\left(p_1^1, p_2^1, w\right)\right) dp_1 . \end{aligned}$$

Alternatively, we could add and subtract $e((p_1^1, p_2^0), v((p_1^1, p_2^1), w))$ such that

$$\begin{aligned} EV((p_1^0, p_2^0), (p_1^1, p_2^1), w) = {} & e((p_1^0, p_2^0), v((p_1^1, p_2^1), w)) - e((p_1^1, p_2^0), v((p_1^1, p_2^1), w)) \\ & + e((p_1^1, p_2^0), v((p_1^1, p_2^1), w)) - e((p_1^1, p_2^1), v((p_1^1, p_2^1), w)) \\ = {} & \int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^0, v\left(p_1^1, p_2^1, w\right)\right) dp_1 + \int_{p_2^1}^{p_2^0} h_2\left(p_1^1, p_2, v\left(p_1^1, p_2^1, w\right)\right) dp_2 . \end{aligned}$$

The Compensating Variation is defined as

$$CV((p_1^0, p_2^0), (p_1^1, p_2^1), w) = w - e((p_1^1, p_2^1), v((p_1^0, p_2^0), w))$$

which can also be written as

$CV((p_1^0, p_2^0), (p_1^1, p_2^1), w) = e((p_1^0, p_2^0), v((p_1^0, p_2^0), w)) - e((p_1^1, p_2^1), v((p_1^0, p_2^0), w))$.

Adding and subtracting $e((p_1^0, p_2^1), v((p_1^0, p_2^0), w))$ yields

$$\begin{aligned} CV((p_1^0, p_2^0), (p_1^1, p_2^1), w) = \; & e((p_1^0, p_2^0), v((p_1^0, p_2^0), w)) - e((p_1^0, p_2^1), v((p_1^0, p_2^0), w)) \\ & + e((p_1^0, p_2^1), v((p_1^0, p_2^0), w)) - e((p_1^1, p_2^1), v((p_1^0, p_2^0), w)) \\ = \; & \int_{p_2^1}^{p_2^0} h_2\left(p_1^0, p_2, v\left(p_1^0, p_2^0, w\right)\right) dp_2 + \int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^1, v\left(p_1^0, p_2^0, w\right)\right) dp_1 . \end{aligned}$$

or adding and subtracting $e((p_1^1, p_2^0), v((p_1^0, p_2^0), w))$ yields

$$\begin{aligned} CV((p_1^0, p_2^0), (p_1^1, p_2^1), w) = \; & e((p_1^0, p_2^0), v((p_1^0, p_2^0), w)) - e((p_1^1, p_2^0), v((p_1^0, p_2^0), w) \\ & + e((p_1^1, p_2^0), v((p_1^0, p_2^0), w) - e((p_1^1, p_2^1), v((p_1^0, p_2^0), w)) \\ = \; & \int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^0, v\left(p_1^0, p_2^0, w\right)\right) dp_1 + \int_{p_2^1}^{p_2^0} h_2\left(p_1^1, p_2, v\left(p_1^0, p_2^0, w\right)\right) dp_2 . \end{aligned}$$

(b)

The proportion of *EV* due to a change in the price of commodity 1 and 2 are

$$\frac{\int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^1, v\left(p_1^1, p_2^1, w\right)\right) dp_1}{\int_{p_2^1}^{p_2^0} h_2\left(p_1^0, p_2, v\left(p_1^1, p_2^1, w\right)\right) dp_2 + \int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^1, v\left(p_1^1, p_2^1, w\right)\right) dp_1} \text{ and}$$

$$\frac{\int_{p_2^1}^{p_2^0} h_2\left(p_1^0, p_2, v\left(p_1^1, p_2^1, w\right)\right) dp_2}{\int_{p_2^1}^{p_2^0} h_2\left(p_1^0, p_2, v\left(p_1^1, p_2^1, w\right)\right) dp_2 + \int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^1, v\left(p_1^1, p_2^1, w\right)\right) dp_1} \text{, or}$$

$$\frac{\int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^0, v\left(p_1^1, p_2^1, w\right)\right) dp_1}{\int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^0, v\left(p_1^1, p_2^1, w\right)\right) dp_1 + \int_{p_2^1}^{p_2^0} h_2\left(p_1^1, p_2, v\left(p_1^1, p_2^1, w\right)\right) dp_2} \text{ and}$$

$$\frac{\int_{p_2^1}^{p_2^0} h_2\left(p_1^1, p_2, v\left(p_1^1, p_2^1, w\right)\right) dp_2}{\int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^0, v\left(p_1^1, p_2^1, w\right)\right) dp_1 + \int_{p_2^1}^{p_2^0} h_2\left(p_1^1, p_2, v\left(p_1^1, p_2^1, w\right)\right) dp_2}.$$

The proportion of *CV* due to a change in the price of commodity 1 and 2 are

$$\frac{\int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^1, v\left(p_1^0, p_2^0, w\right)\right) dp_1}{\int_{p_2^1}^{p_2^0} h_2\left(p_1^0, p_2, v\left(p_1^0, p_2^0, w\right)\right) dp_2 + \int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^1, v\left(p_1^0, p_2^0, w\right)\right) dp_1} \text{ and}$$

$$\frac{\int_{p_2^1}^{p_2^0} h_2\left(p_1^0, p_2, v\left(p_1^0, p_2^0, w\right)\right) dp_2}{\int_{p_2^1}^{p_2^0} h_2\left(p_1^0, p_2, v\left(p_1^0, p_2^0, w\right)\right) dp_2 + \int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^1, v\left(p_1^0, p_2^0, w\right)\right) dp_1}, \text{ or}$$

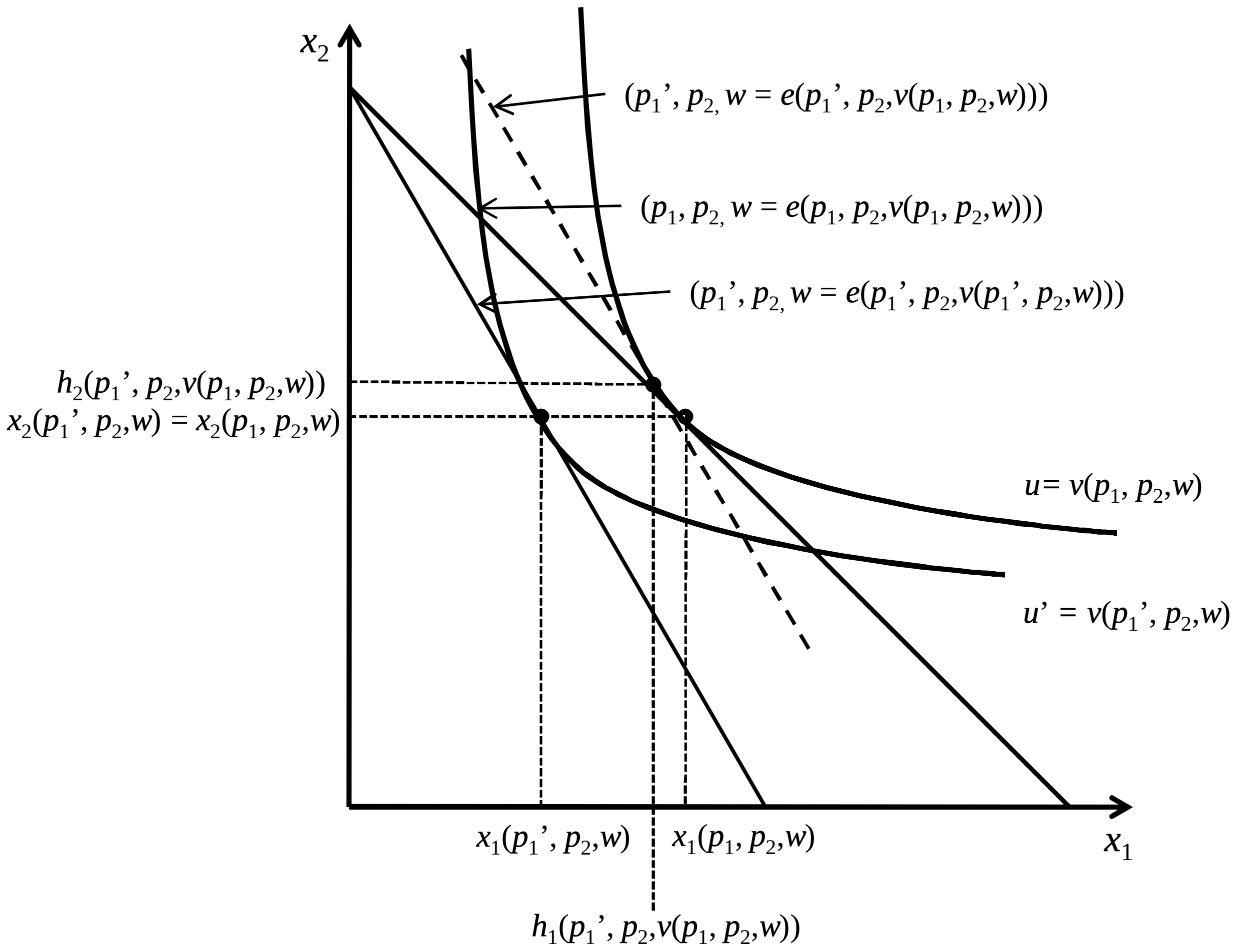
$$\frac{\int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^0, v\left(p_1^0, p_2^0, w\right)\right) dp_1}{\int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^0, v\left(p_1^0, p_2^0, w\right)\right) dp_1 + \int_{p_2^1}^{p_2^0} h_2\left(p_1^1, p_2, v\left(p_1^0, p_2^0, w\right)\right) dp_2} \text{ and}$$

$$\frac{\int_{p_2^1}^{p_2^0} h_2\left(p_1^1, p_2, v\left(p_1^0, p_2^0, w\right)\right) dp_2}{\int_{p_1^1}^{p_1^0} h_1\left(p_1, p_2^0, v\left(p_1^0, p_2^0, w\right)\right) dp_1 + \int_{p_2^1}^{p_2^0} h_2\left(p_1^1, p_2, v\left(p_1^0, p_2^0, w\right)\right) dp_2}.$$

(d)

The decompositions in Problem 2 (b) are theoretically valid. However, they are not unique. Therefore, economic analysts may ultimately disagree on which aspects of a new policy have the greatest impact on a consumer, yet both could be fully justified in their response, which provides another note of caution.

Figure 1: Graphical illustration of Slutsky income and substitution effects for Homework #4, Question 1 (d) assuming $p_1' > p_1$.



Substitution Effects:

$h_1(p_1', p_2, v(p_1, p_2, w)) - x_1(p_1, p_2, w) = h_1(p_1', p_2, v(p_1, p_2, w)) - h_1(p_1, p_2, v(p_1, p_2, w))$

$h_2(p_1', p_2, v(p_1, p_2, w)) - x_2(p_1, p_2, w) = h_2(p_1', p_2, v(p_1, p_2, w)) - h_2(p_1, p_2, v(p_1, p_2, w))$

Income Effects:

$x_1(p_1', p_2, w) - h_1(p_1', p_2, v(p_1, p_2, w)) = x_1(p_1', p_2, w) - x_1(p_1', p_2, e(p_1', p_2, v(p_1, p_2, w)))$

$x_2(p_1', p_2, w) - h_2(p_1', p_2, v(p_1, p_2, w)) = x_2(p_1', p_2, w) - x_2(p_1', p_2, e(p_1', p_2, v(p_1, p_2, w)))$