MICROECONOMIC ANALYSIS
ECON 8001-2
Fall 2009

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HOMEWORK #3

Note: When writing up your answers, carefully define all new notation and terms that you introduce, and write in complete sentences and paragraphs.

1. Let $u(x_1, x_2) = \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)^2$ represent a consumer's continuous, locally non-satiated, strictly convex preference relation for $x \in \Re_+^2$. Assume this consumer has income $w > 0$ and faces prices p_1 and p_2 for x_1 and x_2.
   (a) Formulate the consumer's utility maximization problem.
   (b) Derive the Marshallian demand for x_1 and x_2 assuming the solution is interior.
   (c) Derive the individual's indirect utility function.
   (d) Verify that the indirect utility function is (i) homogenous of degree 0 in p_1, p_2, and w, (ii) strictly increasing in w and nonincreasing in p_1 and p_2, and (iii) quasi-convex in p_1 and p_2.

2. Using the utility function in 1.:
   (a) Formulate the consumer's expenditure minimization problem.
   (b) Derive the Hicksian demands for x_1 and x_2 assuming the solution is interior.
   (c) Substitute the indirect utility function from 1.(c) for utility in the Hicksian demands you derived in 2.(b) and compare the results to the Marshallian demands you found in 1.(b). What is the implication of your result?
   (d) Derive the expenditure function.
   (e) Substitute the indirect utility function you derived in 1.(c) for utility in the expenditure function you derived in 2.(d) and simplify as much as you can (which should actually be quite a bit). What does your result imply about the relationship between the expenditure and indirect utility function?

3. Suppose that there are two possible states of the world denoted by A and B. Also, suppose a consumer's continuous, locally non-satiated, and strictly convex preferences for income in these two states of the world can be represented by the utility function $u(w_A, w_B) = a w_A^t + (1 - a)w_B^t$ where $w_A \geq 0$ is income in state A, $w_B \geq 0$ is income in state B, and $1 > a > 0$ and $1 > t > 0$ are parameters. In state A, the consumer is endowed with income of $w_A^e > 0$ dollars. In state B, the consumer is endowed with income of $w_B^e > 0$ dollars. Furthermore, assume that the consumer can buy or sell insurance that pays an indemnity of x dollars in state B (e.g. for $x > 0$, the consumer is purchasing insurance, while for $x < 0$, the consumer is selling insurance). The cost of this insurance is px regardless of whether the state of the world is A or B where $1 > p > 0$ is the price of insurance per dollar of indemnification.

   (a) Formulate the consumer's budget constraint in terms of w_A and w_B remembering that preferences are locally non-satiated and $w_A \geq 0$ and $w_B \geq 0$.
   (b) Formulate the consumer's utility maximization problem given this budget constraint.
   (c) Solve for the consumer's Marshallian demands in terms of w_A and w_B. Are there any conditions under which that consumer will only consume income in state A (e.g. $w_B = 0$)? Are there any conditions under which that consumer will only consume income in state B (e.g. $w_A = 0$)?
   (d) Are these demands homogeneous of degree 0 in p? Explain.
   (e) Assuming an interior solution, under what condition will the optimal demand for income in state A be greater than the optimal demand for income in state B? If a is the probability of state A and $1 - a$ is the probability of state B, what is the economic intuition of your result?

   Hint: There is a hard way to do part (d) and (e), and an easy way. I encourage you to take some time to look for the easy way.