Applied Game Theory
APEC 8205

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Problem Set #3

1. Consider the two player game where Player 1 can choose either *Up* or *Down* and Player 2 can choose either *Left* or *Right.* Suppose that Player 1 is either cooperative with probability *P* or uncooperative with probability 1 – *P*. Player 1 knows whether or not she is cooperative, but Player 2 doesn't. The tables below show the players payoffs for the game when Player 1 is cooperative and when she is uncooperative.

   a) What are the pure strategy Bayesian Nash equilibria for this game, if any?
   b) Characterize the mixed strategy Bayesian Nash equilibria for this game.

Player 2

Payoffs with a Cooperative Player 1 (*P*)

Player 1		*Left*	*Right*
	Up	1, 1	-1, -1
	Down	-1, -1	1, 1

Payoffs with an Uncooperative Player 1 (1 - *P*)

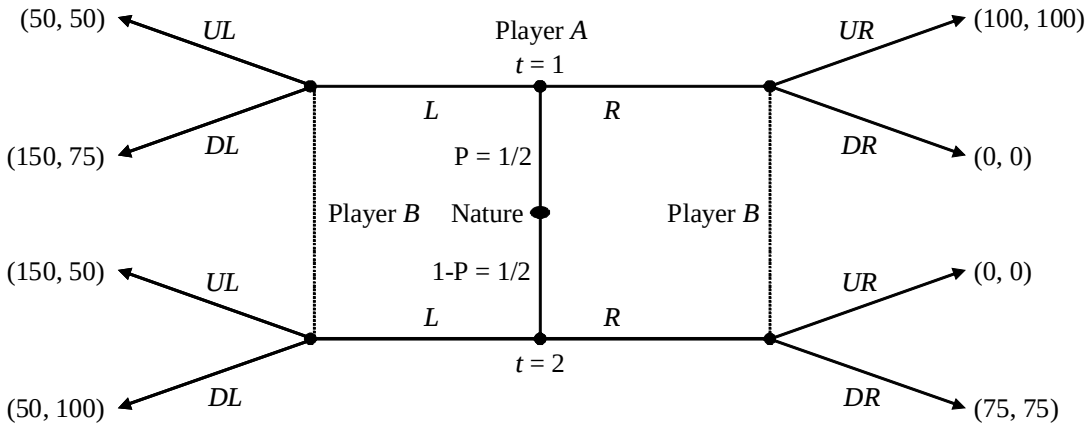
		Left	*Right*
	Up	1, -1	-1, 1
	Down	-1, 1	1, -1

2. Suppose we have two fishermen. Each expends effort x_i for i = 1, 2 to catch fish. The total catch, F, is a quadratic function of total effort: $F(X) = 2(\alpha^2 X - 0.5\beta X^2)$ where $X = x_1 + x_2$ and each fisherman shares in the catch equally. The price of fish is normalized to 1, while the cost of effort is $C(x_i) = 0.5x_i^2$. Finally, suppose fisherman 1 is experienced and knows α, while fisherman 2 is inexperienced and doesn't know α. Furthermore, assume $E(\alpha) = \mu$ and $Var(\alpha) = E(\alpha^2) - \mu^2 = \sigma^2$ completely characterize fisherman 2's beliefs about α and that these beliefs are common information.

   a) Find the Bayesian Nash equilibrium efforts for each fisherman. *Assume that the parameters yield an interior solution and that b > 0.*
   b) How does total effort in this incomplete information game compare to the socially optimal total effort assuming there is no incomplete information? Assuming fisherman 2's beliefs about α are unbiased (i.e. $\alpha = \mu$), does fisherman 2's incomplete information move us closer to the optimal level of effort? That is, does fisherman 2's incomplete information exacerbate or mitigate the free riding incentives of these fishermen?

3. Suppose there are two firms competing for a government contract. Each knows its own value of the contract, but not the value to the other firm. Each also knows that they will win the contract only if they give the most generous bribe to G.W. Hedge. Assume firms are risk neutral. Also assume each firm's value is randomly drawn from the distribution function $F(v)$ where $v \in [0, V]$.
   a) Find the symmetric and monotonic Bayesian Nash bribe strategy. Interpret the meaning of this strategy.
   b) Assuming $F(v)$ is uniform. Find the ex ante expected level of bribes for a firm. Compare this to the ex ante expected value of the contract. What is ex ante rent dissipation?

4. Consider the dynamic incomplete information game in the figure below. There are three players: Nature, *A*, and *B*. Nature starts the game by randomly selecting *A*'s type *t*, either 1 or 2, with equal probability. Nature reveals this information to *A*, but not to *B*. Once *A* has this information, it must choose either *L* or *R*. If *A* chooses *L*, *B* must choose between *UL* or *DL*. If *A* chooses *R*, *B* must choose between *UR* or *DR*. Payoffs are summarized in parentheses, with the first payoff corresponding to *A* and the second to *B*.
   a) What are *A*'s and *B*'s pure strategy spaces?
   b) Find all pure strategy perfect Bayesian equilibria (PBE) for this game?
   c) Which of these equilibria satisfy the intuitive criteria? Justify your answer.



5. Suppose we have two auto company executives going to Capital Hill to get bailout money. Each executive needs a certain minimum amount of bailout funds in order to avoid bankruptcy, but will take as much as it can get— the more the better. How much is needed to avoid bankruptcy is not known to their competitor. Congress has a fixed amount of money it will hand out that is known to both executives. The executives also know that Congress does not want to handout money to a company if the amount it is asking for is not enough for the company to avoid bankruptcy. Finally, the executives know that Congress is mad and will give more of the bailout money to the company that appears to be less greedy. Construct a static game of incomplete information that captures the strategic incentives facing the executives. Derive Bayesian Nash equilibrium strategies for how much bailout money the executives will ask for given the minimum amount of funds they need to need to avoid bankruptcy.