Production

Readings: Ch. 9

Up to now we have focused our attention on describing how individual consumers try to fulfill unlimited wants with limited means. The culmination of this analysis was the derivation of individual and market demand.

We will know turn to the other side of the market and ask the question: What determines supply? As you will see, many of the tools we used to evaluate consumer decisions and demand work equally well for understanding producer decisions and supply. For the most part, we can really just change the name of things.

Objective: Understand what production is and how we can describe production.

Production is the process by which we use things and make new things. To accomplish this task we usually need some sort of raw material, tools to mix the raw material, and people to operate the tools. We call these inputs. What results from the mixing of raw materials with tools and toil is output. Usually, there are lots of different ways we can use tools and toil to mix raw material and make output. The relationship that describes these different possibilities is called a production function.

Definition
Production Function: A relationship that describes how inputs can be transformed into output.

For simplicity, we will assume there are only two inputs, which we will call capital, K, and labor, L. We can write output, Q, as some function of inputs: $Q = F(K, L)$. Alternatively, we could describe this relationship using a table or graph.

Often times the output from one production process is used in another production process. For example, cotton is used to make fabric and fabric is then used to make pants, shirts, or other types of clothing. Output from one production process that is used as an input for another production process is called an intermediate product.

Definition
Intermediate Product: Products that are transformed by a production process into products of greater value.

Often times we will distinguish between fixed and variable inputs. Fixed inputs are inputs in a production process that cannot be changed. Variable inputs are inputs in a production process that can be changed. Ultimately, this distinction really depends on the time frame of interest. For day-to-day production decisions, many inputs will be fixed because there is simply not enough time to change them. However, inputs that cannot change day to day can often be changed over the course of a week, year, or decade. This leads to the distinction economists make between the *Long run* and *Short run*.

Definition
Long run: The shortest period of time required to alter the amount of all inputs used in a production process.

Definition
Short run: The longest period of time during which at least one of the inputs used in the production process cannot be varied.

In the long run, all inputs are variable. In the short run, some inputs are fixed. For example, we can consider a situation where the level of capital available is fixed at K_0, but there is still plenty of time to vary labor all we want. The production function in the short run would then be $Q = F(K_0, L) = F_0(L)$, in which case output in the short run depends only on the amount of labor we choose.

Objective: Understand the characteristics of production in the short run.

In the short run, some inputs are fixed. When some inputs are fixed, our production possibilities are limited. These limits typically imply that we cannot continuously increase production by simply increasing the amount of variable inputs we use. The constraint on production imposed by fixed inputs is referred to as the *Law of Diminishing Returns*.

Definition
Law of Diminishing Returns: If other inputs are fixed, the increase in output from an increase in variable inputs must eventually decline.

The law of diminishing returns implies a short run production function will have a particular shape. This shape is illustrated in Figure 1. When the variable input labor is between 0 and L_0, each additional hour of labor adds more to production than the previous hour. Between L_0 and L_1, each additional hour of labor adds less to production than the previous hour. Finally, each hour of labor above L_1 actually reduces output. Above L_0, the law of diminishing returns begins to operate.

The graph of the short run production function depicted in Figure 1 is also referred to as the total product curve.

Definition
Total Product Curve: A curve showing the amount of output as a function of the amount of variable input.

The slope of the total product curve tells how much additional output we get from adding an additional unit of variable input or the marginal product.

Definition
Marginal Product: Change in total product due to a one-unit change in the variable input.

Specifically, the marginal product of labor for our production function in Figure 1 is MP_L = $\Delta Q/\Delta L = F_0'(L) = \frac{\partial F(K_0, L)}{\partial L}$. Figure 2 illustrates the marginal product curve for the total product curve in Figure 1. Between 0 and L_0, the marginal product curve is increasing, each additional hour adds more to output than the previous addition. Between L_0 and L_1, the marginal product curve decreases. Each additional hour of labor adds less to output than the previous addition. Above L_1, the marginal product curve is negative. Each additional hour of labor actually decreases output.

Another quantity we will find useful is the average product of a variable input.

Definition
Average Product: Total output divided by the quantity of the variable input.

Specifically, the average product of labor is equal to $AP_L = Q/L = F_0(L)/L = F(K_0, L)/L$. Note that the average product will equal the slope of a line from the origin to the total product curve. Figure 3 illustrates. Therefore, to find the maximum average product all we need to do is find the slope of the line through the origin that is just tangent to the total product curve (see Figure 4).

Figure 5 shows the important relationships between the marginal and average product curves. What is important to note from this figure is that the average product increases when it is below the marginal product and decrease when it is above the marginal product. Therefore, the average product curve reaches its highest point (maximum) when it just equals the marginal product.

Objective: Understand production in the long run.

In the short run, production is constrained by fixed inputs. In the long run, this constraint is removed since we are able to vary all inputs in the production process.

When we talked earlier about rational choice, we argued that if a consumer could rank bundles of goods in order of preference and this ranking satisfied the nice properties of completeness, more-is-better, transitivity, and convexity, then we could represent the consumer's preference ordering using a utility function with nice properties.

We can do a similar thing for producers to come up with a production function. For example, suppose we ask a producer to write down all possible combinations of inputs that allow him to produce a certain level of output. We can repeat this question for all possible levels of output. If these input combinations satisfy three nice properties, then it is possible to use the producer's responses to construct a production function.

These properties are:

1) Monotonicity: If it is possible to produce a particular level of output with a particular combination of inputs, it is also possible to produce that level of output when we have more of some inputs. Important note, Monotonicity rules out production in Figure 1

above L_1. The argument is that the firm could always choose to freely dispose of labor above L_1 as oppose to using it and decreasing its level of production.

2) Convexity: If it is possible to produce a particular level of output with either of two different combinations of inputs, then it is also possible to produce that level of output with a mixture of the two combinations of inputs.

3) Regularity: There is some way to produce any particular level of output.

What these three properties imply is that the combinations of inputs capable of producing some level of output, Q_0, will look something like Figure 6.

To get our production function, we need to go one step further and focus on combinations of inputs for a particular level of output that are efficient. That is, we want to use the fewest possible inputs to produce a particular level of output. For example, in Figure 6 we can produce Q_0 with input combination A or B. But, are both the input combinations efficient? For input combination B, we could still produce Q_0 even if we got rid of some of all the inputs. This is not possible however for combination A. If we get rid of some of either input, we will be below Q_0 and not able to produce as much output. Therefore, combination A is efficient and combination B is not. Generally, all points lying on the curve Q_0 will be efficient, while all points above and to the right are inefficient. The curve denoted by Q_0 is called an isoquant.

Definition
Isoquant: The set of all efficient input combinations that yield the same level of output.

Focusing on only those input combinations that are efficient, we are able to construct a production function the same way we construct utility functions.

As mentioned previously, this production function will ultimately rule out production where L is greater than L_1 in Figure 1 because it is not efficient.

While utility and production functions are very similar in nature, there is at least one important difference. Recall that a utility function is ordinal. That is, we can only use it to say which bundles of goods provide more or less satisfaction. Alternatively, a production function is cardinal. It tells us which bundles of inputs produce more and how much more.

Isoquants are similar in nature to indifference curves and just as we found it useful to look at indifference curve maps, we will find it useful to look at maps of isoquants.

Definition
Isoquant map: A representative sample of the set of a firm's isoquants used as a graphical summary of production.

Isoquants that are further from the origin or higher up represent higher levels of production.

Also, isoquants
1. are ubiquitous.
2. are downward sloping.

3. cannot cross.
4. become less steep as we move down and to the right. They bow toward the origin or are convex to the origin.

Figure 7 illustrates an isoquant map when Q = F(K, L) = KL.

A quantity we found useful when talking about indifference curves and a consumer's best feasible bundle was the marginal rate of substitution, which was equal to the absolute value of the slope of indifference curves. Similarly, we will find the absolute value of the slope of an isoquant very useful. We will call this quantity the marginal rate of technical substitution.

Definition
Marginal Rate of Technical Substitution (MRTS): The rate at which one input can be exchanged for another without altering the total level of output.

Specifically, the MRTS of capital for labor is equal to $|\Delta K/\Delta L| = MP_L/MP_K = \dfrac{\frac{\partial F(K,L)}{\partial L}}{\frac{\partial F(K,L)}{\partial K}}$.

Graphically, the MRTS is the absolute value of the slope of the line tangent to an isoquant at a point. Figure 8 illustrates. Note that property 4 above implies the MRTS will be diminishing as we move down and to the right on an isoquant.

Objective: Understand the difference between increasing, constant, and diminishing returns to scale.

One final point to consider before moving on is the notion of increasing, constant, and decreasing returns to scale.

Definition
Increasing Returns to Scale: The property of the production process whereby a proportional increase in every input yields more than a proportional increase in output.

Definition
Constant Returns to Scale: The property of the production process whereby a proportional increase in every input yields an equal proportional increase in output.

Definition
Decreasing Returns to Scale: The property of the production process whereby a proportional increase in every input yields less than a proportional increase in output.

To determine if a production process exhibits increasing, constant or decreasing returns to scale, we need to multiply all inputs by some constant greater than 1 and then see if output increases by more, the same, or less than that constant.

For example, let $\alpha > 1$, if $\alpha Q < F(\alpha K, \alpha L)$, we have increasing returns to scale. If $\alpha Q = F(\alpha K, \alpha L)$, we have constant returns to scale. If $\alpha Q > F(\alpha K, \alpha L)$, we have decreasing returns to scale.

How about a more concrete example? Suppose $Q = K^a L^b$.

Question: When does the production function exhibit increasing, constant, and decreasing returns to scale?

When we multiply K and L by α, output is $(\alpha K)^a(\alpha L)^b = \alpha^a K^a \alpha^b L^b = \alpha^{a+b} K^a L^b = \alpha^{a+b} Q$. Note that for $\alpha > 1$, $\alpha^{a+b}Q > \alpha Q$ if $a + b > 1$, $\alpha^{a+b}Q = \alpha Q$ if $a + b = 1$, and $\alpha^{a+b}Q < \alpha Q$ if $a + b < 1$. Therefore, for $a + b > 1$, we have increasing returns to scale. For $a + b = 1$, we have constant returns to scale. For $a + b < 1$, we have decreasing returns to scale.

Increasing returns to scale implies there are synergies that emerge from increasing inputs. For example, if we can use 1 building and 100 employees to efficiently produce 1,000 cars a year, we can produce more than 2,000 cars a year if we have 2 buildings and 200 employees. Why? Maybe because we can improve efficiency by having one building specialize at assembling motors and the other specialize at building the chassis.

Constant returns to scale say we can always replicate our current production process. For example, if we know how to use 1 building and 100 employees to efficiently produce 1,000 cars a year, why couldn't we just build another identical building and hire another 100 employees in order to produce 2,000 cars.

Always being able to replicate a production process makes it hard to explain why we would ever see an instance of decreasing returns to scale. Essentially, decreasing returns to scale implies we cannot exactly replicate a production process, an implication many economists find unsettling if we can truly identify and accurately measure all inputs in a production process.

Figure 1: Typical Short Run Production Function or Total Product Curve

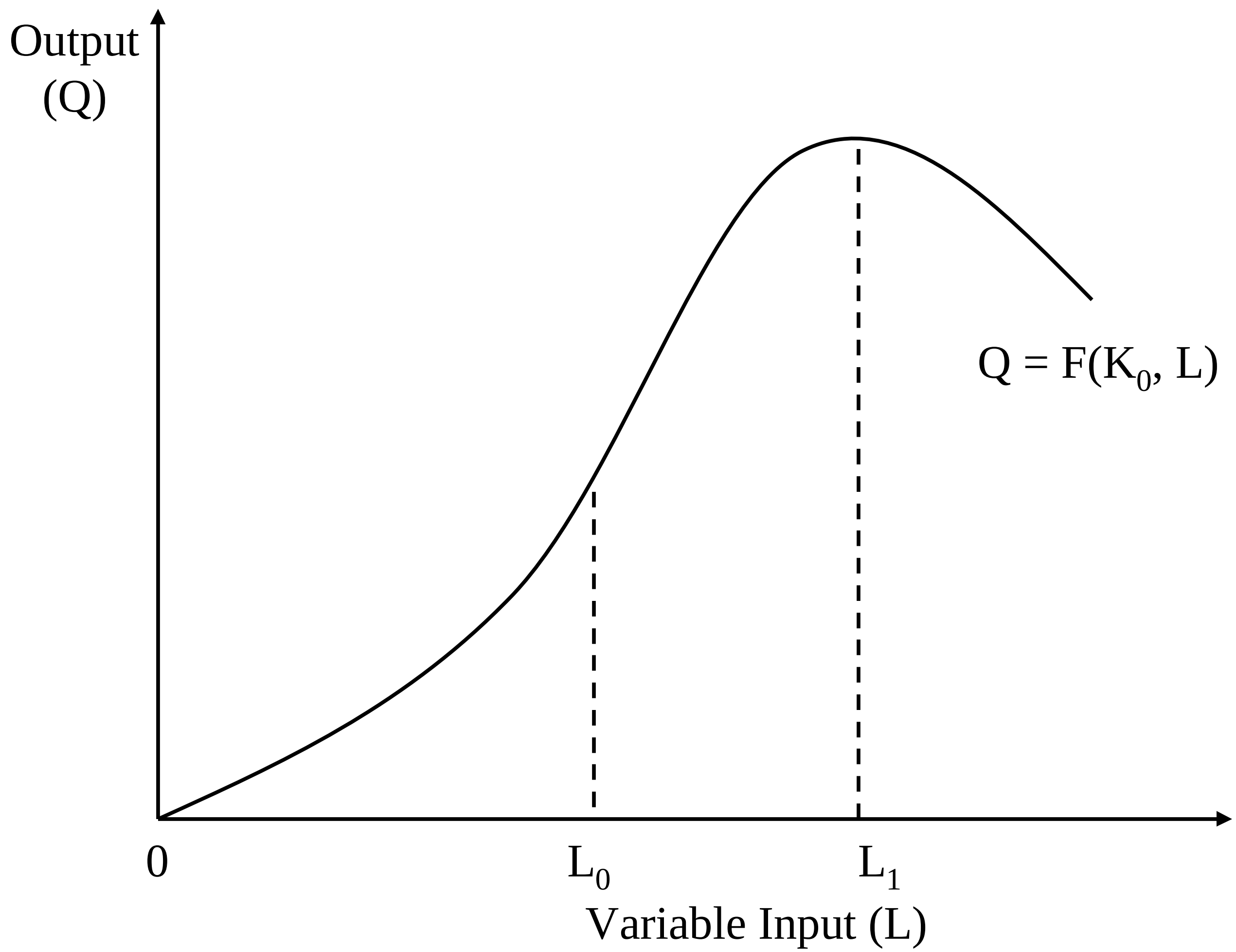

Figure 2: Marginal Product Curve

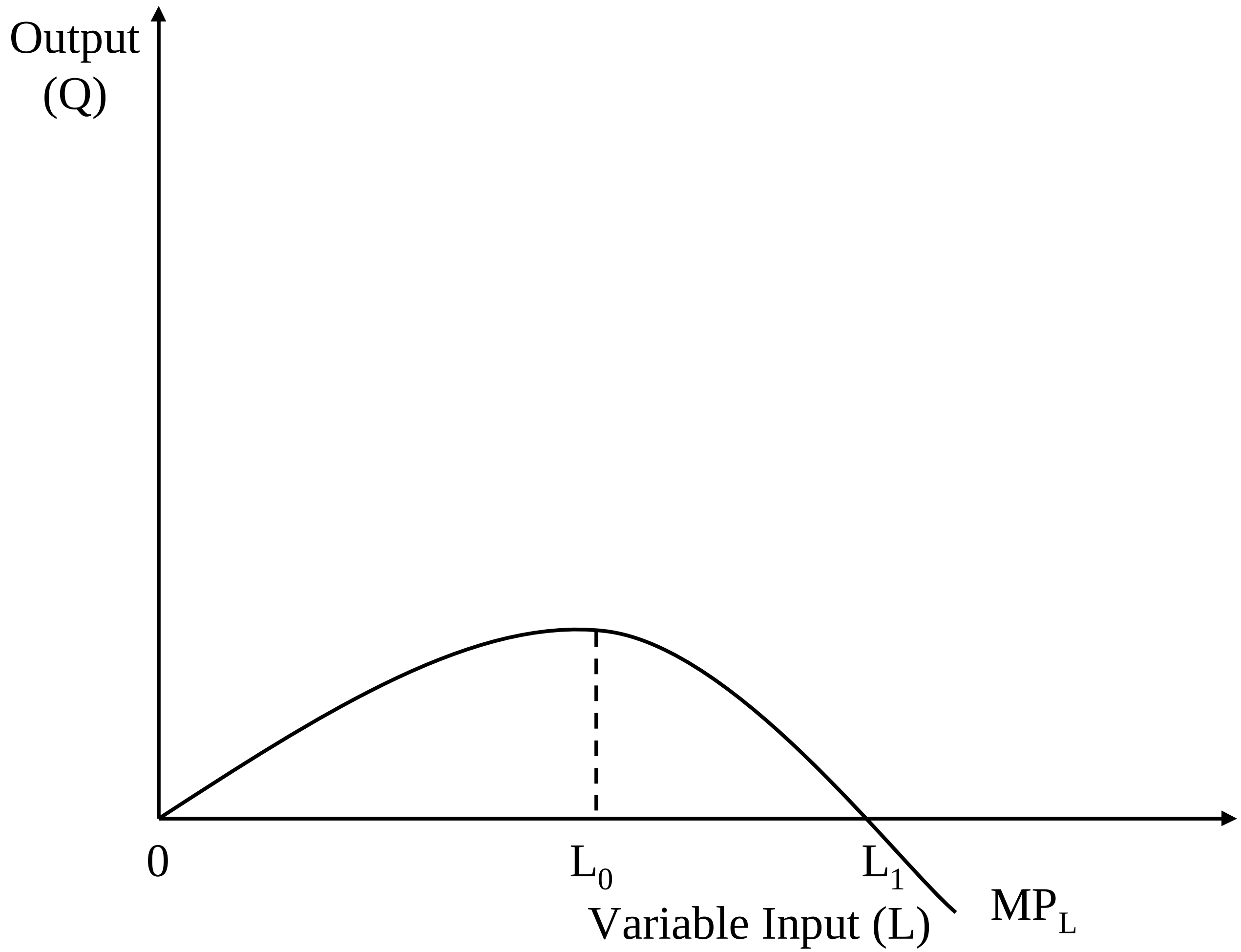

Figure 3: Calculation of Average Product

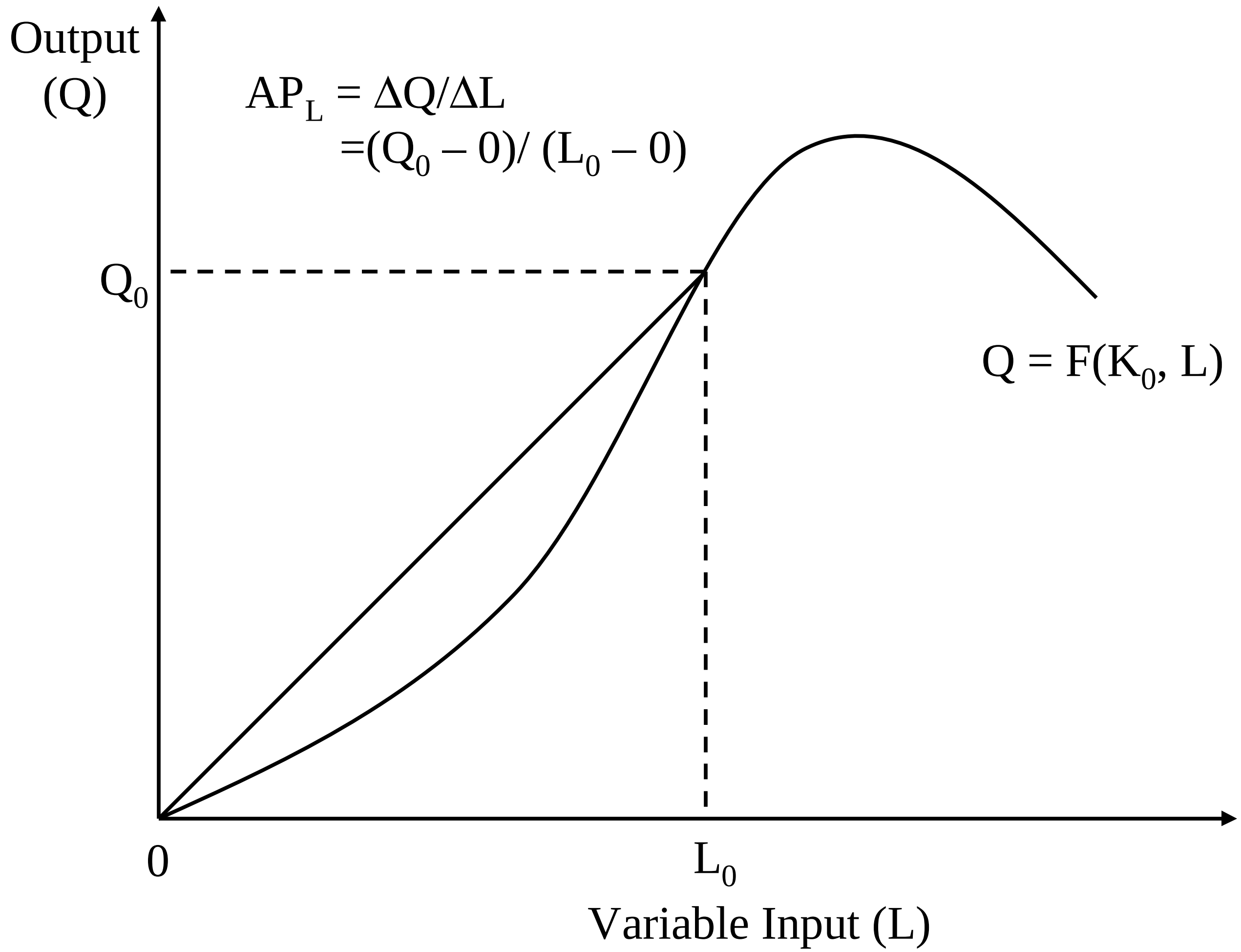

Figure 4: Total Product Curve and Maximum Average Product

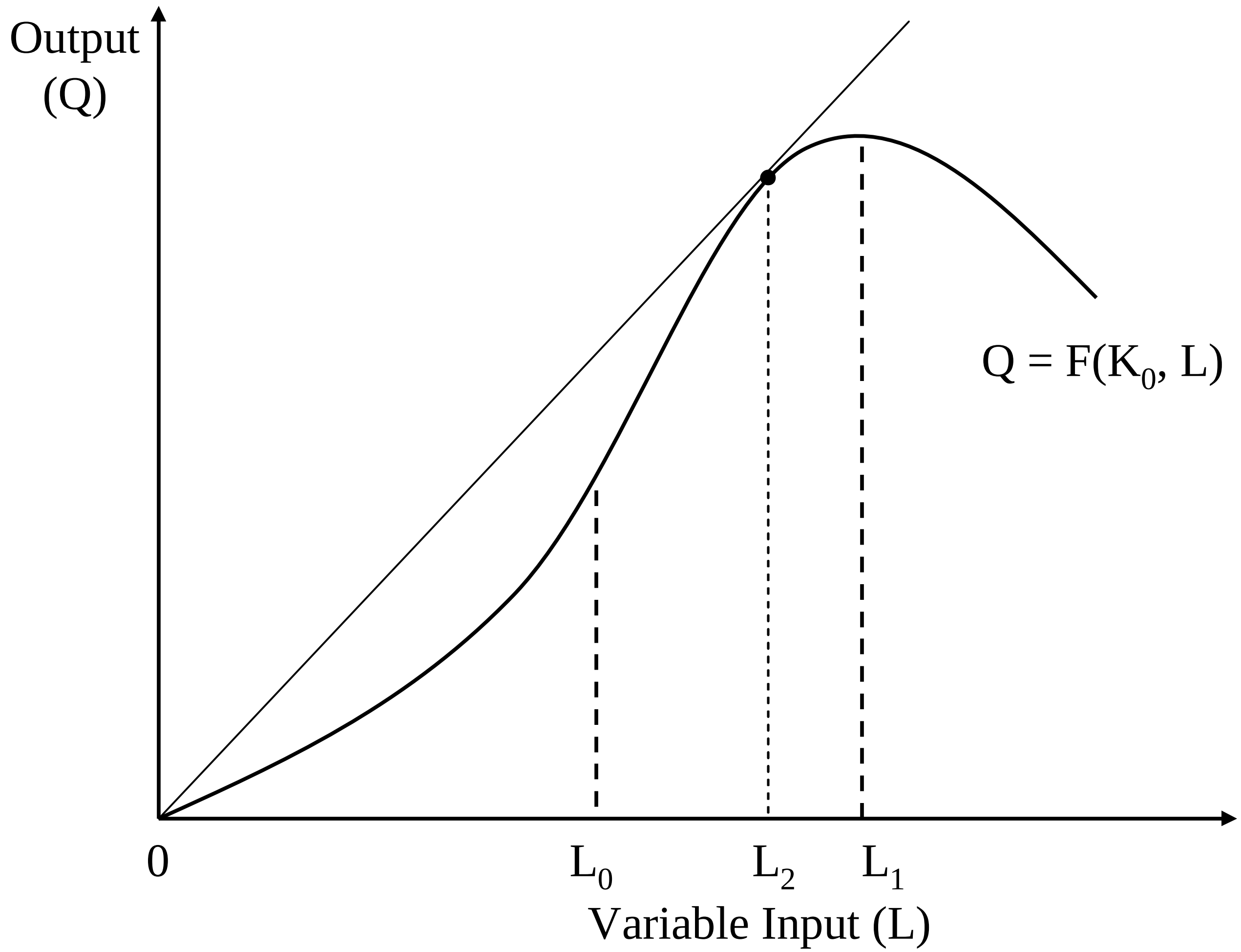

Figure 5: Marginal and Average Product Curves

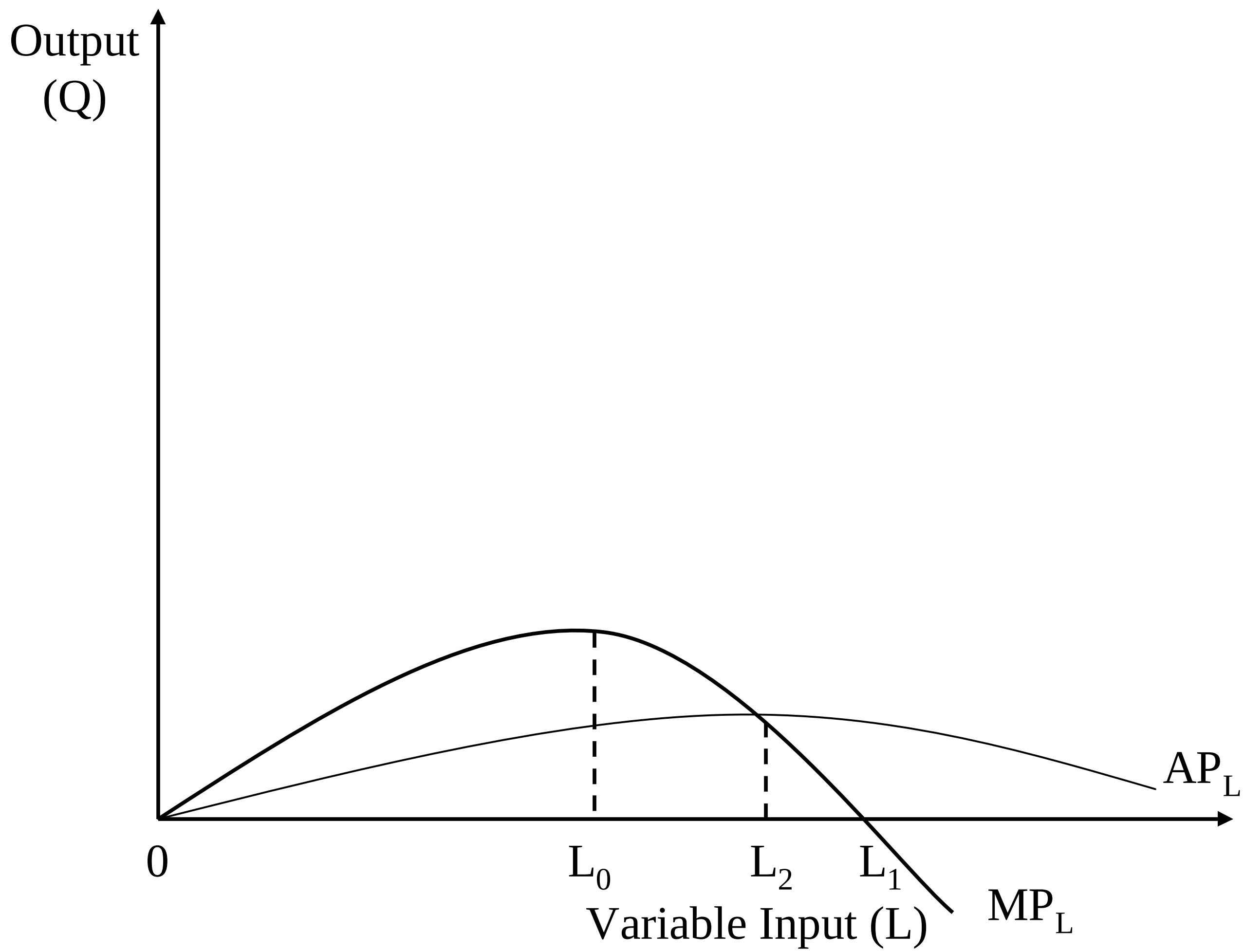

Figure 6: Production Possibilities

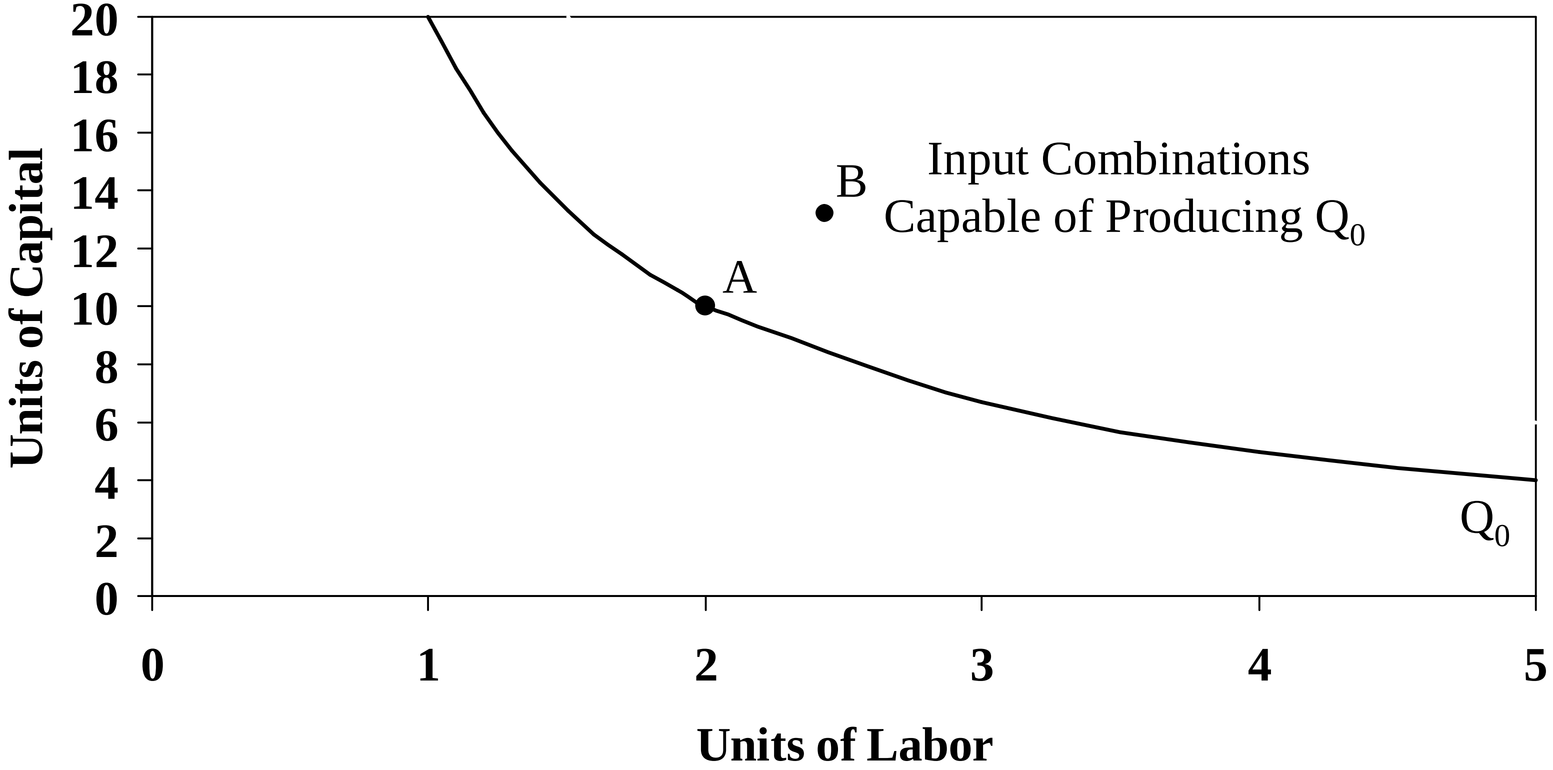

Figure 7: Example Isoquant Map

Figure 8: Marginal Rate of Technical Substitution