

ANSWERS: Quiz #3
APEC 3001
Applied Microeconomics:
Consumers, Producers, and Markets
(Summer 2007)
Instructor: Hurley

Name:

Please show all the work you do to solve a problem.

1. What is an intermediate product? **(1.5 Points)**

Answer: Products that are transformed by a production process into products of greater value.

2. What is a natural monopoly? **(1.5 Points)**

Answer: An industry whose market output is produced at lowest cost when production is concentrated in the hands of a single firm.

3. Consider the production function $Q = K^{0.5}L^{0.4}$ where K is capital and L is labor. Which of the following statements about this production function is true?
- Its exhibits decreasing returns to scale.
 - Its exhibits constant returns to scale.
 - Its exhibits increasing returns to scale.
 - It exhibits decreasing, increasing, or constant returns to scale depending on how much capital and labor are used.

Answer: a. If we multiply both labor and capital by $\alpha > 1$, output will increase by a proportion less than α : $(\alpha K)^{0.5} (\alpha L)^{0.4} = \alpha^{0.5} \alpha^{0.4} K^{0.5} L^{0.4} = \alpha^{0.9} K^{0.5} L^{0.4} = \alpha^{0.9} Q < \alpha Q$.

4. Consider the long-run total cost function $LTC = 500Q + 10Q^2$. Which of the following statements is true?
- Long-run average cost is decreasing.
 - Long-run average cost is increasing.
 - Long-run average cost is constant.
 - Long-run average cost is U-Shaped.

Answer: d. There are two ways to answer this question. First, we can find the long-run average cost and then take the derivative: $LAC = LTC/Q = (500Q + 10Q^2)/Q = 500 + 10Q$, such that $LAC' = 10$. Notice that $LAC' > 0$ for any Q , which means LAC is increasing.

Alternatively, we can compare the long-run average cost to long-run marginal cost. From above, we know $LAC = 500 + 10Q$. Long-run marginal cost is the derivative of long-run total cost: $LMC = LTC' = 500 + 20Q$. Therefore, $LMC > LAC$ because $500 + 20Q > 500 + 10Q$, which implies $20 > 10$. This also implies an increasing long-run average cost curve because long-run marginal cost is always above long-run average cost.

5. Consider the production function $Q = KL$ where K is capital and L is labor.
- Assuming capital is fixed at 250 (i.e. $K = 250$), what are the short-run total, average, and marginal product?
 - Assuming the price of capital is $r = \$20$ and the price of labor is $w = \$10$, what are the short-run fixed, variable, and total cost?
 - What is the marginal rate of technical substitution?
 - Again assuming the price of capital is $r = \$40$ and the price of labor is $w = \$10$, what are the long-run total cost, long-run average cost, and long-run marginal cost?

Answer:

- Short-run total product is $TP = Q = 250L$. Short-run average product is the short-run total product divided by the variable input labor: $TP/L = 250L/L = 250$. Short-run marginal product is the derivative of the short-run total product with respect to the variable input labor: $TP' = 250L^{1-1} = 250$.
- Short-run fixed cost is the quantity of the fixed input multiplied by its price: $FC = \$20 \times 250 = \$5,000$. Short-run variable cost is the quantity of variable input multiplied by its price: $VC = \$10L$. But we want this in terms of output, so recall that $Q = 250L$, which implies $L = Q/250$ or $VC = \$10Q/250 = \$Q/25$. Short-run total cost is the sum of fixed and variable costs: $TC = FC + VC = \$5,000 + \$Q/25$.
- $MRTS = MP_L/MP_K$. $MP_L = \frac{\partial Q}{\partial L} = KL^{1-1} = K$. $MP_K = \frac{\partial Q}{\partial K} = K^{1-1}L = L$. So, $MRTS = K/L$.
- First, we need the output expansion path, which we get by setting the marginal rate of technical substitution equal to the ratio of input prices: $MRTS = w/r \Rightarrow K/L = \$10/\$40$ or $L = 4K$. Now long-run total cost is the sum of the price of inputs multiplied by their quantities: $LTC = rK + wL = \$40K + \$10L$. But again, we want this in terms of quantity. Note that $Q = KL$, but along the output expansion path $L = 4K$, so $Q = K(4K) = 4K^2$ or $K = (Q/4)^{0.5} = Q^{0.5}/2$. Now back to $LTC = \$40K + \$10L = \$40K + \$10(4K) = \$40K + \$40K = \$80K = \$80 Q^{0.5}/2 = \$40Q^{0.5}$. To get long-run average cost, we divide long-run total cost by output: $LAC = LTC/Q = \$40Q^{0.5}/Q = \$40/Q^{0.5}$. To get long-run marginal cost, we take the derivative of long-run total cost: $LTC' = 0.5 \times \$40Q^{0.5-1} = \$20/Q^{0.5}$.