

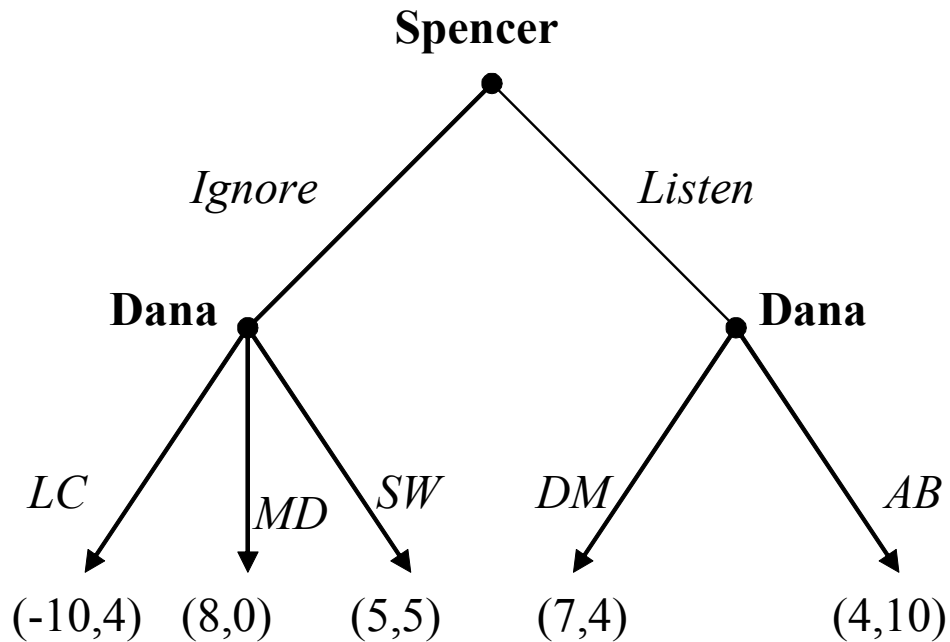
Applied Game Theory
APEC 8205

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Midterm Exam Answers

1. Dana has asked Spencer to get in the car so they can go to dinner. They only have an hour and a half before going to parent/teacher conferences. If Spencer *Ignores* his mom they will only have time for fast food: Leann Chen's (*LC*), McDonald's (*MD*), or Subway (*SW*). If Spencer *Listens* they will have time to eat at one of two nicer restaurants: Damon's (*DM*) or Applebee's (*AB*). Spencer's and Dana's preferences for each restaurant are reflected in the extensive form game below.
 - a) List Spencer's and Dana's strategies for the game.
 - b) Write the game in its normal form.
 - c) Find all pure strategy Nash equilibrium for the game.
 - d) Which of these Nash equilibria are subgame perfect?
 - e) Based on experimental evidence, do you believe that this subgame perfect equilibrium is a good prediction of behavior? Explain.



Note: (Spencer's Utility, Dana's Utility)

Answer:

a) Spencer's Strategy set is $\{Ignore, Listen\}$. Dana's is $\{(LC, DM), (LC, AB), (MD, DM), (MD, AB), (SW, DM), (SW, AB)\}$.

b)

		Dana											
		(LC, DM)	(LC, AB)	(MD, DM)	(MD, AB)	(SW, DM)	(SW, AB)						
Spencer	<i>Ignore</i>	-10	4	-10	4	8*	0	8*	0	5	5*	5*	5*
	<i>Listen</i>	7*	4	4*	10*	7	4	4	10*	7*	4	4	10*

c) In the Table above, I have placed * by each player's best responses, which yields the Nash pure strategy equilibria: $\{Listen, (LC, AB)\}$ and $\{Ignore, (SW, AB)\}$.

d) If Spencer chooses *Ignore*, Dana's best response is *SW*. Spencer chooses *Listen*, Dana's best response is *AB*. Therefore, Spencer can expect to earn 5 from choosing *Ignore* and 4 from choosing *Listen*. He should choose *Listen*. Therefore, the subgame perfect equilibrium is $\{Ignore, (SW, AB)\}$.

e) It is probably not a good prediction of behavior. Why? Because Dana can punish Spencer by choosing *LC* (Spencer despises Chinese food), which reduces his payoff by -14 compared to Dana's preferred outcome. Obviously, this costs Dana, but it only costs her 1 compared to having Spencer *Ignore* her and choosing her optimal strategy of going to *SW*. The subgame perfect equilibrium occurs because Dana's threat of choosing *LC* is deemed incredible. But experimental evidence tells us that incredible threats are often carried out, especially if the cost of carrying them out is not too high.

2. Let C be the total fish catch for a fishing fleet with a total of R vessels such that $C = F(R)$ where $F'(R) > 0$ and $F''(R) < 0$. Normalize the price of fish to one and assume the price of a fishing vessel is p . Suppose that there are n fishermen. Let r_i be the number of fishing vessels employed by the i th fisherman and assume that the i th fisherman's catch is proportional to his share of the fishing fleet and the total catch: $c_i = r_i C/R$.
- Derive and interpret the condition for the socially optimal Fleet Size assuming we want to maximize the net value of the catch.
 - Derive the conditions required for a symmetric Nash equilibrium harvest with n fishermen.
 - Under what circumstances if any will this Nash equilibrium lead to the socially optimal fleet size? Discuss the implications of this result in terms of managing the commons.

Answer:

- a) To maximize the net value of the catch we need to solve: $\max_R F(R) - pR$. The first order condition for this problem is $F'(R) - p = 0$. The second order condition is $F''(R) < 0$. The first order condition implies $F'(R) = p$, or that at the social optimum we should set the marginal productivity of adding one more vessel ($F'(R)$) equal to its cost (p).

- b) The i th fisherman's problem is $\max_{r_i} \frac{r_i F\left(\sum_{j=1}^n r_j\right)}{R} - pr_i$. The first order condition is

$$\frac{(F(R) + r_i F'(R))R - r_i F(R)}{R^2} - p = 0, \text{ which implies } \frac{(F(R) + r_i F'(R))R - r_i F(R)}{R^2} =$$

$$\frac{r_i}{R} F'(R) + \left(1 - \frac{r_i}{R}\right) \frac{F(R)}{R} = p. \text{ I will leave the second order condition to you. Now suppose}$$

we have a symmetric Nash equilibrium such that $r_i = r^*$ for all i :

$$\frac{r^*}{nr^*} F'(R^*) + \left(1 - \frac{r^*}{nr^*}\right) \frac{F(R^*)}{R^*} = \frac{1}{n} F'(R^*) + \frac{n-1}{n} \frac{F(R^*)}{R^*} = p \text{ where } R^* = nr^*.$$

$\frac{1}{n} F'(R^*) + \frac{n-1}{n} \frac{F(R^*)}{R^*} = p$ implies that in a symmetric Nash equilibrium Fishermen equate the price of a fishing vessel with a weighted average of marginal and average productivity, but our socially optimal level of fishing effort equates the price of a fishing vessel to its marginal productivity.

- c) The symmetric Nash equilibrium will be the same as the social optimum only if $n = 1$. As n get very large, fishermen produce where average productivity equals average cost or where there is no economic profit from fishing, a socially inefficient result. These results imply that the tragedy of the commons can be averted by monopolizing the commons.

3. Consider the following two player game.

		Player 2	
		L	R
Player 1	D	4	6*
	U	6*	0

- Find ALL Nash equilibria for this game if it were to be played once.
- Suppose this game is to be played twice, and there is no discounting between the two rounds. What is the highest sum of expected payoffs for the two players that can be achieved as a subgame perfect equilibrium? Explain.
- Now suppose the game is to be repeated an infinite number of periods. Suppose the discount factor is 0.9. Is it possible to support (U, L) as a subgame perfect equilibrium outcome? If so, how (i.e., specify strategies for each player that support this outcome)? If not, why not?

Answer:

- I have placed an * in the Table above for all pure strategy best responses, which imply the pure strategy Nash equilibria: (U, L) and (D, R). Now what about mixed strategies? Let x be the probability Player 1 chooses D and y be the probability Player 2 chooses L. A mixing strategy for Player 1 will make Player 2 indifferent between strategies: $4x + 1(1-x) = 6x + 0(1-x) \Rightarrow x = 1/3$, with $E(\pi_2) = 2$. A mixing strategy for Player 2 will make Player 1 indifferent between strategies: $4y + 1(1-y) = 6y + 0(1-y) \Rightarrow y = 1/3$, with $E(\pi_1) = 2$. Now let's characterize these three Nash equilibrium strategies in terms of x and y : (i) $(x = 0, y = 1)$ such that $E(\pi_1) = 6$ and $E(\pi_2) = 1$; (ii) $(x = 1, y = 0)$ such that $E(\pi_1) = 1$ and $E(\pi_2) = 6$; and (iii) $(x = 1/3, y = 1/3)$ such that $E(\pi_1) = 2$ and $E(\pi_2) = 2$.
- When the game is played twice, given that there are multiple Nash equilibria in the stage game, there is the possibility that many strategy combinations could be part of a subgame perfect equilibrium.

In round 2, the final round, players must play a Nash equilibrium. Both of the pure strategy Nash equilibria yield a total payoff of 7 (6 + 1). The mixed strategy Nash equilibrium yields an expected payoff of 2 for each player (1/9 probability of (D,L), 2/9 probability of (D,R), 2/9 probability of (U,L), 4/9 probability of (U, R): $1/9 \times 4 + 2/9 \times 1 + 2/9 \times 6 + 4/9 \times 0 = 2$), for a total expected payoff of 4.

Going back to round 1, the highest expected total payoff occurs with (D, L), where the total expected payoff is 8. Is there any way to support (D, L) in the first round as part of a subgame perfect equilibrium? The answer is no. Suppose the strategy is to play (D, L) in the first round and the mixed strategy Nash equilibrium in round 2 if (D, L) was played in round 1, otherwise play the Nash equilibrium where the player that deviated in round 1 gets a payoff of 1. By following the strategy, a player receives a payoff of $4 + 2 = 6$, but by

deviating, the player earns $6 + 1 = 7$. So, deviation is more profitable and this is not an equilibrium. Suppose the strategy is to play (D, L) in the first round and one of the two pure strategy Nash equilibrium strategies in round 2. In the pure strategy Nash equilibrium, one of the players receives a payoff of 1. But then why not deviate in the first round? So, this cannot be an equilibrium either.

With standard assumptions, it is not possible to support (D, L) as part of a subgame perfect equilibrium. Therefore, the highest equilibrium total payoff would be 7 in each round, or 14 over the two rounds.

However, if we expand what kind of communication is allowed among players it might be possible to do better. Suppose that the two players agree to a random draw over which pure strategy Nash equilibrium would occur in round 2, which yields an expected payoff of 3.5, as long as players play (D, L) in round 1. If a player deviates in round 1, then the players play the Nash equilibrium in which that player who deviated receives a payoff of 1. In this case, there is no incentive to deviate ($4 + 3.5 > 6 + 1$). But this answer requires some ability to randomize on which Nash equilibrium will be played in round 2 and coordinate play accordingly (also called correlated equilibrium). In this case, we could get a total expected payoff of $8 + 7 = 15$. But, again, this would require abilities beyond Nash equilibrium.

- c) Consider a trigger strategy of the following form: player 1 plays D as long as (D, L) has been played in the prior round, but play U forever after if player 2 ever plays R; player 2 plays L as long as (D, L) has been played in the prior round, but play R forever after if player 1 ever plays U. In this case, the present value of playing the equilibrium strategy is: $4 + 4(0.9) + 4(0.9)^2 + \dots = 4/(1-0.9) = 40$. The payoff from deviation is $6 + 1(0.9) + 1(0.9)^2 + \dots = 6 + 0.9/(1-0.9) = 6 + 9 = 15$. Therefore, no player will ever deviate and (U, L) can be supported in a subgame perfect equilibrium.