

**MICROECONOMIC ANALYSIS**  
ECON 8001-2  
Fall 2009

Instructor: Terry Hurley  
TA: Giovanni Alarcon

Due: 10-6-09

**HOMEWORK #4**

*Note: When writing up your answers, carefully define all new notation and terms that you introduce, and write in complete sentences and paragraphs.*

1. Consider the function

$$f(p_1, p_2, u) = p_1^{\frac{a_1}{a_1+a_2}} p_2^{\frac{a_2}{a_1+a_2}} u^{\frac{1}{a_1+a_2}}$$

where  $p_1 > 0$ ,  $p_2 > 0$ , and  $u > 0$  are the price of good 1, the price of good 2, and utility; and  $a_1$  and  $a_2$  are constant parameters.

- a) Derive the Hicksian demands for good 1 and 2 assuming this function is a valid expenditure function for an individual with a strictly convex, locally nonsatiated, continuous, and rational preference relation  $\underline{f}$  on  $X = \hat{A}_+$ .
- b) List four properties that an expenditure function must satisfy. What restrictions on  $a_1$  and  $a_2$ , if any, are required to satisfy these properties?
- c) Assuming  $a_1$  and  $a_2$  satisfy the conditions necessary for  $f(p_1, p_2, m)$  to be a valid expenditure function, derive the corresponding indirect utility function and Marshallian demands for  $x_1$  and  $x_2$ .
- d) Derive the effect of a change in the price of  $x_1$  on the Marshallian demands for  $x_1$  and  $x_2$ . How much of the change in Marshallian demand is due to an income effect and how much is due to a substitution effect? Illustrate these effects in a figure and discuss their economic implications.
- e) Suppose the government is considering a unit tax of  $t$  on either  $x_1$  or  $x_2$ . Derive conditions under which the consumer would be better off with a tax on good 1 instead of a tax on good 2 (Note: For these alternative tax policies, only consider the partial equilibrium effects and assume that the revenues from the tax are not used to benefit the consumer in any way). Assuming  $a_1 = a_2$ , what is the economic intuition of your result?

2. Consider a two commodity world with the general expenditure and indirect utility functions  $e(p_1, p_2, u)$  and  $v(p_1, p_2, w)$ . In class, we showed how you can integrate the Hicksian Demand for a commodity to evaluate the Equivalent and Compensating Variation when the price of that commodity changed.
  - a) Show how the Hicksian Demands can be integrated to calculate the Equivalent and Compensating Variations when the prices of both commodities decrease.
  - b) Use the results in part (a) to decompose the total effect of a change in the price of both goods on the Equivalent and Compensating Variations into the proportion attributable to a change in the price of commodity 1 and the proportion attributable to a change in the price of commodity 2.
  - c) Show that your answer for part (a) and (b) is not unique.
  - d) What are the implications of this non-uniqueness for welfare analysis?