Rational Choice

Readings: Ch. 3 (including the appendix)

Objective: Understand what economists mean by limited means (scarcity) and how individual choice is constrained by limited means.

Definition

Bundle: A particular combination of products.

Economists assume that all individuals derive satisfaction or utility from the consumption of a bundle of products. This consumption is typically measured as a flow per unit of time: how many pounds of food and square yards of housing are consumed during a week.

Definition

Budget Constraint/Opportunity Set: The set of all affordable bundles given income and prices.

Earlier we discussed how microeconomics is the study of people trying to fulfill unlimited wants with limited means. The budget constraint serves to tell us how limited means restrict the fulfillment of our wants.

For example, if we earn M = \$500 per week and only consume pounds of food (F) and square feet of housing (H) how much food and housing is it possible to consume if the price of food is P_F = \$5 a pound and the price of housing is P_H = \$10 per square foot?

Our expenditure on food is the price of food multiplied by the quantity of food consumed, $P_F F$ = \$5F. Our expenditure on housing is the price of housing multiplied by the quantity of housing consumed, $P_H H$ = \$10H. Total expenditures are equal to the sum of our expenditures on food and housing: $P_F F + P_H H = \$5F + \$10H$. Our means are limited by our weekly income. We cannot spend more in a week than we earn such that

$M = \$500 \geq P_F F + P_H H = \$5F + \$10H$.

Assume for a moment that we spend all our income on food and housing, then

$\$500 = \$5F + \$10H$ or

$M = P_F F + P_H H$.

If we rearrange terms, we get

$F = \$100 - \$2H$ or

$F = M/P_F - (P_H/P_F)H$,

which we should recognize as the equation of a line. Figure 1 shows a graph of this line, which represents all combinations of food and housing that require exactly \$500 to purchase.

Question: Is a combination of food and housing above this line affordable?

No, bundles above this budget constraint have more food, more housing, or more of both food and housing. Since our income is fully exhausted for any bundle on the budget constraint, we have no way to pay for this additional food or housing.

Question: Is a combination of food and housing below this line affordable?

Yes, bundles below this budget constraint have less food, less housing, or less of both. Since our income is fully exhausted for any bundle on the budget constraint, we can get rid of some food or housing and what we will end up with is some income left over.

Therefore, the shaded area in Figure 1 shows all bundles of food and housing that are affordable given our income and prices.

Another interesting feature to note about this budget constraint is its slope. Its slope is the negative of the ratio of prices: - \$2 or $– P_H/P_F$. This slope tells us how much food we must give up if we want to increase the amount of housing we consume and still use up all of our income.

The intercept on the vertical axis tells us how much food we can afford if we only buy food. The intercept on the horizontal axis tells us how much housing we can afford if we only buy housing.

Objective: Understand how changes in prices and income affect the budget constraint.

Question: What happens to the budget constraint in Figure 1 if we increase weekly income from \$500 to \$600?

With an extra \$100 per week we can afford more food, more housing, or more of both food and housing: \$600 = \$5F + \$10H or F = \$120 - \$2H. Since the price of food and housing have not changed, the amount of food we must give up to consume more housing and still use up all of our income doesn't change. The slope of the budget constraint doesn't change. What these two results imply is the parallel shift up in the budget constraint shown in Figure 2. In general, any increase in income will result in a parallel shift up in the budget constraint, while any decrease in income will result in a parallel shift down in the budget constraint.

Question: What happens to the budget constraint in Figure 1 if we increase the price of food from \$5 to \$10 a pound?

With food costing \$10 a pound, our budget constraint becomes \$500 = \$10F + \$10H or F = \$50 - \$1 H. If we spend all our income on housing, we can still purchase 50 square feet. But now if we spend all our money on food we can only buy 50 pounds instead of 100. Also, since the price of food is now higher relative to the price of housing, we must give up less food in order to consume more housing and still use up all our income: the slope of the budget constraint is not as steep. Figure 3 illustrates the net result, which is a downward rotation of the budget constraint about the horizontal axis. Now if we spend all our income on food, we cannot buy as much. But, if we spend all our income on housing, we can still buy just as much. If the price of food decreased, the budget constraint would rotate upward about the horizontal axis.

The effect of an increase or decrease in the price of housing is similar to what happened with a change in the price of food. The only real difference is that the budget constraint rotates about the vertical instead of the horizontal axis. Specifically, if the price of housing increases, the budget constraint rotates downward about the vertical axis. If the price of housing decreases, the budget constraint rotates upward about the vertical axis.

Note that all of this generalizes to scenarios where there are more than two goods. It just becomes more difficult to draw in a two dimensional graph.

Also note that when an economist is only really interested in a single product, say X, he will often define the second product as a ***composite good***, say Y. A ***composite good*** is just an amalgam of all other products other than X or just income spent on something other than the X. With this simplification the budget constraint can be written as $M = Y + P_X X$ or $Y = M - P_X X$.

Budget constraints can get more complicated than this. For example, the food stamp program provides money for food to people with low income.

Question: If an individual earns M = \$300 a week, receives FS = \$100 a week in food stamps that are only good for the purchase of food, the price of food is P_F = \$5, and the price of housing is P_H = \$10, what does the budget constraint look like?

Receiving FS = \$100 in food stamps is like getting $FS/P_F = 100/5 = 20$ pounds of food for free each week. So, if this individual spends all his income on housing he can buy $M/P_H = 300/10 = 30$ square feet and also 20 pounds of food if he wants. If he spends all his money on food, he can buy $M/P_F + FS/P_F = 300/5 + 100/5 = 60 + 20 = 80$. Therefore,

$$
\begin{aligned}
F \le FS/P_F = 20 &\Rightarrow M = P_H H \\
&\Rightarrow 300 = 10H \\
&\Rightarrow H = 30.
\end{aligned}
$$

$$
\begin{aligned}
F > FS/P_F = 20 &\Rightarrow M + FS = P_F F + P_H H \\
&\Rightarrow F = (M + FS)/P_F - (P_H/P_F)H \\
&\Rightarrow F = 80 - 2H.
\end{aligned}
$$

Figure 4 graphs this budget constraint.

Another example that complicates the budget constraint is two-tier pricing, which is common with electricity or phone service for example. Two tier pricing means that the price you pay either declines or increases after you buy a certain amount of a product.

Question: Suppose an individual's income is M = \$500 a week, the price of food is P_F = \$5, and the price of housing is P_{H0} = \$10 for the first H_0 = 20 square feet and P_{H1} = \$5 for each additional square foot above H_0 = 20. What does the budget constraint look like?

If this individual only buys food, he can buy $M/P_F = 500/5 = 100$. If this individual buys H_0 = 20 units of housing, he can afford

$$M = P_F F + P_{H0}H_0 \Rightarrow F = M/P_F - (P_{H0}/P_F)H_0 \text{ or}$$

$$F = 100 - 40 = 60$$

pounds of food. If the individual spends all his money on housing, he can buy

$$M = P_{H0}H_0 + P_{H1}(H - H_0) \Rightarrow H = M/P_{H1} - ((P_{H0}-P_{H1})/ P_{H1})H_0 \text{ or}$$

$$H = 100 - 20 = 80.$$

Therefore,

$$\begin{aligned} H < H_0 = 20 &\Rightarrow M = P_F F + P_{H0}H \\ &\Rightarrow F = M/P_F - (P_{H0}/P_F)H \\ &\Rightarrow F = 100 - 2H \end{aligned}$$

$$\begin{aligned} H > H_0 = 20 &\Rightarrow M = P_F F + P_{H0}H_0 + P_{H1}(H - H_0) \\ &\Rightarrow F = M/P_F - ((P_{H0}-P_{H1})/P_F)H_0 - (P_{H1}/P_F)H \\ &\Rightarrow F = 100 - 20 - H = 80 - H. \end{aligned}$$

Figure 5 shows the graphical representation of this budget constraint.

Objective: Understand what economists mean by unlimited wants and how these unlimited wants affect individual choice.

Definition

Preference Ordering: A scheme whereby the consumer ranks all possible consumption bundles in order of preference.

Economists presume that individuals derive satisfaction or utility by consuming bundles of products. If we think about all possible bundles of products one might consume, a natural question to ask is: which bundles provide greater satisfaction or are preferred? An individual's answer to this question provides the basis for his preference ordering. To keep things simple, suppose we ask an individual to compare two bundles, say (i) and (ii) (e.g. (i) 50 pounds of food and 75 square feet of housing and (ii) 60 pounds of food and 65 square feet of housing), based on the level of satisfaction provided. There are four possible answers to this question: (a) bundle (i) provides greater satisfaction, (b) bundle (ii) provides greater satisfaction, (c) bundles (i) and (ii) are equally satisfying, or (d) the individual does not know which bundle provides greater satisfaction. By repeating this question for all possible pairs of bundles, we can construct the individual's preference ordering.

If an individual's preference ordering exhibits four nice properties, we can conveniently write the individual's level of satisfaction, I, as a function of the amount of each product the individual consumes. For example, $I = U(F,H) = F^{0.5}H^{0.5}$ when the only two products available for consumption are food, F, and housing, H. These properties are

1. Completeness: An individual can answer (a), (b), or (c) for every possible pair of consumption bundles. That is, for any two bundles (i) and (ii), the individual knows that a bundle (i) is more satisfying than (ii), less satisfying than (ii), or as equally satisfying as (ii). The completeness property is violated when an individual responds with "I don't know."
2. More-Is-Better (Nonsatiation): If a bundle (i) has the more of some products and at least the same amount of all other products as bundle (ii), then the individual always answers (a): (i) is more satisfying than (ii).
3. Transitivity: If an individual is asked to compare pairs of any three bundles, say (i), (ii), and (iii), and he responds with (i) is more satisfying than (ii) and (ii) is more satisfying than (iii), then he must also respond with (i) is more satisfying than (iii).
4. Convexity: Mixtures of goods are preferable to extremes. For example, suppose bundle (i) contains 50 pounds of food and 75 square feet of housing and bundle (ii) contains 60 pounds of food and 65 square feet of housing. Also, assume that (i) is just as satisfying as (ii). Convexity then implies that if you take half of (i) and add it to half of (ii) to form a third bundle (iii) with 55 pounds of food and 70 square feet of housing, then bundle (iii) will be more satisfying than either (i) or (ii). More generally, if we draw a straight line between any two equally preferred bundles, the bundles along that line will be preferred. Convexity conveys the idea that individuals like variety.

The function that these properties allow us to construct is called the utility function. Plugging the products contained in a bundle into the utility function gives us a number, but what does this number mean? An important property of the utility function is that it is *ordinal*, which means the values it produces allows us to reconstruct an individual's preference ordering, but nothing more. Therefore, if we use the utility function to evaluate bundle (i) and (ii) and come up with utility values equal to 20 and 10, all we know from this information is that bundle (i) is more satisfying than bundle (ii). Even though 20 is twice as big as 10, we cannot say bundle (i) is twice as satisfying as bundle (ii). By itself, the utility of a bundle tells us absolutely nothing about an individual's level of satisfaction. It only becomes meaningful, when the utility of one bundle is compared to the utility of another bundle. This also means that it is impossible to compare utility or levels of satisfaction between people.

Definition
Indifference Curve (Contour): A set of bundles among which the consumer is indifferent or equally satisfied.

An indifference curve tells us all bundles that are equally satisfying. The utility function we derive from an individual's preference ordering provides us with all the information we need to find an indifference curve. By setting $I = U(F,H) = F^{0.5}H^{0.5}$ equal to some constant, say 5, 5 =

$U(F,H) = F^{0.5}H^{0.5}$ tells us a set of bundles that provide the same level of satisfaction. For graphical purposes, it is convenient to solve for the variable to be plotted on the vertical axis: $F^{0.5}=5/H^{0.5} \Leftrightarrow F=25/H$ or in general $F = f_I(H)$ where I indicates the level of satisfaction and $f_I(H)$ is some function holding I constant.

All this should be familiar. Remember the budget constraint tells us all combinations of bundles that cost the same amount. An indifference curve tells us all combinations of bundles that provide the same level of satisfaction. Both are just contour plots of difficult to draw multi-dimensional figures. The only real difference is that budget constraints are often linear, while indifference curves are not.

If we set $I = U(F,H) = F^{0.5}H^{0.5}$ equal to some other constant, say 10, $10 = U(F,H) = F^{0.5}H^{0.5}$ tells us another set of bundles that all provide the same level of satisfaction. We can repeat this for any value of I we deem useful. The result is an indifference map, which is illustrated in Figure 7 for I = 5, 10, and 15.

Definition
Indifference Map: A representative sample of the set of a consumer's indifference curves, used as a graphical summary of his preference ordering.

The first thing to note about our indifference map is that satisfaction increases as we move to indifference curves that are further from the origin or higher up.

The four properties of preference orderings also imply indifference curves and maps will also satisfy four other important properties:

1. Indifference curves are ubiquitous. Completeness tells us any possible bundle will fall on some indifference curve.
2. Indifference curves are downward sloping. If they were upward sloping, more wouldn't be better.
3. Indifference curves cannot cross. If they did, either more isn't better or transitivity does not work.
4. Indifference curves become less steep as we move down and to the right. They bow toward the origin or are convex to the origin. If not, mixtures of goods would not be preferred to extremes. This property also implies a diminishing the marginal rate of substitution.

The slope of the budget constraint tells us how much of one good we have to give up if we wanted to consume more of another, but didn't want to spend any more money. Similarly, the slope of an indifference curve tells us how much we need to increase the consumption of one good in order to maintain our level of satisfaction if we decrease the consumption of another good. We call this the marginal rate of substitution.

Definition
Marginal Rate of Substitution: The rate at which the consumer is willing to exchange one good for another.

Graphically, the marginal rate of substitution for a bundle is equal to the absolute value of the slope of the line tangent to the indifference curve at that bundle (Figure 8 illustrates).

Definition
Marginal Utility: The increase in utility resulting from a one unit increase in the consumption of a product.

All else equal, if we increase our consumption of a product, the more is better property tells us our level of satisfaction (utility) will increase. Marginal Utility measures by how much.

Here calculus becomes terribly useful:

MRS = $|f_I'(H)|$, or

$$\text{MRS} = \frac{\frac{\partial U(F,H)}{\partial H}}{\frac{\partial U(F,H)}{\partial F}} = \frac{U_H(F,H)}{U_F(F,H)}, \text{ or}$$

MRS = MU_H/MU_F

where MU_H is the marginal utility of housing and MU_F is the marginal utility of food, or

MRS = I^2/H^2 for $I = F^{0.5}H^{0.5}$, or

MRS = 100/64=25/16 = 6.25 for $I = F^{0.5}H^{0.5}$, H = 4, and I = 10.

All of the expressions above represent different ways of saying exactly the same thing, either generally or for a specific utility function.

Objective: Understand how to use the marginal rate of substitution, prices and income to find the best (most satisfying) feasible bundle.

Now that we have a description of our unlimited wants and limited means we can bring them together to find the best (most satisfying) feasible (affordable) bundle. Figure 9 shows the budget constraint from our original example and a representative indifference map. Consider the three bundles denoted as (i), (ii), and (iii) in Figure 9. We know from the budget constraint that only (i) and (ii) are affordable, but bundle (iii) is not. We know from our indifference map that (iii) is more satisfying than (ii) and (ii) is more satisfying than (i). Therefore, even though we would prefer to consume (iii), we can't afford to. We can consume (i), but why would we? Bundle (ii) is more satisfying and affordable.

Question: Is there a bundle that is more satisfying than (ii) and still affordable?

Note that any bundle on the budget constraint to the left of (ii) is affordable, but is also below I_1, so it is not as satisfying. Similarly, any bundle on the budget constraint to the right of (ii) is affordable, but is also below I_1, so it is not as satisfying. Finally, any bundle to the right of I_1 is unaffordable because it is above the budget constraint. Therefore, we cannot find a bundle that is more satisfying than (ii) and still affordable. Bundle (ii) is the best feasible bundle.

There is something important to recognize about bundle (ii), the marginal rate of substitution is equal to the ratio of the prices:

$MRS = P_H/P_F$.

Except in special circumstances, the best feasible bundle will always satisfy this property.

Recall that $MRS = MU_H/MU_F$. Therefore, the best feasible bundle also satisfies the property that

$MU_H/P_H = MU_F/P_F$.

MU_H/P_H tells us the increase in satisfaction from consuming more housing per dollar spent on housing. MU_F/P_F tells us the increase in satisfaction from consuming more food per dollar spent on food. If $MU_H/P_H > MU_F/P_F$, an extra dollar spent on housing increases satisfaction by more than an extra dollar spent on food. Therefore, if at all possible, we should increase our consumption of housing and decrease our consumption of food. Alternatively, if $MU_H/P_H < MU_F/P_F$, an extra dollar spent on food increases satisfaction by more than an extra dollar spent on housing. Therefore, if at all possible, we should increase our consumption of food and decrease our consumption of housing.

This leads us to an important exception to the rule, $MRS = P_H/P_F$. If we are already consuming only housing and $MU_H/P_H > MU_F/P_F$, we cannot afford to consume more housing because we have no food to give up. Therefore, $MRS > P_H/P_F$ if the best feasible bundle is to spend all our income on housing. Similarly, $MRS < P_H/P_F$ if the best feasible bundle is to spend all our money on food. In these types of circumstances, we have what is called a *corner solution*.

For calculus aficionados, what we are talking about here is the constrained optimization problem:

$\underset{F,H}{\text{Max}}\ U(F,H)$ subject to $F \geq 0$, $H \geq 0$, and $M \geq P_F F + P_H H$.

A careful treatment of the problem would use the Kuhn-Tucker conditions with non-negativity constraints. A less careful treatment of the problem assumes $F > 0$, $H > 0$, and $M = P_F F + P_H H$, so we can rewrite the constrained optimization problem as

$$\underset{H}{\text{Max}}\ U\left(\frac{M}{P_F} - \frac{P_H}{P_F}H, H\right),$$

which leads to the first order condition $-\frac{P_H}{P_F}U_F(F,H)+U_H(F,H)=0$ or rearranging terms

$\frac{U_H(F,H)}{U_F(F,H)}=\frac{P_H}{P_F}$. $U_H(F,H)$ is the marginal utility of housing and $U_F(F,H)$ is the marginal utility of food, so $U_H(F,H)$ / $U_F(F,H)$ = MRS. To assure we have an maximum and not a minimum, we must also check the second order condition, which is

$\left(\frac{P_H}{P_F}\right)^2 U_{FF}(F,H)-2\frac{P_H}{P_F}U_{HF}(F,H)+U_{HH}(F,H)<0$. Note that if the preference ordering satisfies our nice properties, this second order condition will hold for any F and H.

To convince you that all of this really does work, let us try an example. Suppose $I = F^{0.5}H^{0.5}$ and $M = P_F F + P_H H$.

Question: What is the best feasible bundle?

Above, we said the MRS for $I = F^{0.5}H^{0.5}$ was I^2/H^2. Substitution yields MRS = $(F^{0.5}H^{0.5})^2/H^2$ = F/H. We know the best feasible bundle satisfies MRS = P_H/P_F, so F/H = P_H/P_F or F = $P_H H/P_F$. Substitution into the budget constraint yields $M = P_F P_H H/P_F + P_H H = 2P_H H$ or $H = M/2P_H$. Substitution again yields $F = P_H(M/2P_H) / P_F$ or $F = M/2P_F$.

Therefore, if the price of food is 5, the price of housing is 10, and weekly earnings are \$500, the best feasible bundle is F = 500/(2×5) = 50 and H = 500/(2×10) =25.

Finally, note that we can substitute our solution for the best feasible bundle back into our utility function: $I = (M/2P_F)^{0.5} (M/2P_H)^{0.5} = M/(2P_F^{0.5}P_H^{0.5}) = V(M,P_F,P_H)$. This function is referred to as the *indirect utility function*. It tells us the most satisfaction we can get for a given level of income and prices. It also tells us that this level of satisfaction increases as income increases and decreases as the price of one of the goods increases.

Figure 1: Example budget constraint.

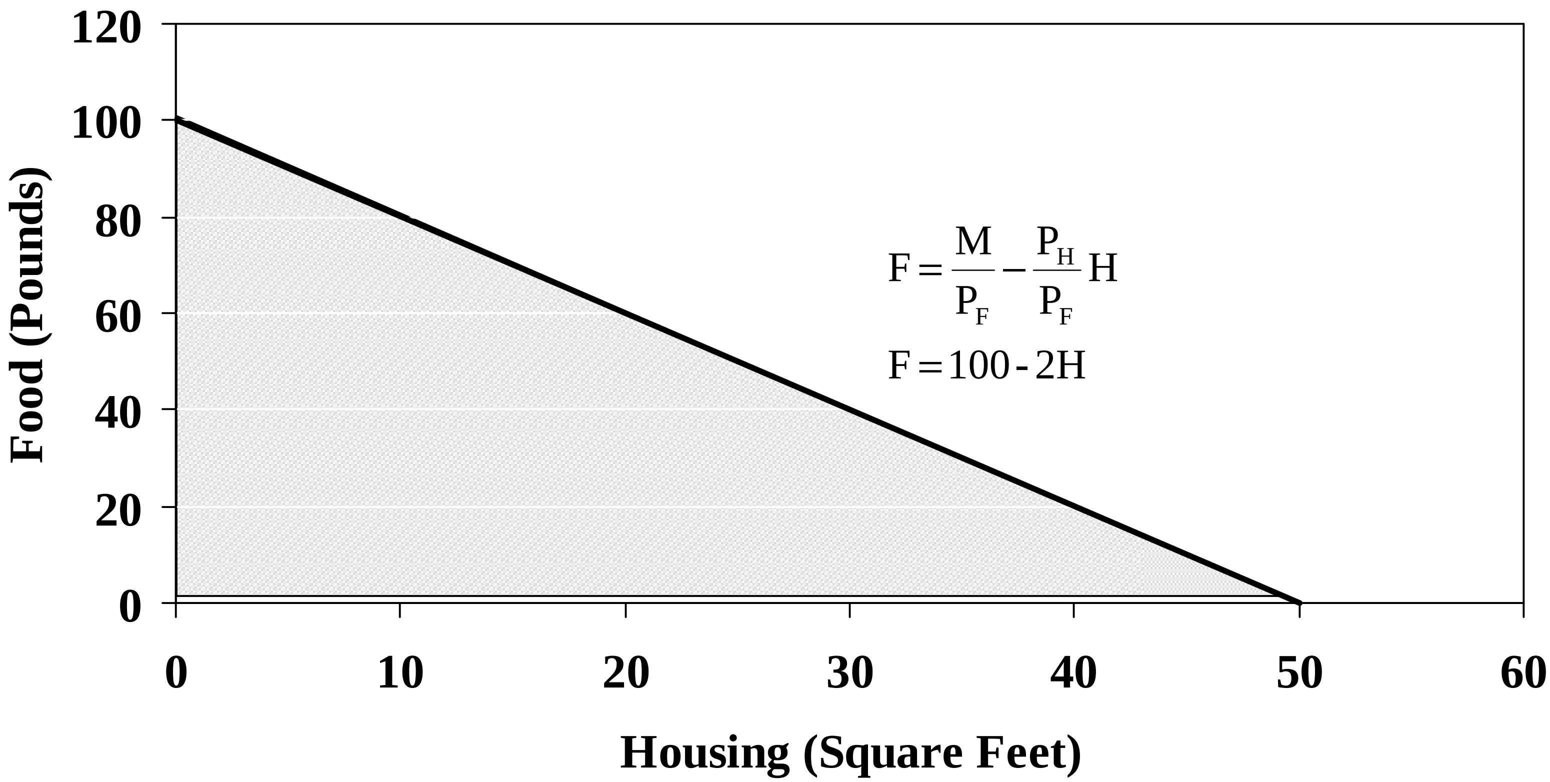

Figure 2: Effect of an increase in income on the budget constraint.

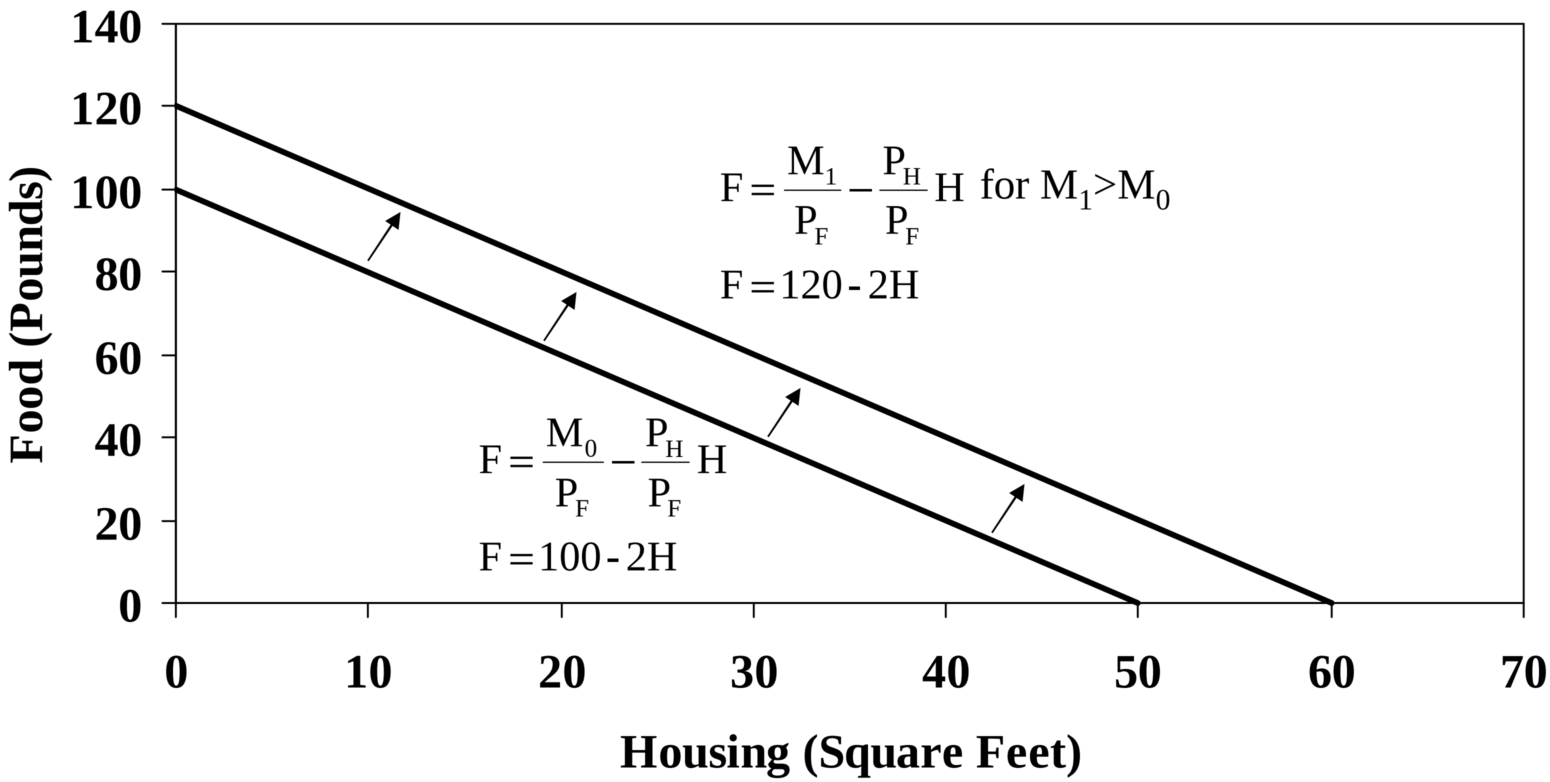

Figure 3: Effect of an increase in the price of food on the budget constraint.

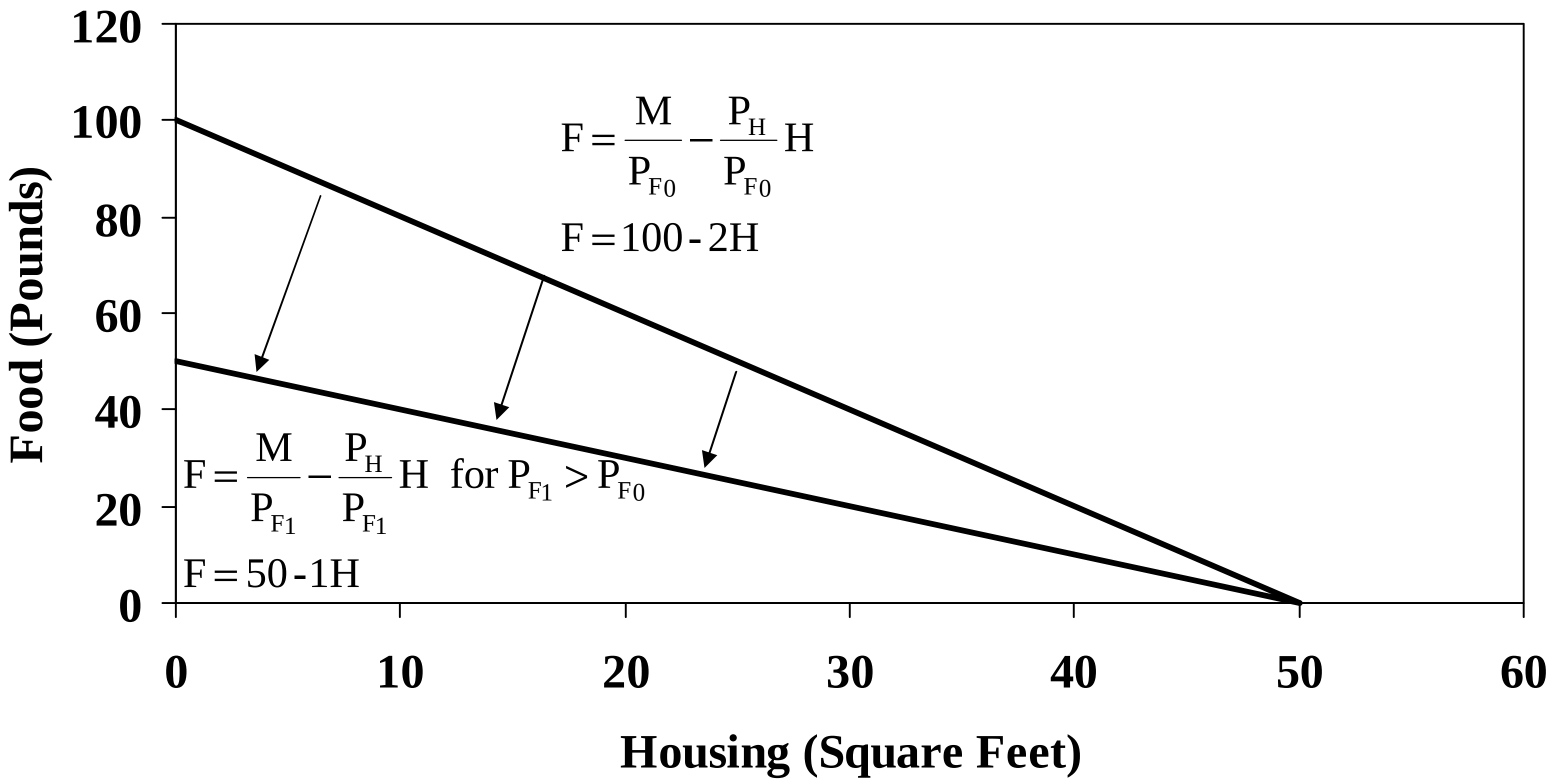

Figure 4: Effect of food stamp program on the budget constraint.

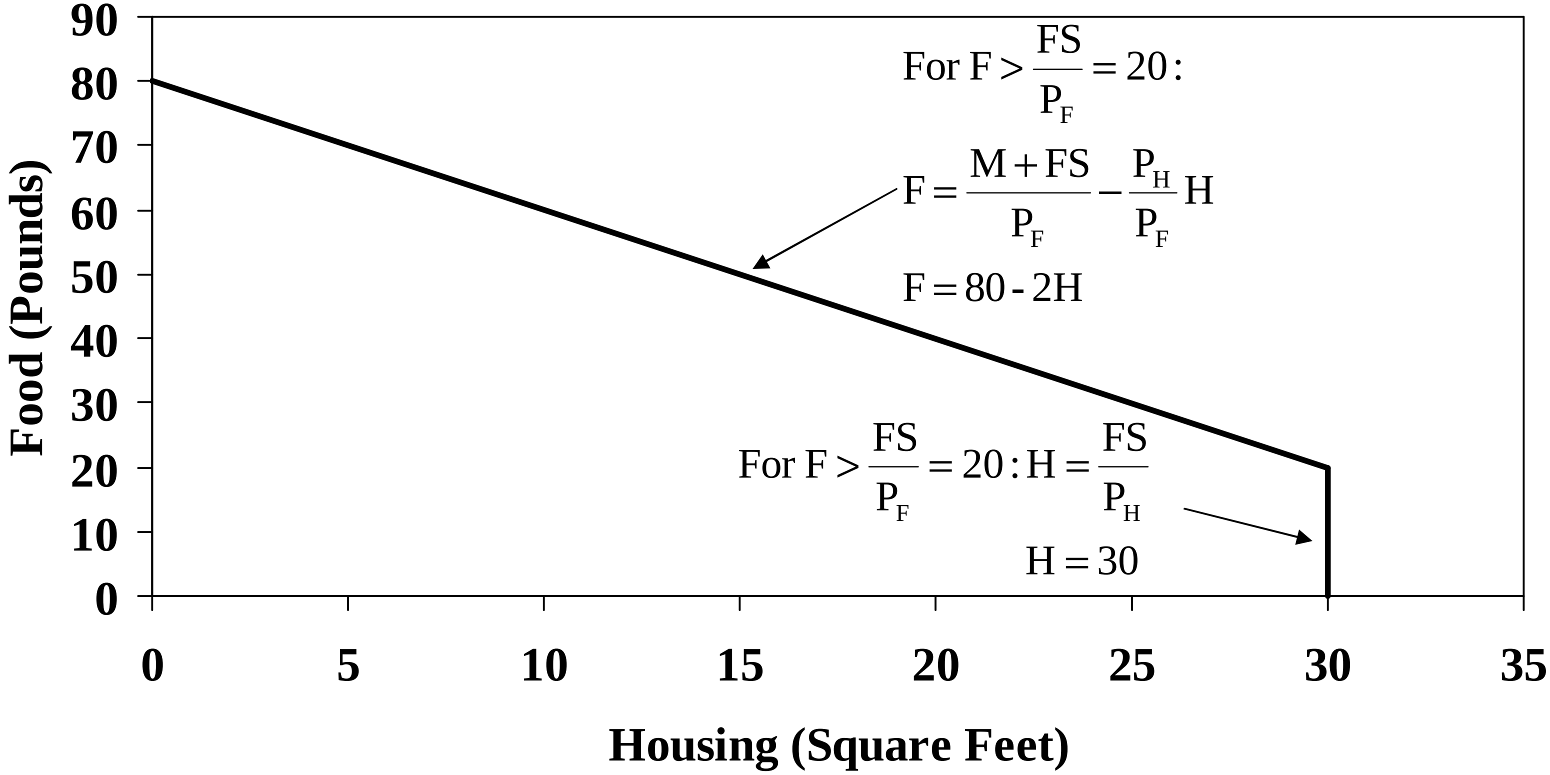

Figure 5: Effect of two tier pricing on the budget constraint.

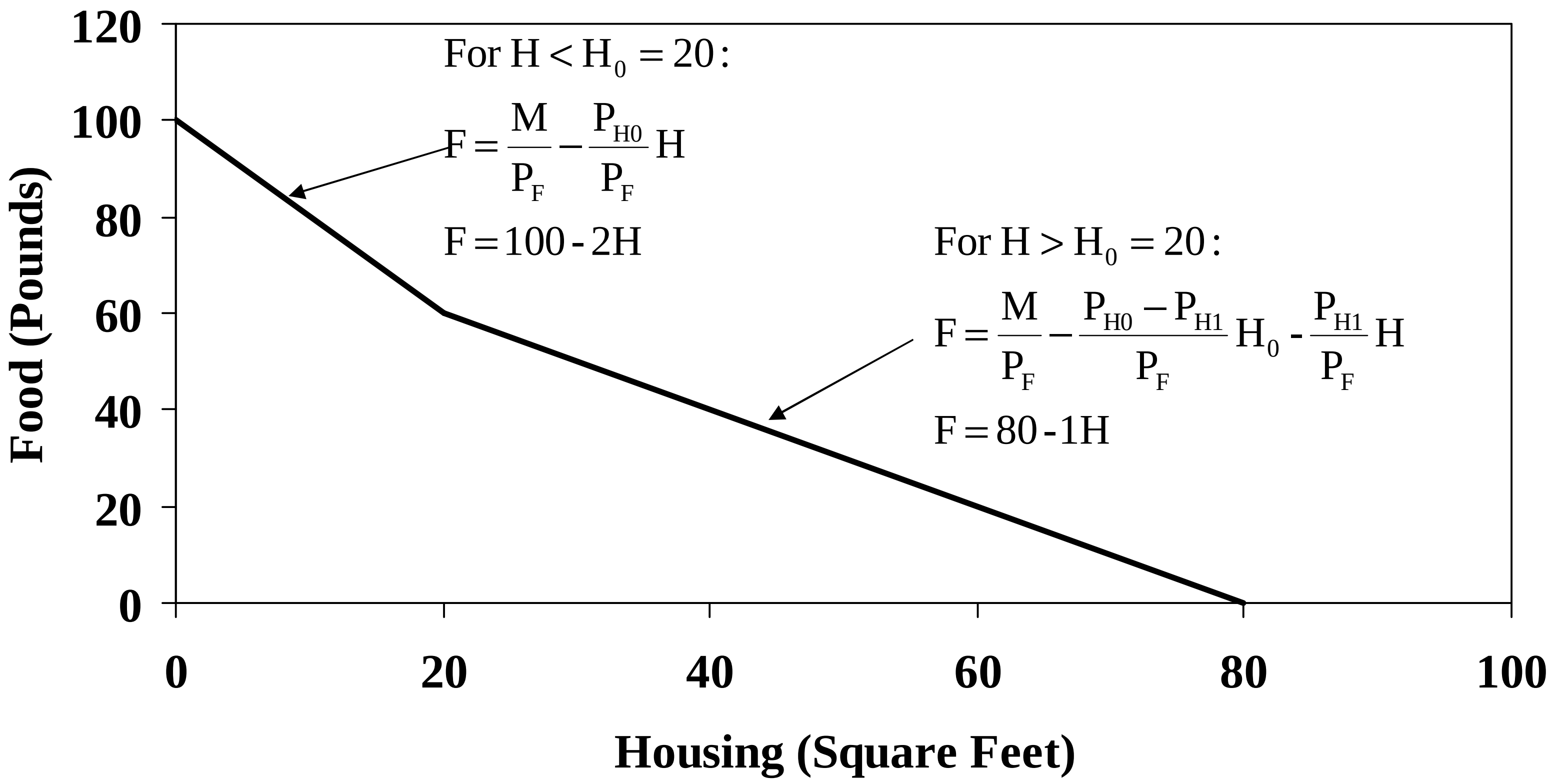

Figure 6: Example Indifference Curve

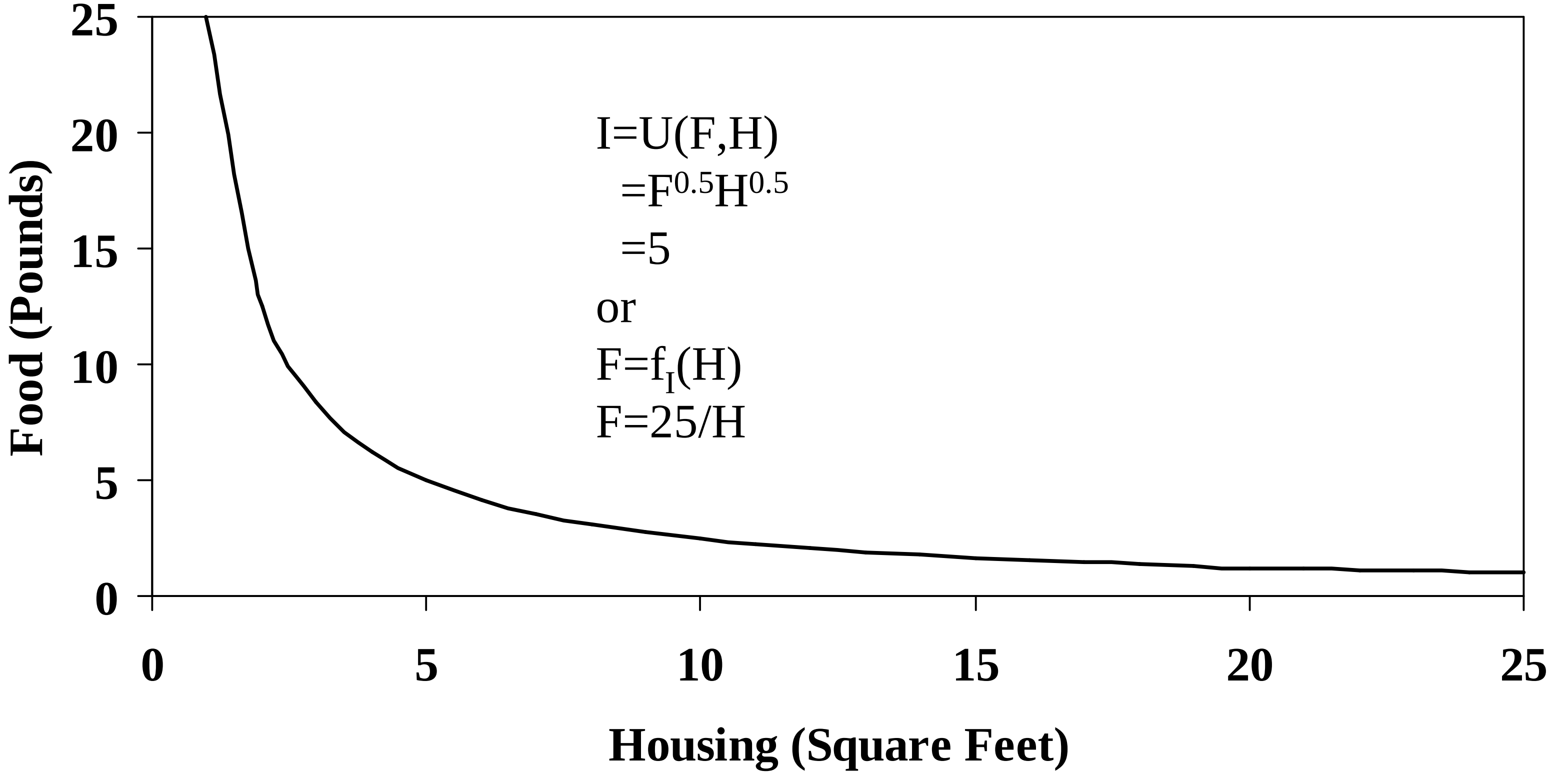

Figure 7: Example Indifference Map

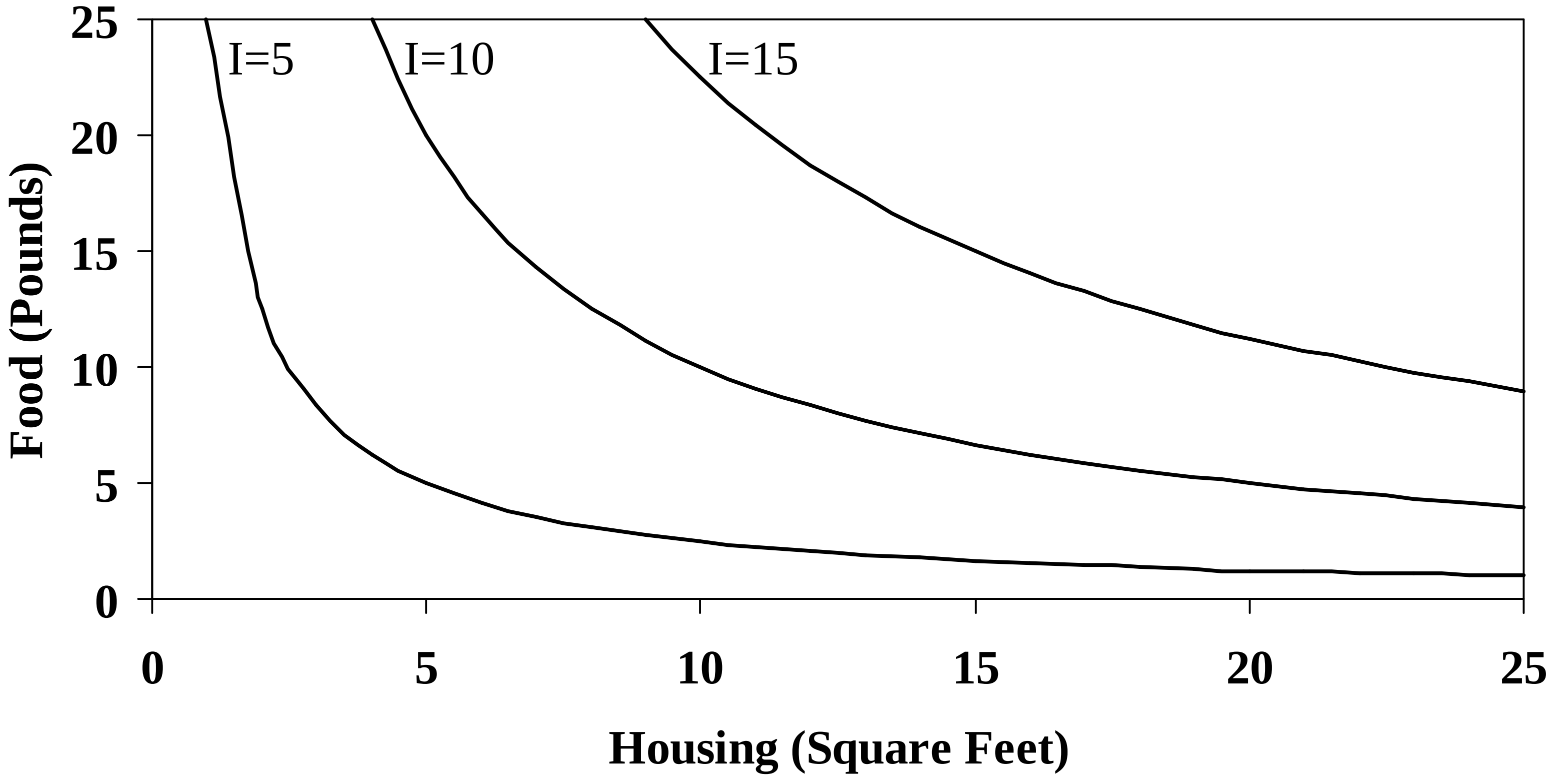

Figure 8: Marginal Rate of Substitution

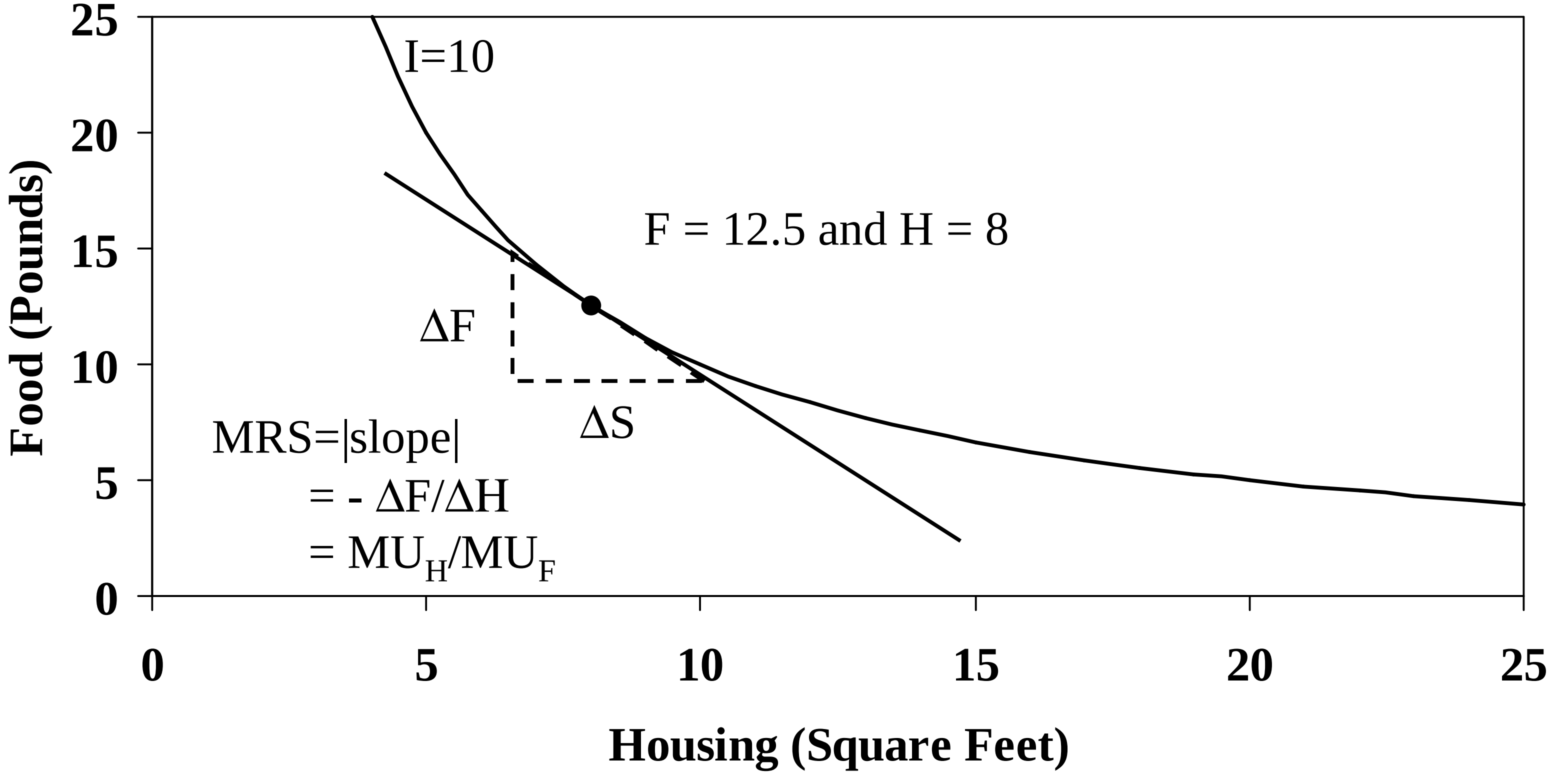

Figure 9: Finding the Best Feasible Bundle