Application: Chain Store Game

The game we will explore now is attributable to Kreps and Wilson (*JET*, 1982). It falls into a class of games referred to as reputation models that are characteristic of a wide variety of economic phenomena. The games are distinct from the signaling types of games we have looked at so far because they are repeated games. Within each period of the repeated game there is no opportunity to convey information, but information can be conveyed across periods. This provides an opportunity for the player with complete information to send information to the other player overtime and build a reputation for how it might play in the future. While we will focus on an example of entry deterrence and the incentives a monopoly might have to build a reputation of playing tough to discourage entry even if it is not, other interesting examples include borrower attempts to build a reputation of creditworthiness or union attempts to build a reputation for being tough even though they might not be.

We will begin with the simple chain store entry deterrence game illustrated in Figure 1. This game has two players: an Entrant (E) and Monopolist (M). The Entrant starts the game by choosing to be *Out* (*O*) or *In* (*I*) the market. If the Entrant chooses to be *In* the market, the Monopolist gets to choose whether to *Fight* (*F*) or *Acquiesce* (*A*). The first payoffs in parentheses are the Entrants, while the second are the Monopolists. To keep the game interesting and make sure it is in the interest for the Monopolist to want to keep the Entrant *Out*, we will assume that $a > 1$, $c < 1$ and $d < 1$.

This game is straight forward to solve using subgame perfection. Let $s_M(F)$ be the probability the Monopolist chooses F given I. The Monopolist's best response function is

$$s_M^{br}(F) = \begin{cases} 0, & for\ \ d > c \\ [0,1], & for\ \ c = d \\ 1, & for\ \ c > d \end{cases}.$$

Let $s_E(I|)$ be the probability that the Entrant chooses in. Then the Entrant's best response is

$$s_E^{br}(I) = \begin{cases} 0, & for\ 0 > s_M(F)(b-1) + (1 - s_M(F))b = b - s_M(F) \\ [0,1], & for\ 0 = b - s_M(F) \\ 1, & for\ b - s_M(F) > 0 \end{cases}$$

To keep this game interesting, we will assume $1 > b > 0$ (if $b < 0$, *Out* would be a dominant or weakly dominant strategy for the Entrant, and if $b \geq 1$, if $b < 0$, *In* would be a dominant or weakly dominant strategy for the Entrant). With this assumption, if $d > c$,

$s_M^{br}(F) = 0$, and $s_E^{br}(I) = 1$. If $c > d$, $s_M^{br}(F) = 1$, and $s_E^{br}(I) = 0$. If $c = d$, $s_M^{br}(F) = [0,$ $1]$ and $s_E^{br}(I)=\begin{cases}0, & for\ s_M^{br}(F)>b\\ [0,1], & for\ s_M(F)=b\\ 1, & for\ b>s_M(F)\end{cases}$.

Now that we have characterized the equilibrium for the complete information Chain Store Game, let us complicate our lives by assuming the Entrant has incomplete information regarding the Monopolist's payoffs. Assume that the Monopolist is *Strong* (S) with probability p and *Weak* (W) with probability $1 - p$. Also, assume that $c = 0$ and $d = -1$ when the Monopolist is *Strong* and $c = -1$ and $d = 0$ when the Monopolist is *Weak*. This incomplete information game is illustrated in Figure 2.

To solve this more complicated game, we can appeal to the Perfect Bayesian Equilibrium (PBE). Let us begin by considering the Monopolist's best response given it is *Strong* and *Weak*. Let $s_M(F|t)$ for $t = S, W$ be the Monopolist's probability of choosing to *Fight* given its type. Since $0 > -1$, $s_M^{br}(F|S) = 1$ and $s_M^{br}(F|W) = 0$. Now let us ask ourselves about the Entrant's best response. If the Entrant chooses *Out*, it earns $0\times p + 0\times(1 - p) = 0$. If the Entrant chooses *In*, it earns $(s_M(F|S)(b - 1) + (1 - s_M(F|S))b)p + (s_M(F|W)(b - 1) + (1 - s_M(F|W))b)(1 - p) = b - s_M(F|S)\,p - s_M(F|W)(1 - p)$. Let $s_E(I|\,p)$ be the probability that the Entrant chooses *In* given the probability of a *Strong* Monopolist and the probabilities that a *Strong* and *Weak* Monopolist choose to *Fight*. The Entrant's best response is then

$$s_E^{br}(I\mid p)=\begin{cases}0, & for\ s_M(F\mid S)p+s_M(F\mid W)(1-p)>b\\ [0,1], & for\ s_M(F\mid S)p+s_M(F\mid W)(1-p)=b\\ 1, & for\ b>s_M(F\mid S)p+s_M(F\mid W)(1-p)\end{cases}.$$

Given the Monopolist's best responses, the BPE is

$$s_E^{br}(I\mid p)=\begin{cases}0, & for\ \ p>b\\ [0,1], & for\ \ p=b\\ 1, & for\ b>p\end{cases},\ s_M^{br}(F|S) = 1,\ s_M^{br}(F|W) = 0, \text{ and } p.$$

The last game was still pretty straightforward, but now let us suppose that the monopolist faces two different Entrants in succession and that the second Entrant has the opportunity to see how the Monopolist played against the first Entrant. That is, we are considering a two period game where the Monopolist plays the game in Figure 1 against one Entrant and then plays the game in Figure 1 against another Entrant.

The appropriate solution concept is still the PBE, which implies starting in the second period of the game. But we have already solved this game for the second period with one exception. In the second period game, the Entrant has possibly had a chance to observe what the Monopolist did in the first period game. Therefore, it may have a chance to update it beliefs regarding the probability that the Monopolist is *Strong*. Of course, there will be no opportunity for the second period Entrant to update its beliefs if the Monopolist does not have a chance to move. So, in the interest of keeping the game interesting, we will assume $b > p$.

Let $m(S|a)$ be the second period Entrant's belief regarding the probability that the Monopolist is *Strong*, given that the Monopolist chose the action $a = F, A$ in the first period. The second period Entrant's best response is then

$$s_{E_2}^{br}(I \mid m(S \mid a)) = \begin{cases} 0, & \textit{for } m(S \mid a) > b \\ [0,1], & \textit{for } m(S \mid a) = b \\ 1, & \textit{for } b > m(S \mid a) \end{cases}$$

where subscripts are used to denote the period and Bayes Rule implies

$$m(S \mid a) = \frac{p s_{M_1}(a \mid S)}{p s_{M_1}(a \mid S) + (1-p) s_{M_1}(a \mid W)}.$$

So, how can we proceed from here?

Let us start by asking ourselves if it is possible to have a separating equilibrium. To have a separating equilibrium, the *Strong* and *Weak* Monopolists must choose distinctly different actions in period 1. Suppose these actions are $s_{M_1}(F \mid S)$ = 1 and $s_{M_1}(F \mid W)$ = 0. These actions imply that that a *Strong* Monopolist will earn 0 + a = a in equilibrium, while a *Weak* Monopolist will earn 0 + 0 = 0 in equilibrium. Notice however, that if the *Weak* Monopolist deviated in period 1 and chose $s_{M_1}(F \mid W)$ = 1 instead of 0, he would earn -1 + a, which is greater than 0 by assumption, so he would be better off deviating. Therefore, this cannot be a PBE. We could also have a separating equilibrium where $s_{M_1}(F \mid S)$ = 0 and $s_{M_1}(F \mid W)$ = 1 resulting in the equilibrium payoffs of -1 + 0 = -1 and -1 + a for the *Strong* and *Weak* types. Notice however that if the *Strong* type deviated in period 1 and chose $s_{M_1}(F \mid S)$ = 1, he would earn 0 + a = a, which is higher than if he chose $s_{M_1}(F \mid S)$ = 0 in equilibrium, so this cannot be a PBE. The bottom line is that a separating equilibrium is not possible.

What about a pooling equilibrium? For a pooling equilibrium, $s_{M_1}(F \mid S)$ = $s_{M_1}(F \mid W)$ such that the equilibrium payoffs for the *Strong* and *Weak* type are 0 $s_{M_1}(F \mid S)$ -1(1 - $s_{M_1}(F \mid S)$) + 0 and -1 $s_{M_1}(F \mid W)$ + 0(1 - $s_{M_1}(F \mid W)$) + 0 because the second period Entrant will choose *In* given our assumption that $p > b$ and the fact that $m(S|F) = p$. Notice that if the *Strong* Monopolist deviated and chose $s_{M_1}(F \mid S)$ = 1, it would earn 0 + 0 which is greater than the pooling payoff for any $s_{M_1}(F \mid S)$ < 1. Also notice that if the *Weak* monopolist deviated and chose $s_{M_1}(F \mid W)$ = 0, it would earn 0 + 0, which is greater than the pooling payoff for all $s_{M_1}(F \mid W) > 0$. Therefore, for any possible pooling strategies for period 1, at least one type of Monopolist will have an incentive to deviate, so this cannot be a PBE.

What we have left to investigate is the possibility of a partially separating equilibrium. Let us take a stab in the dark (not really) and suppose that the *Strong* type of Monopolist always chooses to $s_{M_1}(F \mid S) = 1$ and the *Weak* type chooses $s_{M_1}(F \mid W) < 1$. Bayes rule and these first period strategies imply $m(S \mid F) = \frac{p}{p + (1-p)s_{M_1}(F \mid W)}$ and $m(S \mid A) = 0$. Therefore, the Entrant knows that the Monopolist is *Weak* and will always choose *In* if he sees the Monopolist *Acquiesce*. Now for the *Weak* Monopolist to be willing to mix its expected payoff from *Fight* and *Acquiesce* must be the same:

$$-1 + 0\, s_{E_2}^{br}(I \mid m(S \mid F)) + (1 - s_{E_2}^{br}(I \mid m(S \mid F)))a = 0 + 0,$$

which implies $s_{E_2}^{br}(I \mid m(S \mid F)) = \frac{a-1}{a}$. But, this means that the Entrant in the second period must play a mixing strategy when it sees *Fight* if there is to be an equilibrium. For the second period Entrant to be willing to play this mixing strategy, its payoffs from choosing *In* and *Out* must be equal or

$$m(S \mid F) = \frac{p}{p + (1-p)s_{M_1}(F \mid W)} = b,$$

such that $s_{M_1}(F \mid W) = \frac{p(1-b)}{b(1-p)}$.

We have one last thing to check before declaring an equilibrium. We must make sure that a *Strong* Monopolist does not want to deviate from choosing $s_{M_1}(F \mid S) = 1$, which yields the payoff $0 + \frac{a-1}{a}0 + \frac{1}{a}a = 1$. If the *Strong* Monopolist deviated and chose $s_{M_1}(F \mid S) < 1$, its payoff would be $s_{M_1}(F \mid S)(0 + \frac{a-1}{a}0 + \frac{1}{a}a) + (1 - s_{M_1}(F \mid S))(-1 + 0) = 2\, s_{M_1}(F \mid S) - 1 < 1$. Therefore, the *Strong* Monopolist cannot do better by deviating and we have found a PBE. This PBE is such that

$$s_{E_1}(I \mid p) = \begin{cases} 0, & for\ p > b^2 \\ [0,1], & for\ p = b^2 \\ 1, & for\ p < b^2 \end{cases},\ s_{E_2}(I \mid m(S \mid A)) = 1,\ s_{E_2}(I \mid m(S \mid F)) = \frac{a-1}{a},\ s_{M_1}(F \mid S)$$

$= 1$, $s_{M_2}(F \mid S) = 1$, $s_{M_1}(F \mid W) = \frac{p(1-b)}{b(1-p)}$, $s_{M_2}(F \mid W) = 0$, $m(S \mid F) = b$ and $m(S \mid A)$ $= 0$.

Two periods was indeed more challenging, but what about *T* periods? This is actually the problem solved by Kreps and Wilson. What did they find? Let m_t be the Entrant's belief in period *t* given the Monopolist chose to *Fight* in the previous period and s_{E_t} be the probability the Entrant is *In* in period *t*, then

- $s_{E_t} = 1$ if $m_t < b^{T-t+1}$,
- $s_{E_t} = \frac{a-1}{a}$ if $m_t = b^{T-t+1}$,
- $s_{E_t} = 0$ if $m_t > b^{T-t+1}$,
- $s_{M_t}(F \mid S) = 1$ for all *t*,
- $s_{M_t}(F \mid W) = 1$ if $m_t \geq b^{T-t}$,
- $s_{M_t}(F \mid W) = \frac{m_t(1-b^{T-t})}{b^{T-t}(1-m_t)}$ if $m_t < b^{T-t}$,
- $s_{M_t}(F \mid W) = 0$ if *Acquiesce* in $t - 1$,
- $s_{M_T}(F \mid W) = 0$,
- $m_1 = p$,
- $m_T = m_{T-1} = \ldots = m_{t+2} = m_{t+1} = 0$ if the Monopolist chooses *Acquiesce* in period *t*,
- $m_{t+1} = m_t$ if there is no entry in period *t* or if the Monopolist chooses *Fight* and $s_{M_t}(F \mid S) = s_{M_t}(F \mid W) = 1$ in period *t*, and
- $m_{t+1} = b^{T-t}$ if the Monopolist chooses *Fight* in period *t* and $s_{M_t}(F \mid S) = 1$ and $1 > s_{M_t}(F \mid W) > 0$.

I will let you look through their paper for the details of how they got here since our time is short.

The basic idea is that in early periods a *Weak* Monopolist should behave just like a strong one. As we begin to approach the end period, the *Weak* Monopolist should start mixing things up between *Fight* and *Acquiesce*. With the end period very near, the *Weak* Monopolist should just *Acquiesce*. In early periods, the Entrant should always be in unless it thinks there is a relatively high probability of a *High* type Monopolist. Over time, if the Monopolist has not chosen to *Acquiesce*, it is increasing likely that the Monopolist is *Strong*, so the Entrant should eventually switch to choosing between *In* and *Out* and should ultimately only choose *Out*.

It is obvious that equilibria for these types of games are really quite complicated. Therefore, one might suspect that subjects would not perform so well in experiments. Using a similar borrowing game in an experiment, Camerer and Weigelt conclude that the “sequential equilibrium predicts pretty well, given its complexity.” Jung, Kagel, Levin (1994) are also surprised at how well experienced subjects do in an entry deterrence game similar to the one above, but find inexperience players do not perform so well. Therefore, even though the solution to the games is very complex, subjects with complete information seem to grasp pretty well how building a favorable reputation can benefit them — at least for a period of time.

Figure 1: Chain Store Game

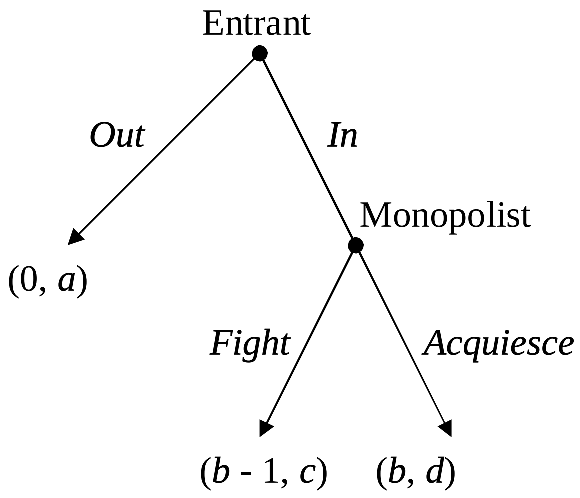

Figure 2: Chain Store Game With Incomplete Information