MICROECONOMIC ANALYSIS
ECON 8001-2
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HOMEWORK #2: ANSWERS

Note: When writing up your answers, carefully define all new notation and terms that you introduce, and write in complete sentences and paragraphs.

1. Define the choice set $\boldsymbol{B} = (\{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2, x_4\})$ and define a rule

$$C(b) = \begin{cases} \{x_2\}, & for\ \{x_1, x_2\} \\ \{x_3\}, & for\ \{x_1, x_3\} \\ \{x_3\}, & for\ \{x_2, x_3\} \\ \{x_2, x_4\}, & for\ \{x_1, x_2, x_4\} \end{cases}.$$

Use this choice rule to define the set of revealed preference relations for the different sets of alternatives in $\boldsymbol{B}$. Taken together, is this revealed preference relation consistent with a rational preference relation defined on $\{x_1, x_2, x_3, x_4\}$? Explain.

ANSWER: For $\{x_1, x_2\}$, $C(\{x_1, x_2\}) = \{x_2\}$ implying $x_2 \succsim^* x_1$ and not $x_1 \succsim^* x_2$, or $x_2 \succ^* x_1$. For $\{x_1, x_3\}$, $C(\{x_1, x_3\}) = \{x_3\}$ implying $x_3 \succsim^* x_1$ and not $x_1 \succsim^* x_3$, or $x_3 \succ^* x_1$. For $\{x_2, x_3\}$, $C(\{x_2, x_3\}) = \{x_3\}$ implying $x_3 \succsim^* x_2$ and not $x_2 \succsim^* x_3$, or $x_3 \succ^* x_2$. For $\{x_1, x_2, x_4\}$, $C(\{x_1, x_2, x_4\}) = \{x_2, x_4\}$ implying $x_2 \succsim^* x_4$, $x_4 \succsim^* x_2$, $x_2 \succsim^* x_1$, not $x_2 \succsim^* x_1$, $x_4 \succsim^* x_1$ and not $x_1 \succsim^* x_4$ or $x_2 \sim^* x_4$, $x_2 \succ^* x_1$ and $x_4 \succ^* x_1$. Together these results imply that

(i) $x_2 \succsim^* x_1$,
(ii) $x_3 \succsim^* x_1$,
(iii) $x_3 \succsim^* x_2$
(iv) $x_4 \succsim^* x_1$,
(v) $x_2 \succsim^* x_4$,
(vi) $x_4 \succsim^* x_2$
(vii) not $x_1 \succsim^* x_2$,
(viii) not $x_1 \succsim^* x_3$,
(ix) not $x_2 \succsim^* x_3$,
(x) not $x_1 \succsim^* x_4$

Now, are these revealed preference relations consistent with a rational preference relation defined on the set of all possible alternatives, $\{x_1, x_2, x_3, x_4\}$? A preference relation is rational if it is complete and transitive. Completeness requires that for all $s, s' \in X$ where X is an individual's set of alternatives, $s \succsim s'$ or $s' \succsim s$. For x_1 and x_2, (i) and (vii) say $x_2 \succsim^* x_1$ and not $x_1 \succsim^* x_2$. For x_1 and x_3, (ii) and (viii) say $x_3 \succsim^* x_1$ and not $x_1 \succsim^* x_3$. For x_1 and x_4, (iv) and (x) say $x_4 \succsim^* x_1$ and not $x_1 \succsim^* x_4$. For x_2 and x_3, (iii) and (ix) say $x_3 \succsim^* x_2$ and not $x_2 \succsim^* x_3$. For x_2 and x_4, (v) and (vi) say $x_2 \succsim^* x_4$ and $x_4 \succsim^* x_2$. For x_3 and x_4, none of the conditions (i) – (x) provide any guidance regarding whether $x_3 \succsim^* x_4$ or $x_4 \succsim^* x_3$. Therefore, this revealed preference relation is not complete.

Transitivity requires that for all $s, s', s'' \in X$, if $s \succsim s'$ and $s' \succsim s''$ then $s \succsim s''$. Notice that conditions (iii) and (i) imply $x_3 \succsim^* x_2$ and $x_2 \succsim^* x_1$, while condition (ii) implies $x_3 \succsim^* x_1$. Similarly, conditions (v) and (iv) imply $x_2 \succsim^* x_4$ and $x_4 \succsim^* x_1$, while condition (i) implies $x_2 \succsim^* x_1$, and conditions (vi) and (i) imply $x_4 \succsim^* x_2$ and $x_2 \succsim^* x_1$, while condition (iv) implies $x_4 \succsim^* x_1$. Therefore, transitivity does hold.

Since the revealed preference relation is not complete, it is not rational even though it is transitive.

2. Consider the complete preference relation $\succsim$ defined by $z' \succsim z''$ if and only if min $\{z_1', z_2'\} \geq$ min $\{z_1'', z_2''\}$ for all $z', z'' \in \Re_+^2$ (e.g. $z_l \geq 0$ for $l = 1, 2$).

   (a) Illustrate the indifference contour sets, ICS(z), for this preference relation.
   (b) Prove that this preference relation is monotone, convex, and transitive.
   (c) Is the preference relation strictly monotone and strictly convex? Explain.
   (d) What are the economic implications of this preference relation?

ANSWER:

(a) The definition of indifference implies that if $x, y \in X$ and $x \sim y$, then $x \succsim y$ and $y \succsim x$. Therefore, $z' \sim z''$ if $z' \succsim z''$ and $z'' \succsim z'$ or min $\{z_1', z_2'\} \geq$ min $\{z_1'', z_2''\}$ and min $\{z_1'', z_2''\} \geq$ min $\{z_1', z_2'\}$, which implies min $\{z_1', z_2'\}$ = min $\{z_1'', z_2''\}$. Let z_{min} = min $\{z_1', z_2'\}$ = min $\{z_1'', z_2''\}$. Figure 1 (a) divides $\Re_+^2$ into four regions relative to the consumption bundle (z_{min}, z_{min}). Consider an alternative consumption bundle from Region I, (z_1, z_2) such that $z_{min} \geq z_1$ and $z_{min} \geq z_2$, but $(z_1, z_2) \neq (z_{min}, z_{min})$. Question: Can this consumption bundle be part of ICS(z') = ICS(z'')? The answer is no because $z_{min} \geq z_1$, $z_{min} \geq z_2$, and $(z_1, z_2) \neq (z_{min}, z_{min})$ imply min $\{ z_1, z_2 \}$ < min $\{ z_{min}, z_{min}\}$, so $(z_{min}, z_{min}) \succsim (z_1, z_2)$ and not $(z_1, z_2) \succsim (z_{min}, z_{min})$. Consider an alternative consumption bundle from Region II, (z_1, z_2) such that $z_{min} > z_1$ and $z_{min} < z_2$ (so we are not including the boundary of Region II with Region I and IV). Question: Can this consumption bundle be part of ICS(z') = ICS(z'')? The answer is no again because $z_{min} > z_1$ and $z_{min} < z_2$ imply min $\{ z_1, z_2\}$ < min $\{ z_{min}, z_{min}\}$, so $(z_{min}, z_{min}) \succsim (z_1, z_2)$ and not $(z_1, z_2) \succsim (z_{min}, z_{min})$. A similar argument can be made for bundles in Region III exclusive of its boundaries with Regions I and IV (e.g. $z_{min} < z_1$ and $z_{min} > z_2$). Now consider an alternative consumption bundle from Region IV, (z_1, z_2), such that $z_{min} < z_1$ and $z_{min} < z_2$ (so we are again not including the horizontal or vertical boundary of Region IV with Regions II and III). Question: Can this consumption bundle be part of ICS(z') = ICS(z'')? The answer is no again because $z_{min} < z_1$ and $z_{min} < z_2$ implies min $\{z_1, z_2\}$ > min $\{z_{min}, z_{min}\}$, so $(z_1, z_2) \succsim (z_{min}, z_{min})$ and not $(z_{min}, z_{min}) \succsim (z_1, z_2)$. What we are now left to consider are bundles on the boundary between Regions II and IV ($z_1 = z_{min}$ and $z_2 > z_{min}$) and between Regions III and IV ($z_1 > z_{min}$ and $z_2 = z_{min}$). For $z_1 = z_{min}$ and $z_2 > z_{min}$, min $\{z_1, z_2\}$ = min

$\{z_{min}, z_{min}\}$, so $(z_1, z_2) \in$ ICS(z') = ICS(z''). For $z_1 > z_{min}$ and $z_2 = z_{min}$, min $\{z_1, z_2\}$ = min $\{z_{min}, z_{min}\}$, so $(z_1, z_2) \in$ ICS(z') = ICS(z''). Combining these results, yields the ICS illustrated in Figure 1 (b). What is important to note about these ICSs is that they are at right angles relative to the 45^o line (e.g. the line where $z_1 = z_2$). This type of preference relation is referred to as a Leontieff preference relation.

(b) We begin with monotonicity. A preference relation $\succsim$ on $\Re^2_+$ is monotone if for all $x, y \in \Re^2_+$, if $y >> x$ ($y_l > x_l$ for l = 1, 2), then $y \succ x$. Consider two consumption bundles $z', z'' \in \Re^2_+$ such that $z' >> z''$. Let z_{min}' = min $\{z_1', z_2'\}$ and z_{min}'' = min $\{z_1'', z_2''\}$. Note that the $z_{min}' > z_{min}''$ because $z' >> z''$ implies $z_1' > z_1''$ and $z_2' > z_2''$, z_{min}'' = min $\{z_1'', z_2''\}$ implies $z_1'' \geq z_{min}''$ and $z_2'' \geq z_{min}''$, and transitivity of real numbers implies $z_1' > z_{min}''$ and $z_2' > z_{min}''$. Since $z_{min}' > z_{min}''$, $z' \succ z''$ as required by monotonicity.

Now let us consider convexity. A preference relation $\succsim$ on $\Re^2_+$ is convex if for any $a \in [0, 1]$ and all x, y, z, $ay + (1 - a)z \in X$, if $y \succsim x$ and $z \succsim x$, then $ay + (1 - a)z \succsim x$. Note that for any $z', z'', z''' \in \Re^2_+$ $z' \succsim z'''$ implies z_{min}' = min $\{z_1', z_2'\} \geq z_{min}'''$ = min $\{z_1''', z_2'''\}$, and $z'' \succsim z'''$ implies z_{min}'' = min $\{z_1'', z_2''\} \geq z_{min}'''$ = min $\{z_1''', z_2'''\}$ by definition of the preference relation. Since min $\{z_1', z_2'\} \geq z_{min}'''$ and min $\{z_1'', z_2''\} \geq z_{min}'''$, min$\{z_1', z_1'', z_2', z_2''\} \geq z_{min}'''$. Now let z_{min} = min$\{az_1' + (1 - a)z_1'', az_2' + (1 - a)z_2''\}$ and note that $az_1' + (1 - a)z_1'' \geq$ min$\{z_1', z_1''\} \geq$ min$\{z_1', z_1'', z_2', z_2''\}$ and $az_2' + (1 - a)z_2'' \geq$ min$\{z_2', z_2''\} \geq$ min$\{z_1', z_1'', z_2', z_2''\}$. Therefore, $z_{min} \geq$ min$\{z_1', z_1'', z_2', z_2''\}$. Since $z_{min} \geq$ min$\{z_1', z_1'', z_2', z_2''\}$ and min$\{z_1', z_1'', z_2', z_2''\} \geq z_{min}'''$ transitivity implies $z_{min} \geq z_{min}'''$. But, $z_{min} \geq z_{min}'''$ implies $az' + (1 - a)z'' \succsim z'''$ by the definition of the preference relation as required for convexity.

Finally, we will consider transitivity. A preference relation is transitive if for all $x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$. Consider any $z', z'', z''' \in \Re^2_+$ such that $z' \succsim z''$ and $z'' \succsim z'''$. $z' \succsim z''$ implies z_{min}' = min $\{z_1', z_2'\} \geq z_{min}''$ = min $\{z_1'', z_2''\}$, and $z'' \succsim z'''$ implies z_{min}'' = min $\{z_1'', z_2''\} \geq z_{min}'''$ = min $\{z_1''', z_2'''\}$ by definition of the preference relation. Then by transitivity of real numbers, $z_{min}' \geq z_{min}''$ and $z_{min}'' \geq z_{min}'''$ implies $z_{min}' \geq z_{min}'''$. By the definition of the preference relation, $z_{min}' \geq z_{min}'''$ implies $z' \succsim z'''$ as required for transitivity.

(c) It should be clear from Figure 1 (b) that this preference relation is not strongly monotone or strictly convex. For strong monotonicity, for all $x, y \in X$, if $y \geq x$ ($y_l \geq x_l$ for $l = 1, 2, \ldots, L$) and $y \neq x$, then $y \succ x$. Consider z' = (1, 2) and z'' = (1,4). Note that $z'' \geq z'$, but min{1, 2} = min{1, 4} such that $z' \sim z''$ by the definition of the preference relation, which contradicts $z' \succ z''$. For strict convexity, for any $a \in (0, 1)$ and all x, y, z, $ay + (1 - a)z \in X$, if $y \succsim x$, $z \succsim x$, and $y \neq z$, then $ay + (1 - a)z \succ x$. Consider a = 0.5, z' = (1, 2), z'' = (1,4), and z''' = (1,6): min{1, 2} ≥ min{1, 4}, min{1, 6} ≥ min{1, 4}, min{0.5×1 + 0.5×1, 0.5×2 + 0.5×6.} = min{1, 4} = min{1, 4}. Therefore, $0.5z' + 0.5z''' \sim z''$, which contradicts $0.5z' + 0.5z''' \succ z''$.

(d) The economic implications of this preference relation is that goods are perfect complements or not substitutable. The only way for an individual to increase satisfaction given these preferences is to consume more of both goods.

3. Suppose $u(\cdot)$ is a continuous utility function representing a strongly monotone, continuous, and rational preference relation $\succsim$ on $X = \Re^L_+$. Prove that the Marshallian demand correspondence $x(p, w)$ satisfies *Walras Law*: $p \cdot x = w$ for all $x \in x(p, w)$.

ANSWER: By the definition of a strongly monotone preference relation, we know that for all x, $y \in X$, if $y \geq x$ ($y_l \geq x_l$ for $l = 1, 2, \ldots, L$) and $y \neq x$, then $y \succ x$, or $y \succsim x$ and not $x \succsim y$. By the definition of a utility function, we know that for all $x, y \in X$, if $y \succsim x$ then $u(y) \geq u(x)$. By definition of the demand correspondence, we know that for all $x \in x(p, w)$, $x \in B_{p,w}$ and $u(x) \geq u(z)$ for all $z \in B_{p,w}$ where $B_{p,w} = \{z \in X\text{: } w \geq p \cdot z\}$. Given these definitions, we want to show that $p \cdot x = w$ for all $x \in x(p, w)$.

Assume this is not the case such that there is some $x' \in x(p, w)$ where $w \neq p \cdot x'$. First note that $p \cdot x' > w$ contradicts $x' \in B_{p,w}$ and therefore, $x' \in x(p, w)$. Now if $w > p \cdot x'$, there must be some $y \geq x$ and $y \neq x$ such that $w \geq p \cdot y > p \cdot x'$. Since $w \geq p \cdot y$, $y \in B_{p,w}$. By strong monotonicity, $y \succsim x$ and not $x \succsim y$. By the definition of a utility function, $u(y) \geq u(x)$ and not $u(x) \geq u(y)$, which implies $u(y) > u(x)$. But $x' \in x(p, w)$ also implies $u(x) \geq u(z)$ for all $z \in B_{p,w}$ by the definition of demand, which contradicts $u(y) > u(x)$ and $y \in B_{p,w}$.

The economic implication of this result is that if preferences are strongly monotonic, then consumers will fully exhaust their budget when choosing their preferred bundle of commodities.

4. In the classical consumer problem, consumption is constrained only by income and prices. That is, consumer can consume as much as they want of a particular good, provided their total expenditures do not exceed their income. However, there are lots of examples where a consumer's choices are constrained by more than just income. For example, the U.S. Food Stamp Program provides income in the form of Food Stamps to low income individuals, but restricts what these individuals can spend this income on.

   Consider a two commodity world, $x = (x_1, {}_2) \in \Re_+^2$, with prices $p = (p_1, p_2) \in \Re_{++}^2$ (e.g. $p_l > 0$ for $l = 1,2$). Assume the individual has income $w > 0$ from working which can be spent freely on either commodity. Additionally, the individual receives income $w_{FS} > 0$ from the government which can only be spent on commodity x_2.

   (a) Describe the budget set for this individual, $B_{p,w,w_{FS}}$, mathematically and illustrate it in a figure.
   (b) Prove that this budget set is convex.
   (c) Prove that for $w' > w''$, $B_{p,w'',w_{FS}} \subset B_{p,w',w_{FS}}$. What is the economic implication of this result?

(d) Suppose the individual's preference relation is strictly convex, locally nonsatiated, continuous, and rational. Can we guarantee demand will be a unique? Explain.

(e) Again, suppose an individual's preference relation is strictly convex, locally nonsatiated, continuous, and rational. Will *Walras Law* hold? Explain.

ANSWER:

(a) The budget constraint can be written as $B_{p,w,w_{FS}} \equiv \{x \in \Re^2_+ : w + w_{FS} \geq p \cdot x \text{ and } w \geq p_1 x_1\}$. Figure 2 illustrates.

(b) By definition $B_{p,w,w_{FS}}$ is convex if for any x', $x'' \in B_{p,w,w_{FS}}$ and any $a \in [0, 1]$, $ax' + (1 - a) x'' \in B_{p,w,w_{FS}}$. Suppose to the contrary that $ax' + (1 - a) x'' \notin B_{p,w,w_{FS}}$ for some $a \in [0, 1]$. Note that x', $x'' \in B_{p,w,w_{FS}}$ implies $w + w_{FS} \geq px'$ and $w \geq p_1x_1'$, and $w + w_{FS} \geq px''$ and $w \geq p_1x_1''$. For any $a \in [0, 1]$, $a(w + w_{FS}) \geq apx'$, $aw \geq ap_1x_1'$, $(1 - a)(w + w_{FS}) \geq (1 - a)px''$ and $(1 - a)w \geq (1 - a)p_1x_1''$. Summing then implies $a(w + w_{FS}) + (1 - a)(w + w_{FS}) \geq apx' + (1 - a)px''$ and $aw + (1 - a)w \geq ap_1x_1' + (1 - a)p_1x_1''$, or $w + w_{FS} \geq p(ax' + (1 - a)x'')$ and $w \geq p_1(ax_1' + (1 - a)x_1'')$. By definition, $ax' + (1 - a) x'' \notin B_{p,w,w_{FS}}$ implies $w + w_{FS} < p(ax' + (1 - a)x'')$ or $w < p_1(ax_1' + (1 - a)x_1'')$, a contradiction.

(c) To prove that the set $B_{p,w'',w_{FS}} \subset B_{p,w',w_{FS}}$ for $w' > w''$, we must show that (i) if $y \in B_{p,w'',w_{FS}}$, then $y \in B_{p,w',w_{FS}}$ and (ii) for some $y \in B_{p,w',w_{FS}}$, $y \notin B_{p,w'',w_{FS}}$. Let us start with (i). Assume to the contrary such that for some $y \in B_{p,w'',w_{FS}}$, $y \notin B_{p,w',w_{FS}}$. By definition, $y \in B_{p,w'',w_{FS}}$ implies $w'' + w_{FS} \geq py$ and $w'' \geq p_1y_1$, while $y \notin B_{p,w',w_{FS}}$ implies $w' + w_{FS} < py$ or $w' < p_1y_1$. Transitivity of real numbers then implies $w'' + w_{FS} > w' + w_{FS}$ or $w'' > w'$, but both of these statements contradict $w' > w''$. Now for (ii). Choose y such that $w' + w_{FS} = p{\cdot}y$ and $w' \geq p_1{\cdot}y_1$. By definition, $y \in B_{p,w',w_{FS}}$. Since $w' > w''$, $p{\cdot}y > w''$ which implies $y \notin B_{p,w'',w_{FS}}$ by definition. The economic implication of the result is that if wealth decreases so do our consumption opportunities (in two dimensions, a decrease in our wealth shifts our budget constraint Southwest).

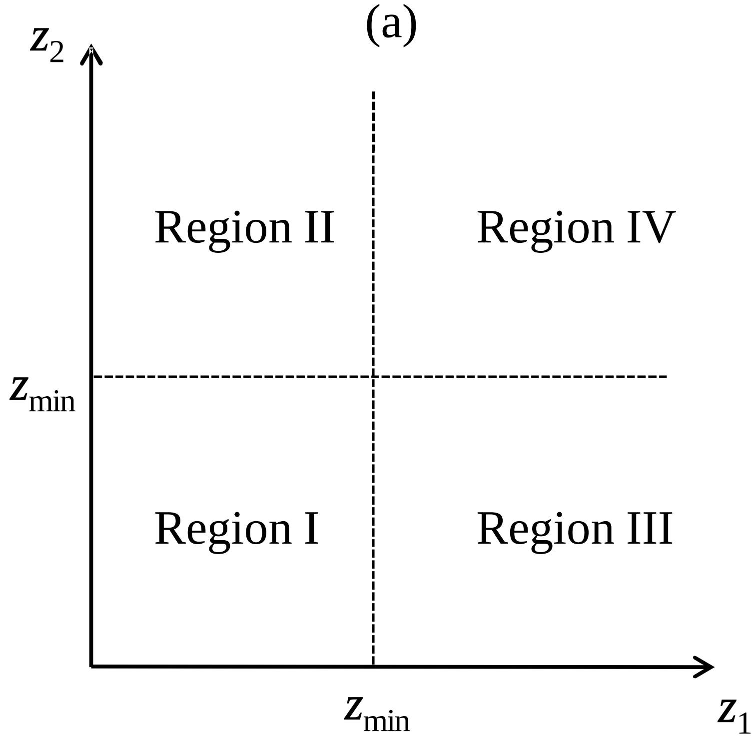
(d) Strictly convex, locally nonsatiated, continuous, and rational preferences give use a utility function that generates strictly convex (bowed toward the origin) indifference curves. I have drawn an example in Figure 3. In Figure 3, the indifference curve is just tangent to the budget set at a unique point. But this is just one of several possibilities. Two other interesting possibilities are an indifference curve that intersects the budget constraint at $(0, (w + w_{FS})/p_2)$ and one that intersects the budget constraint at $(w/p_1, w_{FS}/p_2)$. Note that strict convexity will rule out any point on the budget line connecting $(0, (w + w_{FS})/p_2)$ and $(w/p_1, w_{FS}/p_2)$. The bottom line is we know it will be unique because our *UCS* is strictly convex and our budget set is convex.

(e) Our arguments in part (d) are enough to establish that Walras Law will be satisfied. The reason why this is true relates to our assumption that preferences are locally nonsatiated.

You do not need convexity of the budget set or convexity of the Upper Contour sets for people to want to consume all their wealth. You just need to be able to say that there is always a bundle nearby that is strictly preferred and that preferences are continuous.

Figure 1

(a)


z_2
Region II
Region IV
z_{min}
Region I
Region III
z_{min}
z_1

(b)

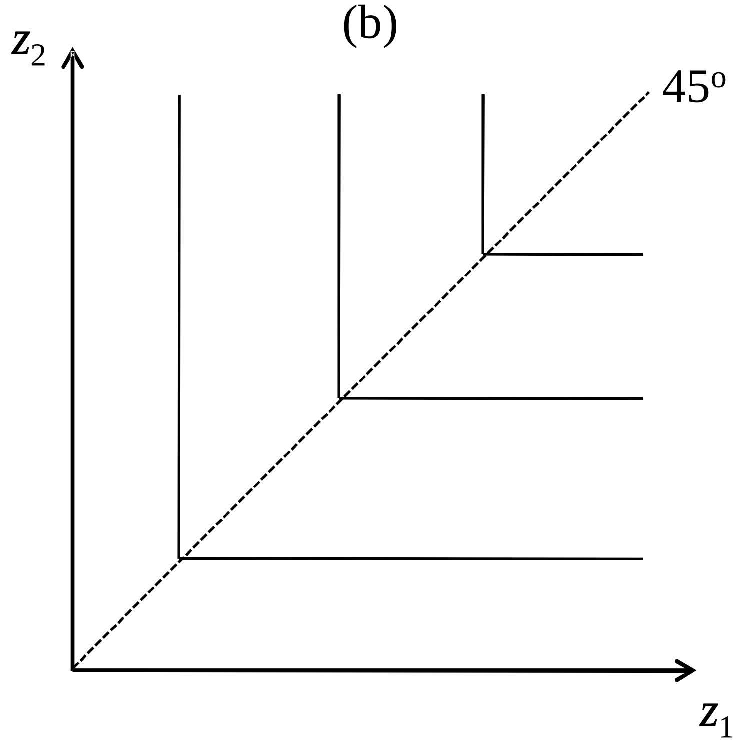

z_2
45°
z_1

Figure 2

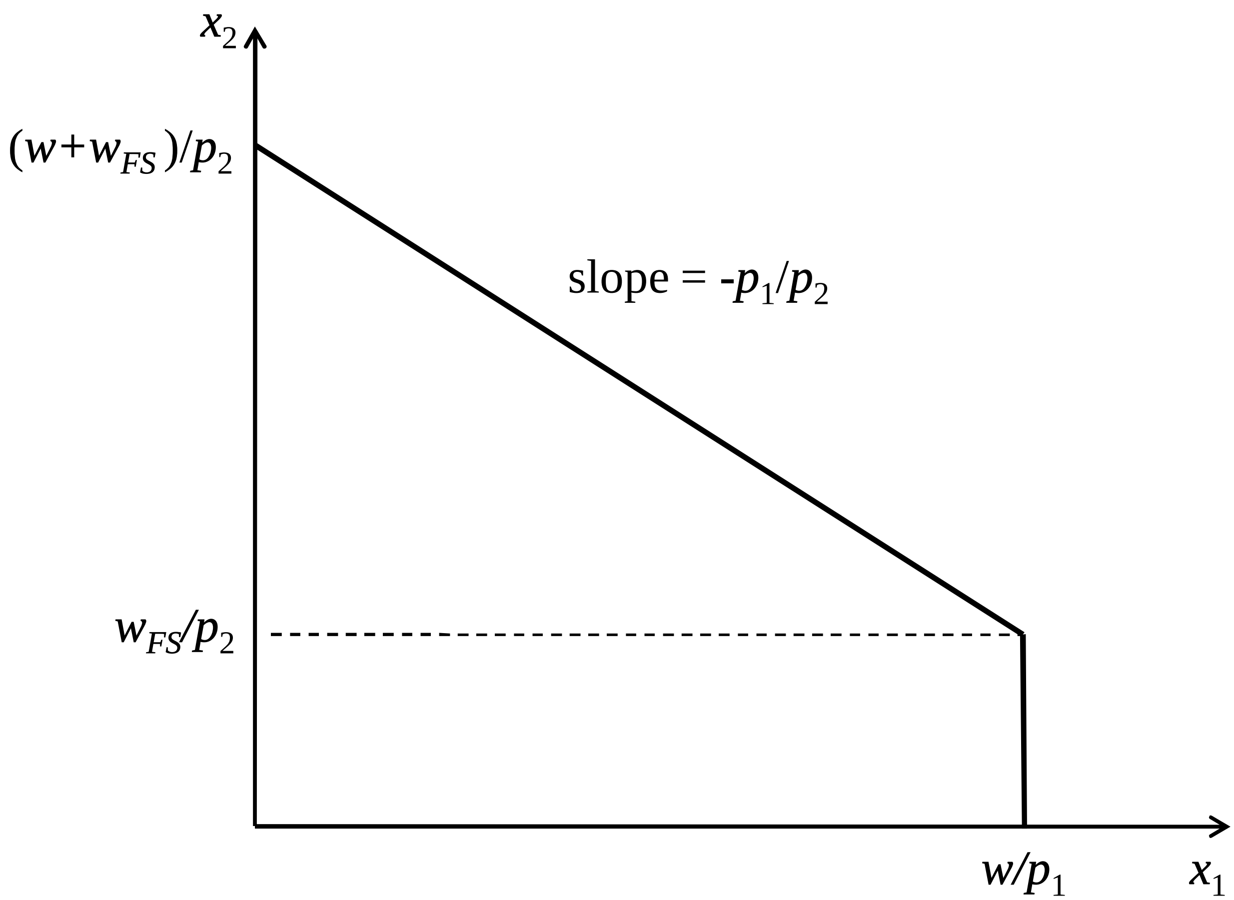


Figure 3