

Production

APEC 3001

Summer 2007

Readings: Chapter 9 & Appendix in Frank

Objectives

- Describing Production
- Short-Run Production
- Long-Run Production
- Returns to Scale

Describing Production Definitions

- Output:
 - Good or service produced by an individual or firm.
- Inputs:
 - Resources used in the production of output.
- Production Function:
 - A relationship that describes how inputs can be transformed into output.
 - e.g. $Q = F(K,L)$ where K is capital & L is labor.
- Intermediate Product:
 - Products that are transformed by a production process into products of greater value.

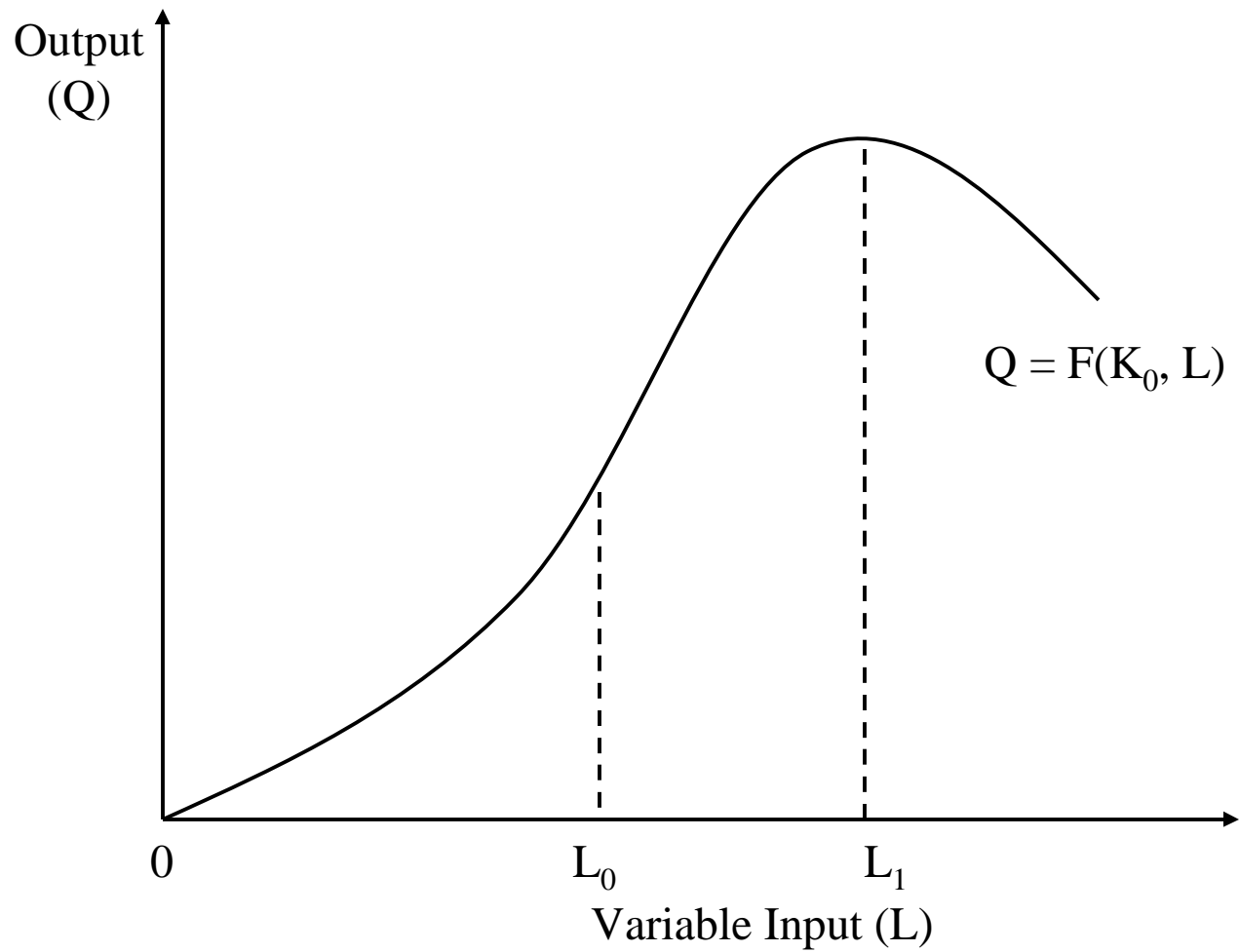
Inputs Definitions

- Variable Inputs:
 - Inputs in a production process that can be changed.
- Fixed Inputs:
 - Inputs in a production process that can not be changed.

Short-Run Production Definitions & Example

- Definition
 - The longest period of time during which at least one of the inputs used in the production process cannot be varied.
- Example
 - Suppose $K = K_0$, such that $Q = F(K_0, L) = F_0(L)$.
 - Output in the short-run only depends on the amount of labor we choose.
- Some More Definitions
 - Total Product Curve:
 - A curve showing the amount of output as a function of the amount of variable input.
 - Law of Diminishing Returns:
 - If other inputs are fixed, the increase in output from an increase in variable inputs must eventually decline.

Typical Short Run Production Function or Total Product Curve



Short-Run Production Regions of Production

- Region I:
 - Increasing Returns - 0 to L_0
- Region II:
 - Decreasing (Positive) Returns - L_0 to L_1
- Region III:
 - Decreasing (Negative) Returns - Above L_1

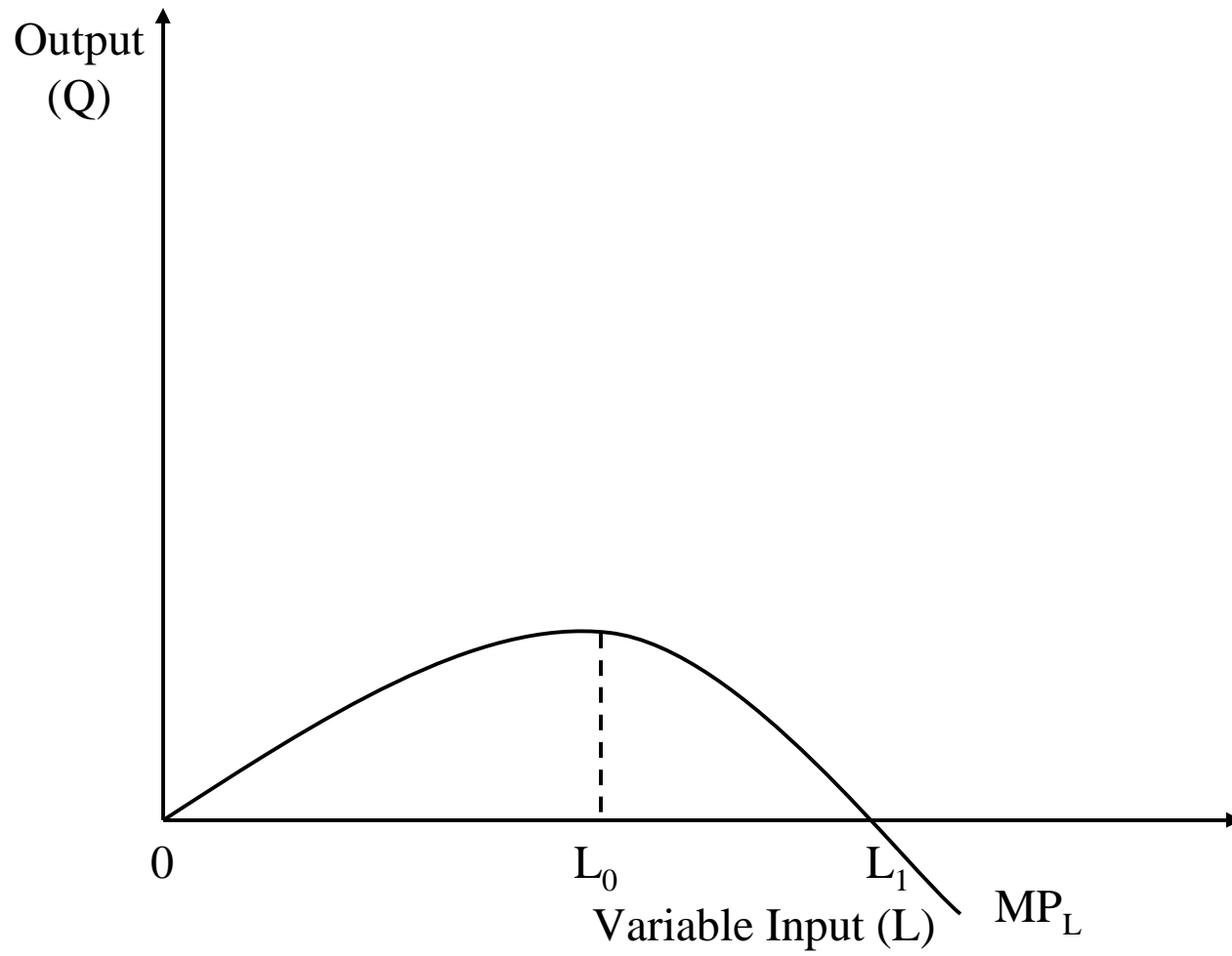
The Law of Diminishing Returns means that Region II must exist.

Short-Run Production

More Definitions

- Marginal Product:
 - Change in total product due to a one-unit change in the variable input:
 $MP_L = \Delta Q / \Delta L = F_0'(L)$.
- Average Product:
 - Total output divided by the quantity of the variable input: $AP_L = Q/L = F_0(L)/L$.
- Example
 - Suppose $Q = KL^2$ and $K_0 = 100$.
 - $F_0(L) = 100L^2$
 - $MP_L = 200L$
 - $AP_L = 100L^2/L = 100L$

Marginal Product Curve

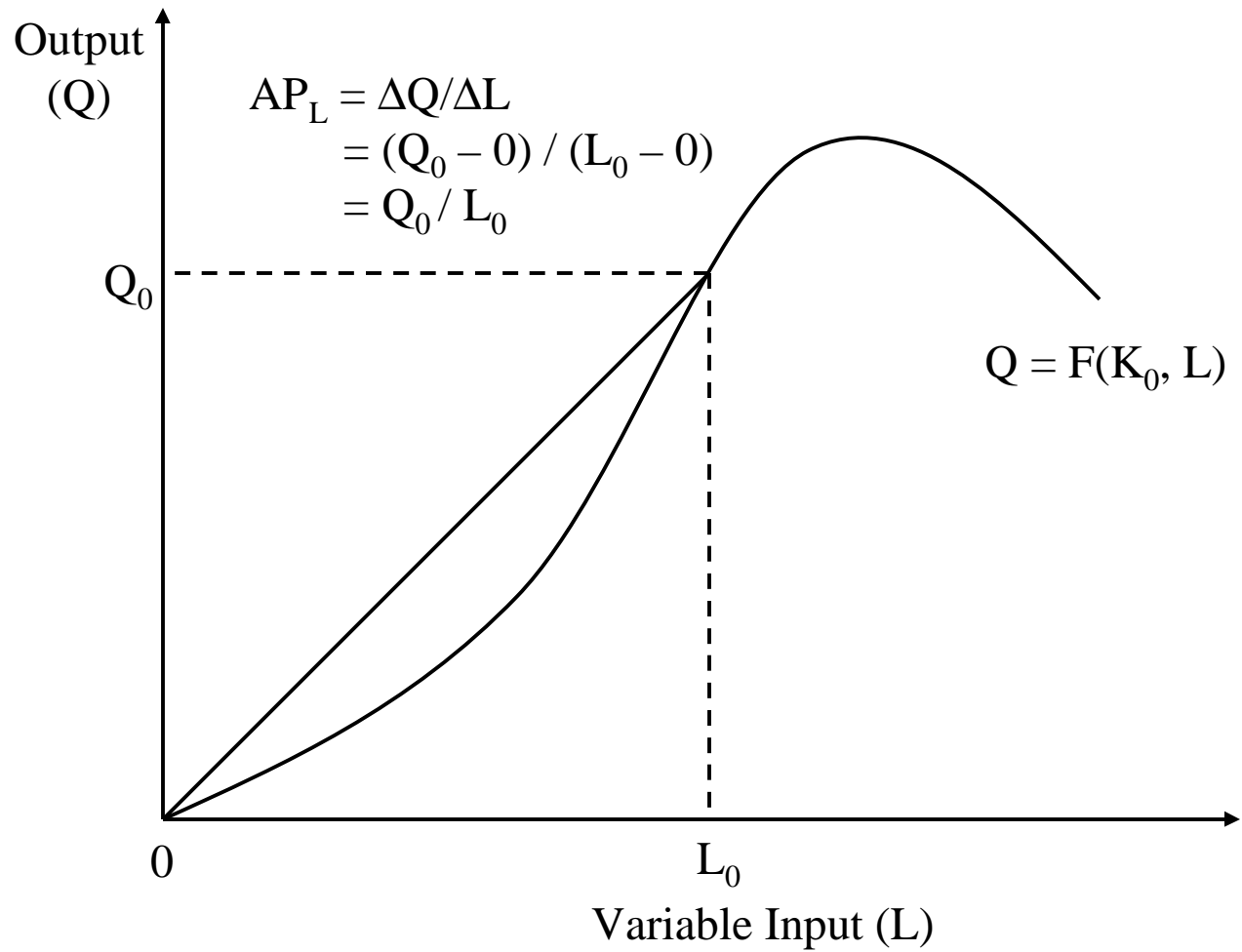


Short-Run Production

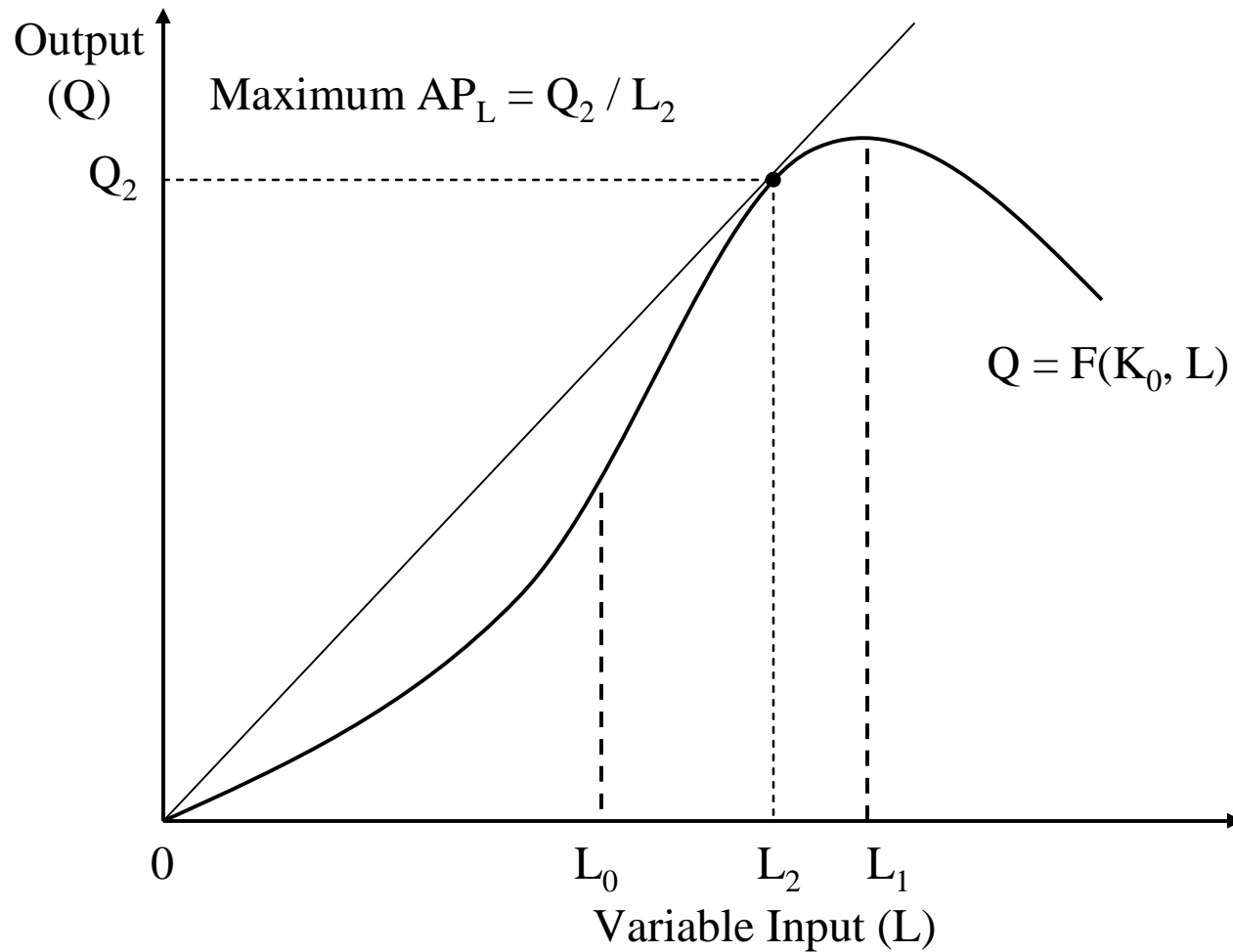
Production Regions & Marginal Products

- Region I:
 - Increasing Marginal Product - 0 to L_0
- Region II:
 - Decreasing (Positive) Marginal Product - L_0 to L_1
- Region III:
 - Decreasing (Negative) Marginal Product - Above L_1

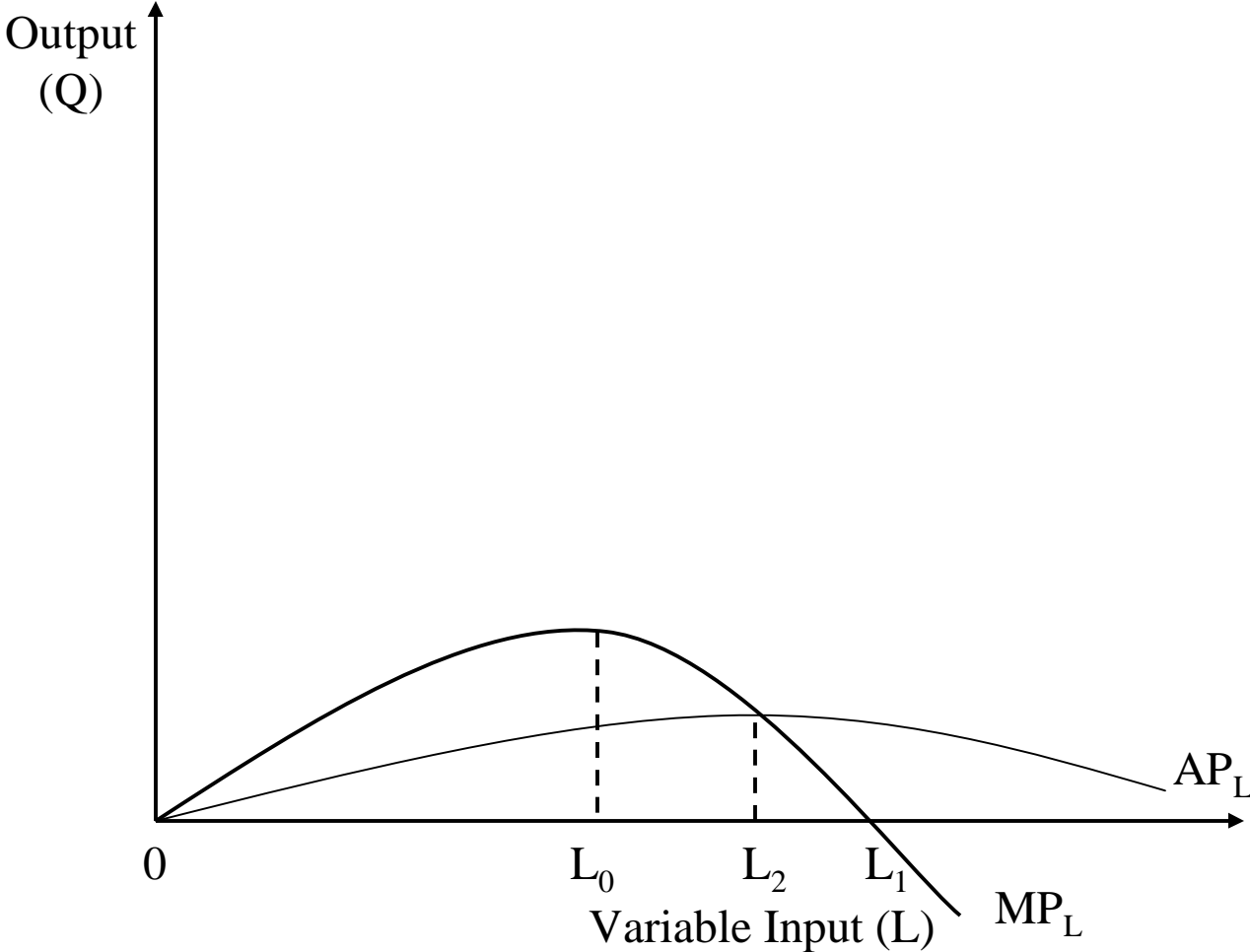
Calculation of Average Product



Total Product Curve and Maximum Average Product



Marginal and Average Product Curves



Relationship Between Marginal & Average Products

- $MP_L > AP_L$
⇒ AP_L is increasing (e.g. below L_2).
- $MP_L < AP_L$
⇒ AP_L is decreasing (e.g. above L_2).
- $MP_L = AP_L$
⇒ AP_L is maximized (e.g. at L_2).

Long-Run Production

- Definition
 - The shortest period of time required to alter the amount of all inputs used in a production process.
- Assumptions
 - Regularity
 - Monotonicity
 - Convexity

Regularity

- There is some way to produce any particular level of output.

Similar to the completeness assumption for rational choice theory.

Monotonicity

- If it is possible to produce a particular level of output with a particular combination of inputs, it is also possible to produce that level of output when we have more of some inputs.

Similar to the more-is-better assumption for rational choice theory.

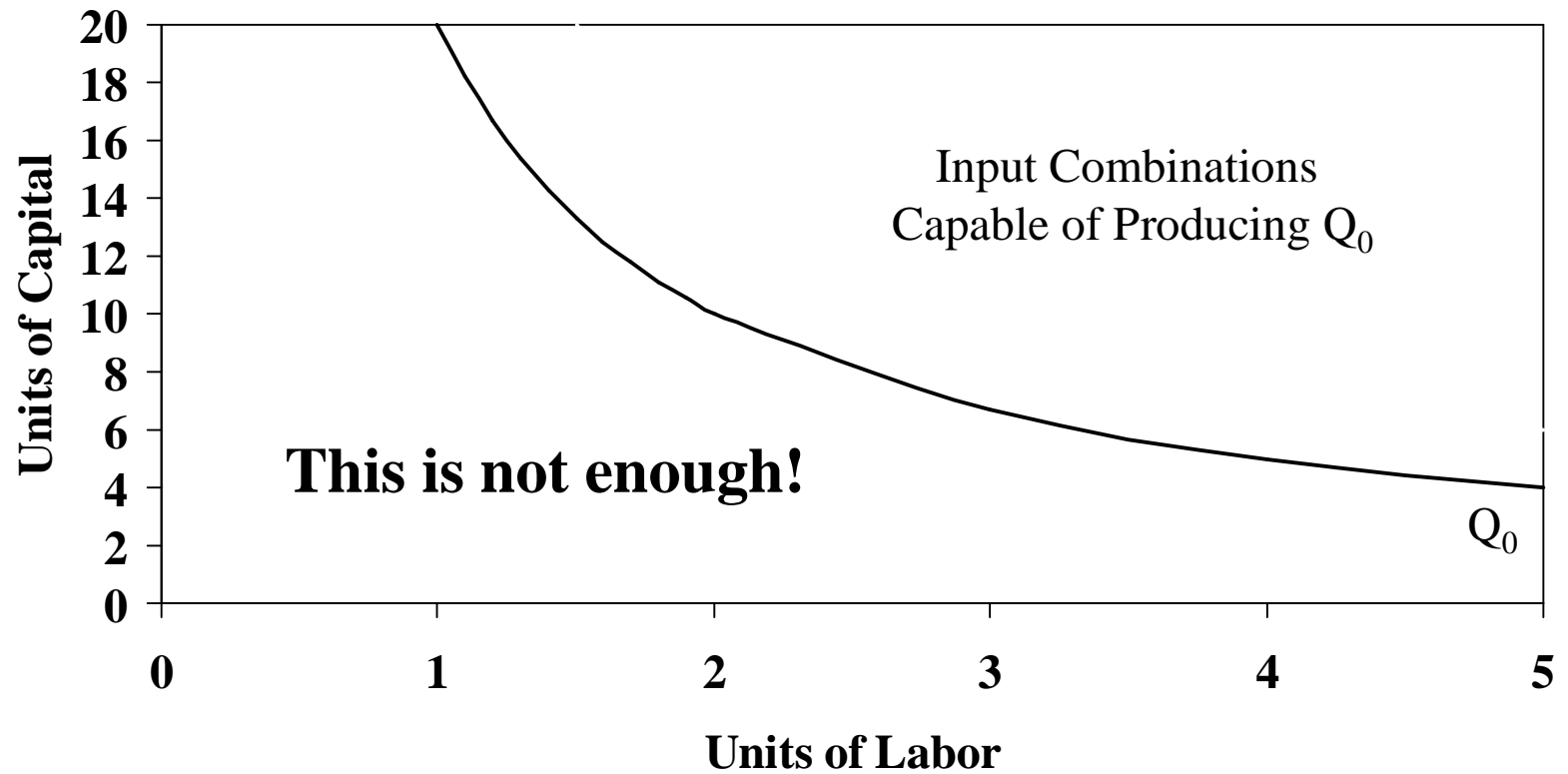
Convexity

- If it is possible to produce a particular level of output with either of two different combinations of inputs, then it is also possible to produce that level of output with a mixture of the two combinations of inputs.

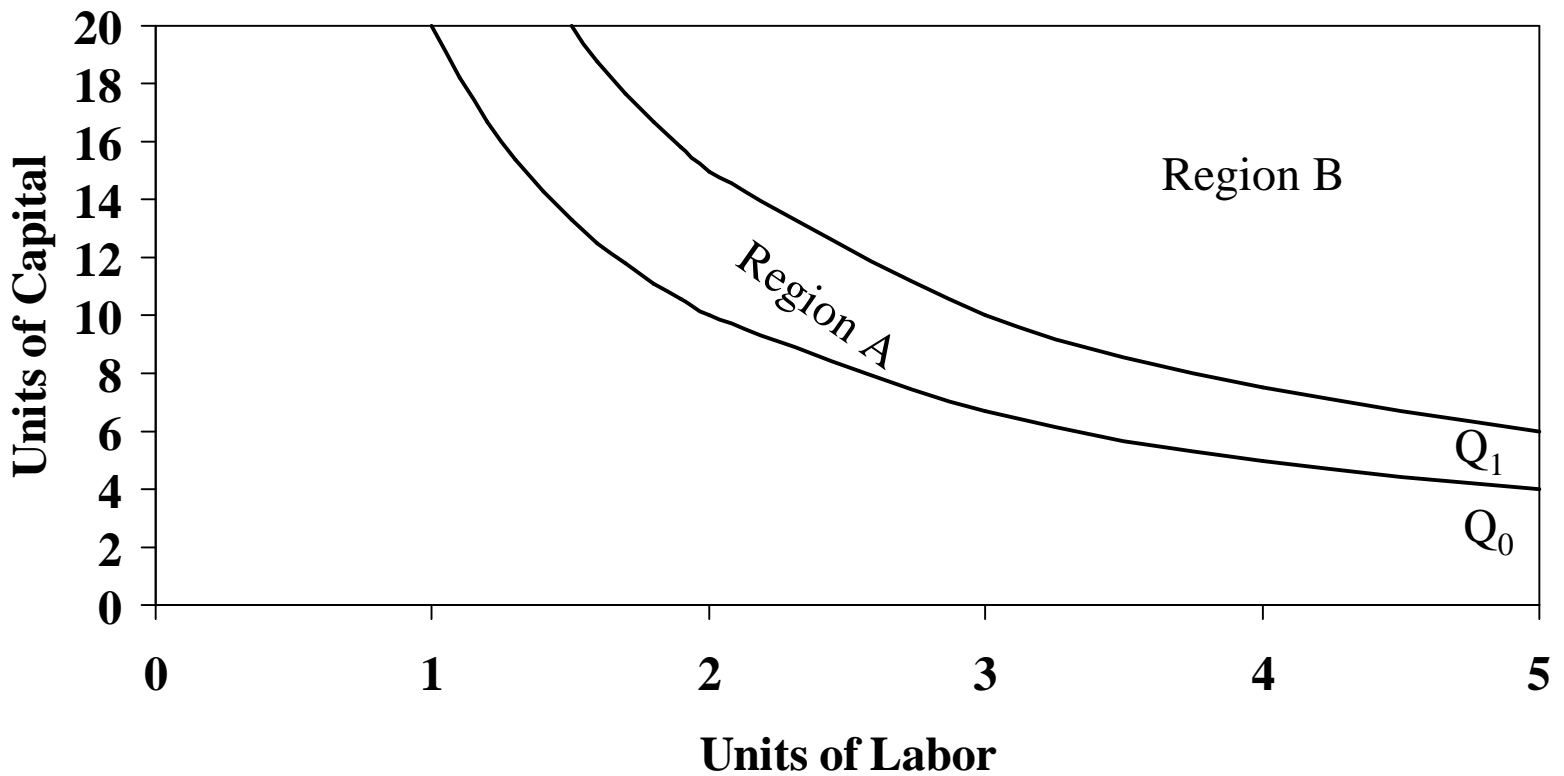
Similar to the convexity assumption for rational choice theory.

These assumptions imply that we have very specific production possibilities!

Figure 6: Production Possibilities



Region A: Input Combinations Capable of Producing Q_0
Region B: Input Combinations Capable of Producing Q_0 & Q_1

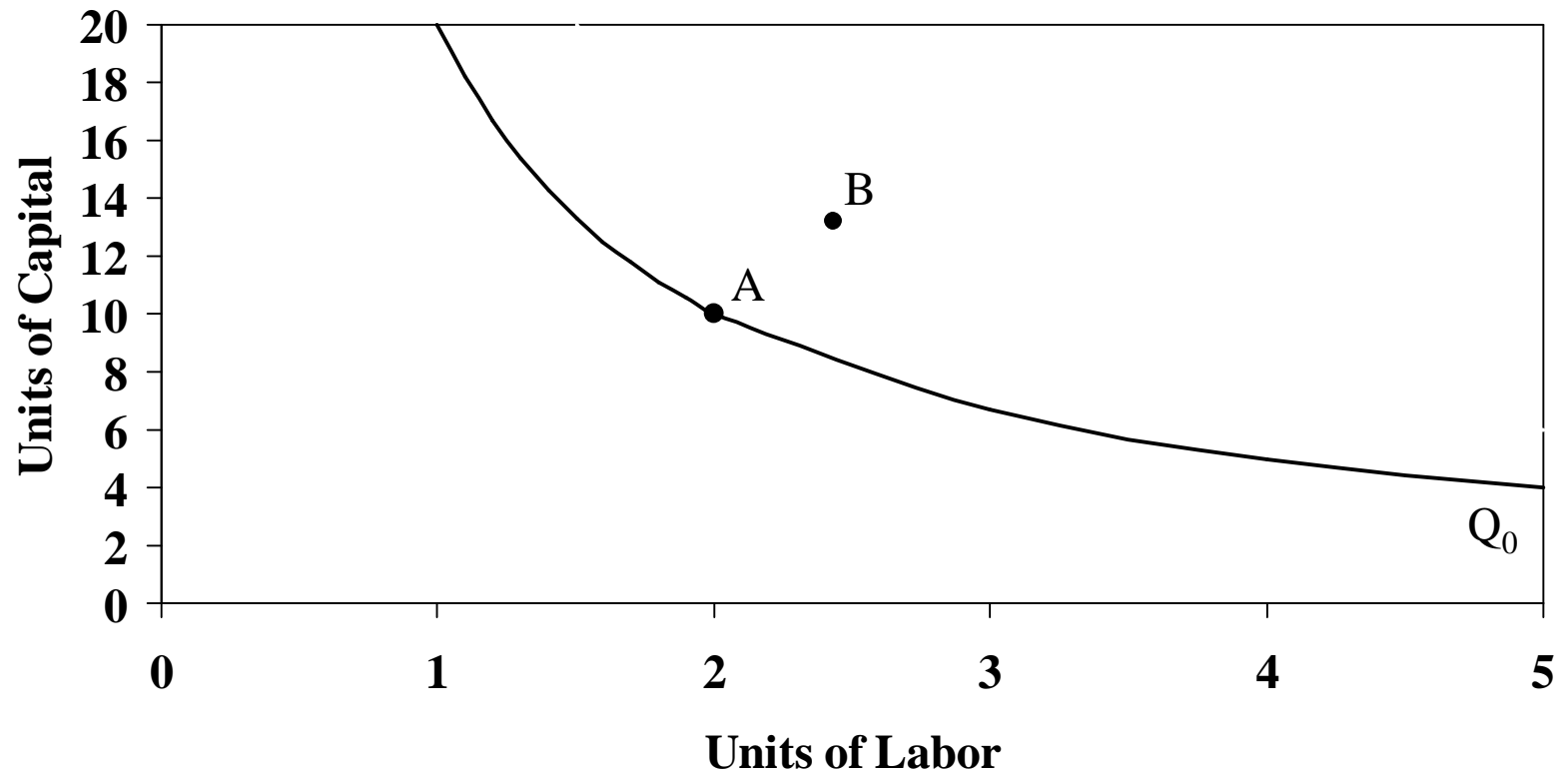


The Problem

- Combinations of capital & labor in region A are capable of producing Q_0 .
- Combinations of capital & labor in region B are capable of producing Q_1 & Q_0 .
- Without being more specific, the production function will not yield a unique output for different combinations of capital & labor.
- Question: What other assumptions can we make to be sure a combination of capital & labor gives us a unique level of output?
 - We can assume production is efficient!

We can use combination B to produce Q_0 , but we would not be doing the best we can with what we have!

We can use combination A to produce Q_0 , and we would be doing the best we can with what we have!



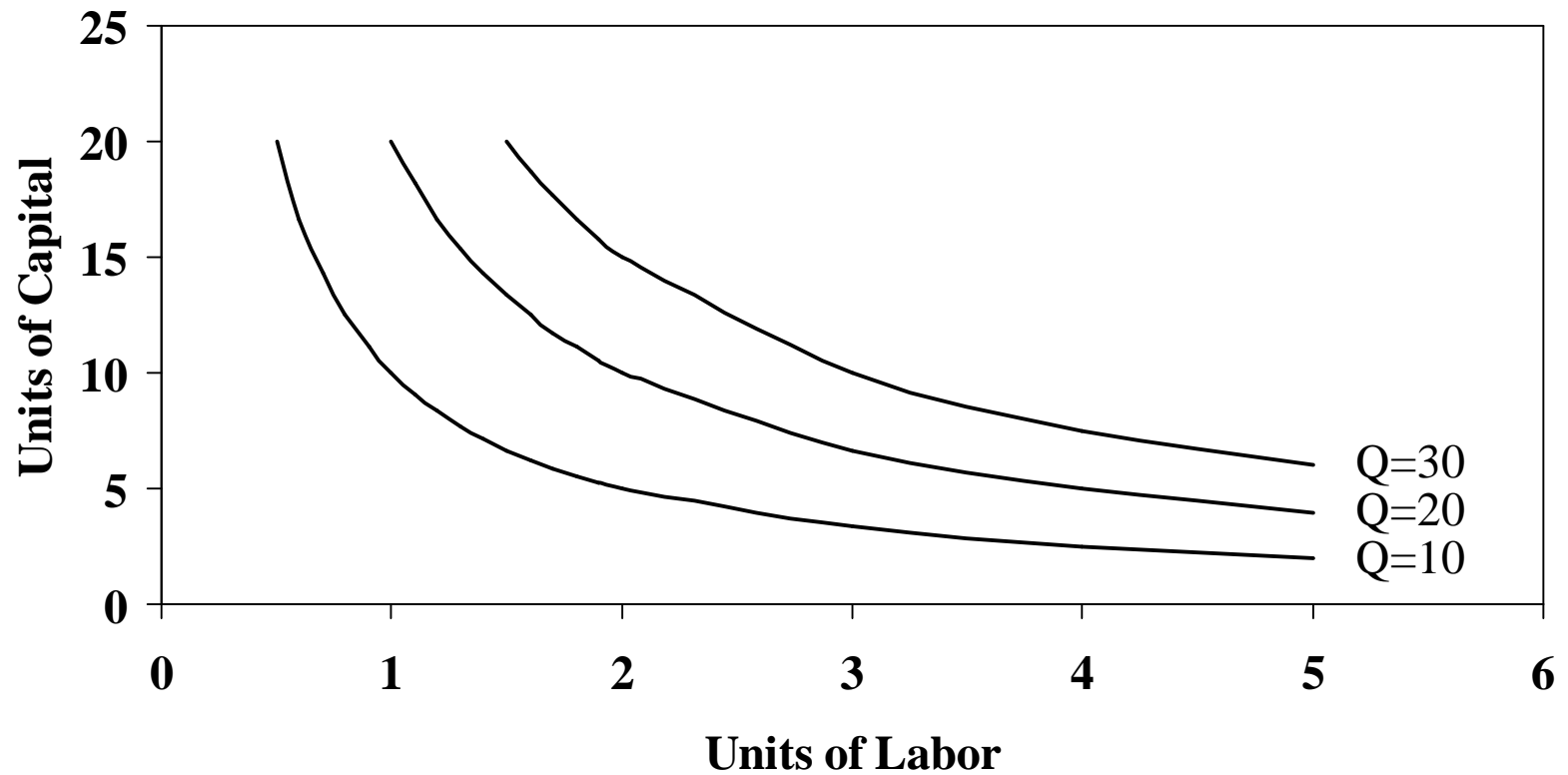
Regularity, Monotonicity, Convexity, & Efficiency

- We can construct a production function: $Q = F(K,L)$.
- Unlike the utility function, the production function is cardinal.
 - 2,000 Units of Output is Twice as Much as 1,000
- Definitions
 - Isoquant:
 - The set of all efficient input combinations that yield the same level of output.
 - Isoquant map:
 - A representative sample of the set of a firm's isoquants used as a graphical summary of production.

Properties of Isoquants & Isoquant Maps

- Higher Isoquants (Isoquants to the Northeast) represent higher levels of output.
- Ubiquitous
- Downward Sloping
- Cannot Cross
- Become Less Steep Moving Down & Right (Bowed Toward the Origin)

Figure 7: Example Isoquant Map

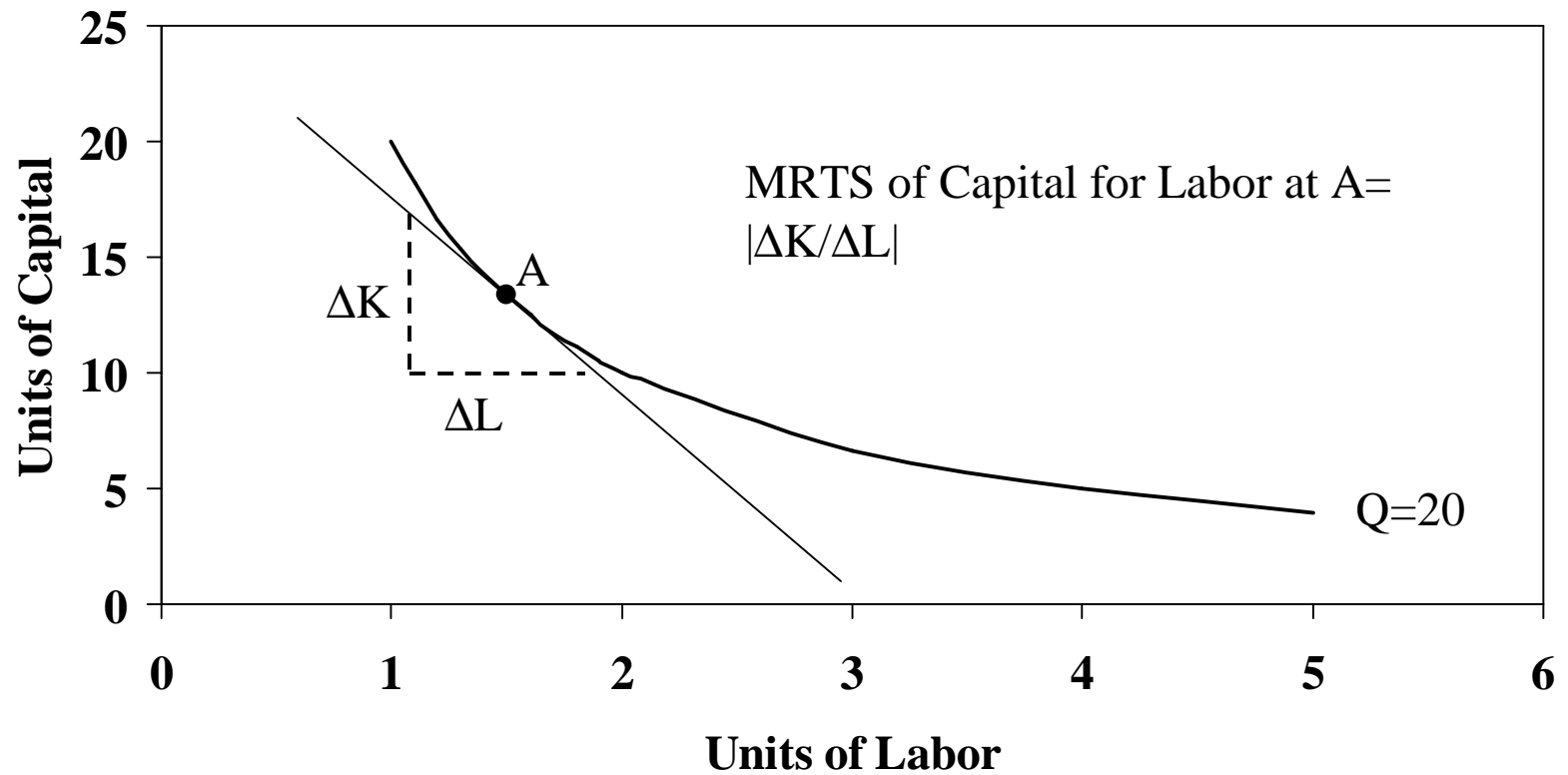


Marginal Rate of Technical Substitution (MRTS)

- Definition:
 - The rate at which one input can be exchanged for another without altering the total level of output:

$$|\Delta K/\Delta L| = MP_L/MP_K = \frac{\frac{\partial F(K, L)}{\partial L}}{\frac{\partial F(K, L)}{\partial K}}$$

Figure 8: Marginal Rate of Technical Substitution



Returns to Scale

Definitions

- **Increasing Returns to Scale:**
 - A proportional increase in every input yields more than a proportional increase in output.
- **Constant Returns to Scale:**
 - A proportional increase in every input yields an equal proportional increase in output.
- **Decreasing Returns to Scale:**
 - A proportional increase in every input yields less than a proportional increase in output.

Identifying Returns to Scale

- Let $\alpha > 1$
 - If $\alpha Q < F(\alpha K, \alpha L)$, returns to scale are increasing.
 - If $\alpha Q = F(\alpha K, \alpha L)$, returns to scale are constant.
 - If $\alpha Q > F(\alpha K, \alpha L)$, returns to scale are decreasing.
- Example
 - Suppose $Q = F(K, L) = K^a L^b$.
 - Then $F(\alpha K, \alpha L) = (\alpha K)^a (\alpha L)^b = \alpha^a K^a \alpha^b L^b = \alpha^{a+b} K^a L^b = \alpha^{a+b} Q$
 - $a + b > 1 \Rightarrow \alpha Q < \alpha^{a+b} Q$ or increasing returns to scale.
 - $a + b = 1 \Rightarrow \alpha Q = \alpha^{a+b} Q$ or constant returns to scale.
 - $a + b < 1 \Rightarrow \alpha Q > \alpha^{a+b} Q$ or decreasing returns to scale.

What You Should Know

- What a production function is?
- Short-Run versus Long Run Production
- Short-Run Production
 - Total Product, Marginal Product, & Average Product
 - Law of Diminishing Returns
- Long-Run Production
 - Assumptions
 - Isoquants & Isoquant Maps: What they are & properties.
 - Marginal Rate of Technical Substitution
- Returns to Scale: What they are and how to test.