Applied Game Theory
APEC 8205

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Problem Set #4

1. Suppose there are two firms in an industry and that the firms act as Cournot competitors. Firm 1 has constant per unit costs of 12. Firm 2 can have either high constant per unit costs of 24, or low constant per unit costs of 12. Assume that the probability of high cost and low cost is each 0.5. Firm 2 knows whether it has high or low cost but firm 1 does not. The inverse industry demand curve is $P(X) = 72 - X$, where $X = x_1 + x_2$, where x_i is the output of firm i, i = 1, 2.

   a) Find the ex ante expected profit for firms 1 and 2 when information about firm 2's costs is private information.
   b) Find the ex ante expected profit for firms 1 and 2 when information about firms 2's cost is known by both firms prior to production.
   c) Compare the answers in parts (a) and (b). Is it better for firm 2 to keep information private or to reveal information?
   d) Now suppose that firm 2, after it has learned its own cost, can choose whether to reveal information to firm 1. Further suppose that firm 1 knows that this is the case. Find the perfect Bayesian equilibrium.

2. Two partners must dissolve their partnership. Partner 1 currently owns share s of the partnership, while Partner 2 owns share $1 - s$. The partners agree to play the following game: (i) Partner 1 names a price, p, for the whole partnership, and (ii) Partner 2 then either buys Partner 1's shares for ps or sells his shares to Partner 1 for $p(1 - s)$. Assume each partner's value for owning the whole partnership is a random independent draw from a uniform distribution on the interval [0. 1]. Also assume each partner's valuation is private information. What is the perfect Bayesian equilibrium? (Gibbons 4.10)

3. There are two types of workers. High productivity workers produce output of α, while low productivity workers produce output of β, with $\alpha > \beta > 0$. The probability of a worker being the high type is λ and the probability of a worker being the low type is $(1 - \lambda)$. In the first stage of the game, a worker can choose an education level, e. Education does not increase productivity. In the second stage, firms offer contracts to pay a wage based on the observed level of education: $w(e)$. Assume that firms are competitive so that in equilibrium, firms will earn zero profit. The utility function of high productivity workers is: $U_H = w - e^2$. The utility function of low productivity workers is: $U_L = w - 2e^2$.
   a) Find all perfect Bayesian separating equilibria for this game.
   b) Find all perfect Bayesian pooling equilibria for this game.
   c) Find one hybrid (semi-pooling/semi-separating) perfect Bayesian equilibrium for this game.

4. Consider the following simple game in which all players $i = 1, 2, \ldots, N$, simultaneously choose between strategy *A* and strategy *B*. The game is symmetric so that only the payoffs to a single player need to be represented. The payoffs given in the table below are for the row player. Let p be the proportion of players that play strategy *A* and $(1 - p)$ be the proportion of players that play strategy *B*. Suppose that players do not necessarily start out at equilibrium but that their strategy evolves over time based on the relative expected payoffs to playing different strategies. For this problem, assume that the evolution of strategies is given by continuous-time replicator dynamics:

$$dp/dt = p(\Pi(p) - \overline{\Pi}(p)),$$

where $\Pi(p)$ is the expected payoff from playing strategy *A* given p, and $\overline{\Pi}(p)$ is the expected payoff for all players given p.

	A	B
A	3	4
B	0	9

a) Solve the equilibrium dynamics for this game, i.e., solve for the equation that determines dp/dt.

b) What are the stationary points of dp/dt (i.e., equilibrium values of p)? Which of these stationary points are stable? Explain.

c) Suppose the initial value of p was 0.4. What is the value of p to which the system would evolve?

d) Replicator dynamics have been criticized as leaving out important elements of learning and behavior by strategic players in games. Briefly discuss the major limitations of replicator dynamics and explain an alternative to approach that addresses at least one major limitation.

5. Many homeowners in the U.S. are not making the housing payment and facing foreclosure do to the weak economy. In some instance, it could be better for lenders to reduce a homeowner's payments so they can stay in their home and the lender does not incur the high cost of trying to liquidate the house. Unfortunately, it is difficult for lenders to know for certain which homeowners really just need a lower payment to make ends meet and which may simply be trying to get lower payments even though they can afford their current payments and would never go into foreclosure.

   a) Design and solve a simple dynamic game of incomplete information to help inform lenders what factors they should consider when deciding whether or not to lower a homeowner's house payment if they are asked.

   b) The current administration is thinking of providing incentives to lenders to reduce a homeowner's housing payments instead of sending them into foreclosure. Modify your game in part a) and use your modified game to show how lenders and homeowners might respond to this type of policy. Who will be the winners and who will be the losers?