Oligopoly and Monopolistic Competition

APEC 3001

Summer 2007

Readings: Chapter 13

Objectives

- Characteristics of Oligopoly & Monopolistic Competition
- Cournot Duopoly Model
- Strategic Behavior In Cournot Duopoly Model
- Reaction Functions & Nash Equilibrium
- Bertrand Duopoly Model
- Stackelberg Duopoly Model
- Effect of Industrial Organization on Prices, Output, & Profit
- Monopolistic Competition Model
- Basic Concepts of Economic Games & Their Solutions

Oligopoly & Monopolistic Competition Definitions

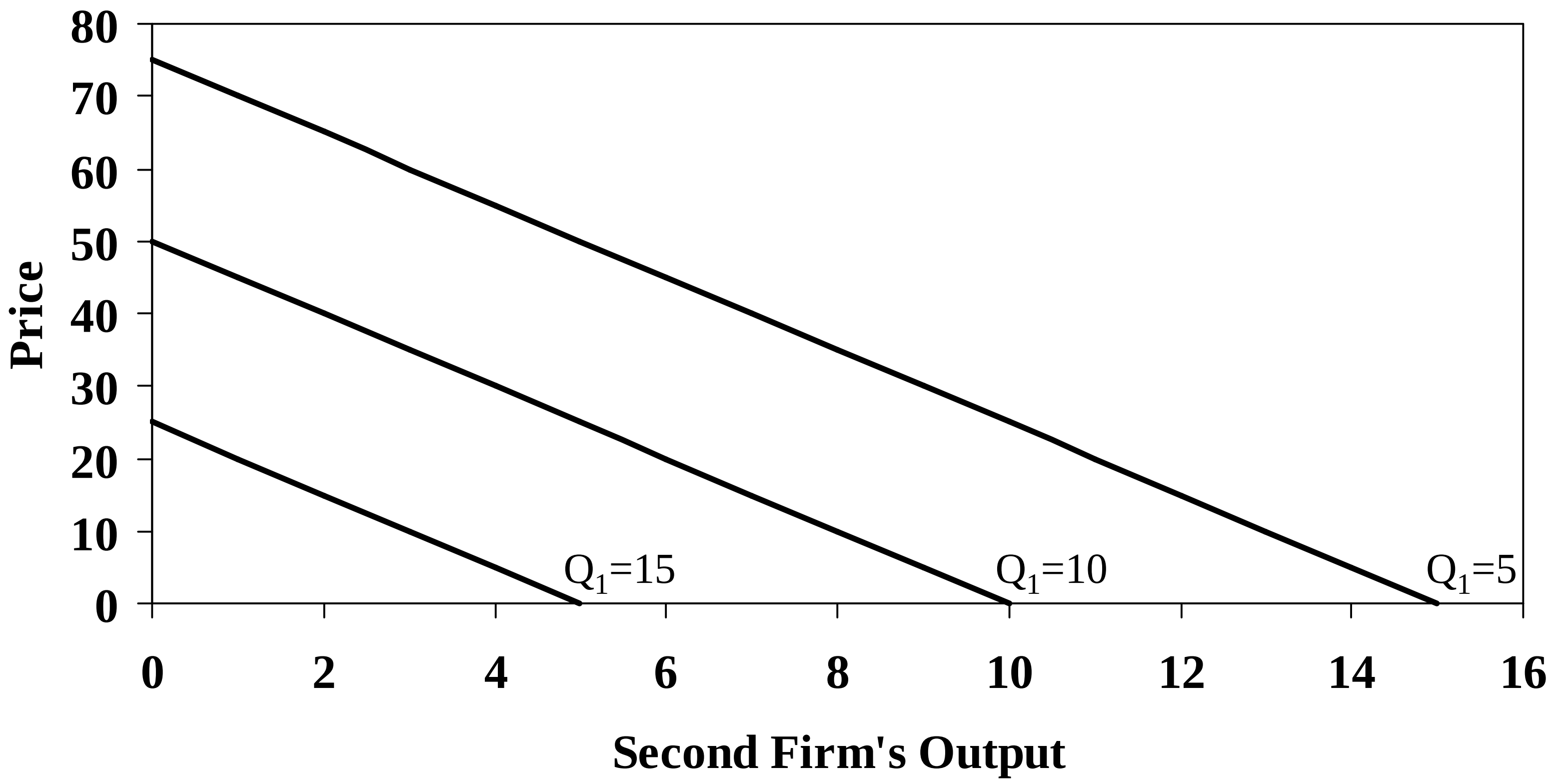
- Oligopoly:
  - An industry in which there are only a few important sellers of an identical product.
- Monopolistic Competition:
  - An industry in which there are (1) numerous firms each providing different but very similar products (close substitutes) and (2) free entry and exit.

Important: One firm's choices affects the profit potential of other firms, which results in strategic interactions among firms!

Cournot Duopoly Model

- Assumptions
  - $P = a – bQ$ where Q is industry output.
  - Two firms produce identical product: $Q = Q_1 + Q_2$.
  - Marginal Costs: $MC_1 = MC_2 = 0$.
- Question: How does Firm 1's choice of output affect the demand for the Firm 2's output?
  - $P = a – bQ = a – b(Q_1 + Q_2) = (a – bQ_1) – bQ_2$
    - Linear Equation: Intercept = $(a – bQ_1)$ & slope = $-b$

Demand For Firm 2's Output Given Firm 1's Output



Important Implications

- Demand for Firm 2 depends on Firm 1's output!
- Likewise, demand for Firm 1 depends on Firm 2's output!

Profit Maximization for Duopolist

- Short Run Conditions:
  - MC = MR
  - MC’ > MR’
  - P* > AVC
- Long Run Conditions:
  - LMC = MR
  - LMC’ > MR’
  - P* > LAC

Nothing new here!

To keep things simple, we will assume MC’ > MR’ & P* > AVC in the short run & LMC’ > MR’ & P* > LAC in the long run.

What is marginal revenue for a Cournot Duopolist?

- Firm 1
  - $TR_1 = P(Q)Q_1 = (a – bQ)Q_1$
    $= aQ_1 – bQQ_1$
    $= aQ_1 – b(Q_1 + Q_2)Q_1$
    $= aQ_1 – bQ_1^2 – bQ_1Q_2$
  - $MR_1 = \Delta TR_1/ \Delta Q_1 = TR_1' =$
    $a – 2bQ_1 – bQ_2$

- Firm 2
  - $TR_2 = P(Q)Q_2 = (a – bQ)Q_2$
    $= aQ_2 – bQQ_2$
    $= aQ_2 – b(Q_1 + Q_2)Q_1$
    $= aQ_1 – bQ_1Q_2 – bQ_2^2$
  - $MR_2 = \Delta TR_2/ \Delta Q_2 = TR_2' =$
    $a – bQ_1 – 2bQ_2$

What is the profit maximizing output for a Cournot Duopolist?

- Firm 1

$$MC_1 = MR_1$$
$$0 = a - 2bQ_1^* - bQ_2$$
$$Q_1^* = \frac{a - bQ_2}{2b}$$

- Firm 2

$$MC_2 = MR_2$$
$$0 = a - bQ_1 - 2bQ_2^*$$
$$Q_2^* = \frac{a - bQ_1}{2b}$$

But now what do we do?

The two firm's are identical, so lets assume they behave identically: $Q_1^* = Q_2^*$!

Firm Output:

$$Q_1^* = \frac{a - bQ_1^*}{2b}$$

$$2bQ_1^* = a - bQ_1^*$$

$$3bQ_1^* = a$$

$$Q_1^* = Q_2^* = \frac{a}{3b}$$

Industry Output:

$$Q^* = Q_1^* + Q_2^*$$

$$= \frac{a}{3b} + \frac{a}{3b}$$

$$= \frac{2a}{3b}$$

Price:

$$P^* = a - bQ^*$$

$$= a - b\frac{2a}{3b}$$

$$= \frac{3a}{3} - \frac{2a}{3}$$

$$= \frac{a}{3}$$

What about firm & industry profit?

Firm & Industry Profit

Firm Profit:

$$\Pi_1^* = P(Q^*)Q_1^*$$

$$= \left(\frac{a}{3}\right)\frac{a}{3b}$$

$$= \frac{a^2}{9b}$$

Industry Profit:

$$\Pi^* = \Pi_1^* + \Pi_2^*$$

$$= \frac{a^2}{9b} + \frac{a^2}{9b}$$

$$= \frac{2a^2}{9b}$$

So, what does all this mean?

Question: What would happen if the two firms merged into a monopoly?

- $TR = P(Q)Q = (a - bQ)Q = aQ - bQ^2$
- $MR = TR' = a - 2bQ^*$
- $MC = MR \Rightarrow 0 = a - 2bQ$ or $Q^* = a/2b$
- $P^* = P(Q^*) = a - b(a/2b) = a/2$
- $\Pi^* = P(Q^*)Q^* = (a/2)(a/2b) = a^2/4b$

Notice that $$\frac{a^2}{4b} > \frac{2a^2}{9b}$$

Industry profit with a monopoly is higher!

So, why would a Cournot Duopoly ever exist?

Here is a Game

- Suppose a = 100 & b = 5
- Each firm can choose
  - the optimal Cournot Output: a/3b = 20/3 or
  - half the monopoly output: a/4b = 20/4.
- Each firm must choose its output before knowing the other firm's choice.

The Profit Matrix

		Firm 2's Output	
		Q_2 = 20/4	*Q_2 = 20/3*
Firm 1's Output	Q_1 = 20/4	**250**, *250*	**208.3**, *277.7*
	Q_1 = 20/3	**277.7**, *208.3*	**222.2**, *222.2*

Firm 1 gets to choose the row, while Firm 2 gets to choose the column.

The profits for the game are determined by the row & column that is chosen.

Firm 1's profit is in **bold**, Firm 2's profit is in *italics*.

What is a firm's best strategy, given the other firm's choice?

- Firm 1 maximizes profit by choosing $Q_1 = 20/3$!
  - If Firm 2 chooses $Q_2 = 20/4$, Firm 1's profits are higher if it chooses $Q_1 = 20/3$ (277.7 > 250).
  - If Firm 2 chooses $Q_2 = 20/3$, Firm 1's profits are higher if it chooses $Q_1 = 20/3$ (222.2 > 208.3).
- Firm 2 maximizes profit by choosing $Q_2 = 20/3$!
  - If Firm 1 chooses $Q_1 = 20/4$, Firm 2's profits are higher if it chooses $Q_2 = 20/3$ (277.7 > 250).
  - If Firm 1 chooses $Q_1 = 20/3$, Firm 2's profits are higher if it chooses $Q_2 = 20/3$ (222.2 > 208.3).

The Prisoner's Dilemma

- Both Firm's would be better off agreeing to produce half the monopoly output compared to the Cournot output.
- Yet, both firm's maximize their own profit by choosing the Cournot output regardless of what the other firm chooses to do.
- Therefore, choosing half the monopoly output seems to make little sense.

Reaction Functions & Nash Equilibrium
An Asymmetric Cournot Duopoly

- Assumptions
  - $P = a - bQ$ where Q is industry output.
  - Two firms produce identical product: $Q = Q_1 + Q_2$.
  - Marginal Costs: $MC_1 = c_1$ & $MC_2 = c_2$ such that $c_1 \neq c_2$.

What is the profit maximizing output for asymmetric Cournot Duopolists?

- Firm 1

$$MC_1 = MR_1$$
$$c_1 = a - 2bQ_1 * - bQ_2$$
$$R_1(Q_2) = Q_1 * = \frac{a - c_1 - bQ_2}{2b}$$

- Firm 2

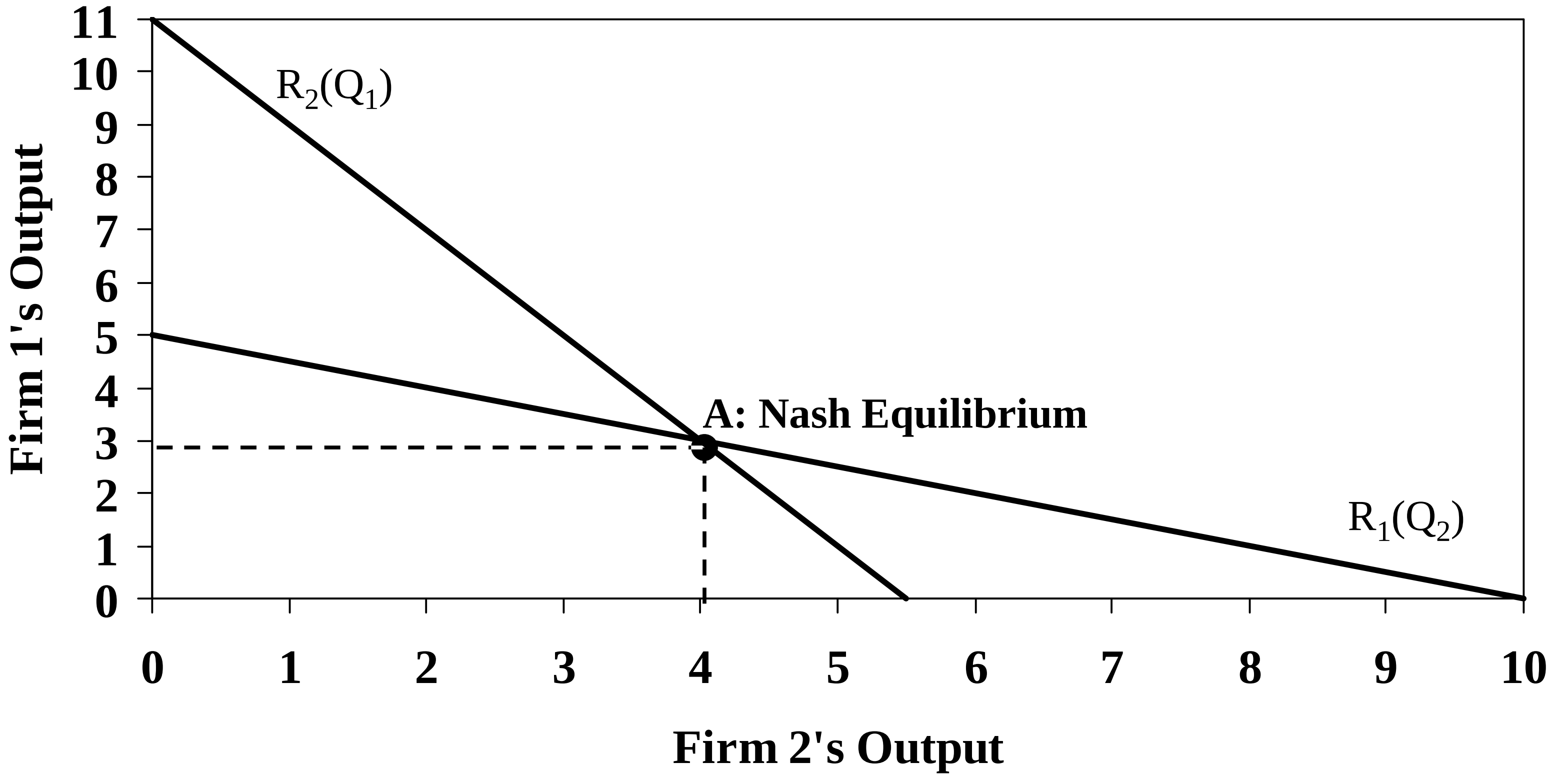
$$MC_2 = MR_2$$
$$c_2 = a - bQ_1 - 2bQ_2 *$$
$$R_2(Q_1) = Q_2 * = \frac{a - c_2 - bQ_1}{2b}$$

But now what do we do?

Reaction Functions & Nash Equilibrium Definitions

- Reaction/Best Response Function:
  - A curve that tells the profit maximizing level of output for one oligopolist for each quantity supplied by others.
- Nash Equilibrium:
  - A combination of outputs such that each firm's output maximizes its profit given the output chosen by other firms.

Example Asymmetric Duopoly Reaction Functions Assuming a = 100, b = 5, c_1 = 50, & c_2 = 45



General Solution to the Problem

Starting with $Q_1* = \frac{a - c_1 - bQ_2*}{2b}$ & $Q_2* = \frac{a - c_2 - bQ_1*}{2b}$

substitution implies

$$Q_1* = \frac{a - c_1 - b\frac{a - c_2 - bQ_1*}{2b}}{2b} = \frac{\frac{2a - 2c_1}{2} - \frac{a - c_2 - bQ_1*}{2}}{2b} = \frac{a - 2c_1 + c_2 + bQ_1*}{4b}$$

Or

$$4bQ_1^* = a - 2c_1 + c_2 + bQ_1^*$$

$$4bQ_1^* - bQ_1^* = a - 2c_1 + c_2$$

$$3bQ_1^* = a - 2c_1 + c_2$$

$$Q_1^* = \frac{a - 2c_1 + c_2}{3b}$$

$$Q_2^* = \frac{a - c_2 - b\dfrac{a - 2c_1 + c_2}{3b}}{2b}$$

$$= \frac{\dfrac{3a - 3c_2}{3} - \dfrac{a - 2c_1 + c_2}{3}}{2b}$$

$$= \frac{\dfrac{2a + 2c_1 - 4c_2}{3}}{2b} = \frac{a + c_1 - 2c_2}{3b}$$

For a = 100, b = 5, c_1 = 50, & c_2 = 45,

$$Q_1^* = \frac{100 - 2 \times 50 + 45}{3 \times 5} = 3$$

$$Q_2^* = \frac{100 + 50 - 2 \times 45}{3 \times 5} = 4$$

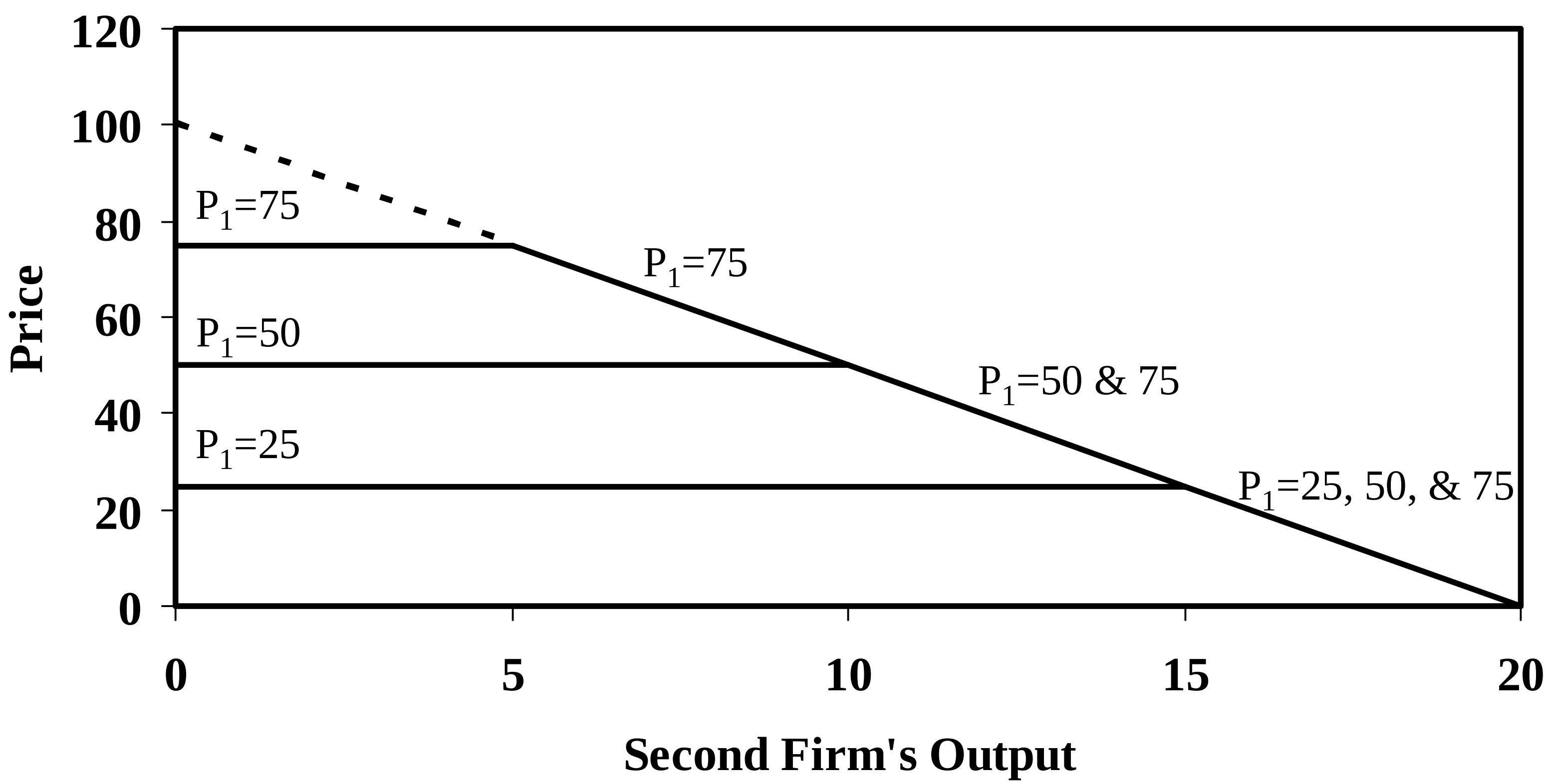
Bertrand Duopoly Model

- Firms choose price simultaneously, instead of quantity.
- Question: Does this matter?
- Yes, or we probably would not be talking about it!

Bertrand Duopolist Strategy

- Question: If I know my competitor will choose some price P_0, say \$50, what price should I choose?
- Assumptions
  - Two Firms
  - Demand: $P = a – bQ$
  - Marginal Costs: $MC = MC_1 = MC_2 = 0$
- Question: What does Firm 2's demand look like given Firm 1's choice of price?

Firm 2's Demand Given Firm 1's Price



Implications

- Firms have an incentive to undercut their competitor's price as long as they can make a profit.
- This behavior will drive the price down to the marginal cost:
  - $P^* = MC \Rightarrow 0 = a - bQ^* \Rightarrow Q^* = a/b$
  - $\Pi^* = P^*Q^* = (a - b(a/b))(a/b) = (a - a)(a/b) = 0$
  - Bertrand outcome is same as perfect competition!

Stackelberg Duopoly Model

- Firms choose quantities sequentially rather than simultaneously.
- Question: Does this matter?
- Yes, or we probably would not be talking about it!
- Assumptions
  - Two Firms
  - Demand: $P = a - bQ$
  - Marginal Costs: $MC = MC_1 = MC_2 = 0$
  - Firm 1 chooses output Q_1 first.
  - Firm 2 chooses output Q_2 second after seeing Firm 1's choice.
  - $Q = Q_1 + Q_2$

How do we find Firm 1 & 2's profit maximizing outputs?

- In the Cournot Model, neither firm got the see the other's output before making its choice.
- In the Stackelberg Model, Firm 2 gets to see Firm 1's output before making its choice.
  - Question: How can Firm 1 use this to its advantage?
    - Firm 1 should consider how Firm 2 will respond to its choice of output.

Given Firm 1's choice of output, what is Firm 2's profit maximizing output?

- It is again optimal for Firm 2 to set marginal cost equal to marginal revenue: $MC_2 = MR_2$.
- Firm 2's Total Revenue:
  - $TR_2 = P(Q)Q_2 = (a - b(Q_1 + Q_2))Q_2 = aQ_2 - bQ_1Q_2 - bQ_2^2$.
- Firm 2's Marginal Revenue:
  - $MR_2 = TR_2' = a - bQ_1 - 2bQ_2$
- $MC_2 = MR_2 \Rightarrow 0 = a - bQ_1 - 2bQ_2^* \Rightarrow 2bQ_2^* = a - bQ_1 \Rightarrow Q_2^* = (a - bQ_1) / (2b) = R_2(Q_1)$.

Given Firm 2's best response, what is Firm 1's profit maximizing output?

- It is optimal for Firm 1 to set marginal costs equal to marginal revenue: $MC_1 = MR_1$.
- Firm 1's Total Revenue:
  - $TR_1 = P(Q)Q_1 = (a – b(Q_1 + Q_2))Q_1 = aQ_1 – bQ_1^2 – bQ_1Q_2$.
    - But $Q_2 = R_2(Q_1)$, so $TR_1 = aQ_1 – bQ_1^2 – bQ_1R_2(Q_1)$.
- Firm 1's Marginal Revenue:
  - $MR_1 = TR_1' = a – 2bQ_1 – bR_2(Q_1) – bQ_1R_2'(Q_1)$
    - But $R_2(Q_1) = (a – bQ_1) / (2b)$ & $R_2'(Q_1) = -b/(2b) = -1/2$, so $MR_1 = a – 2bQ_1 – b(a – bQ_1) / (2b) – bQ_1 (-1/2) = a – 2bQ_1 – a/2 + bQ_1/2 + bQ_1/2 = a/2 – bQ_1$
- $MC_1 = MR_1 \Rightarrow 0 = a/2 – bQ_1^* \Rightarrow bQ_1^* = a/2 \Rightarrow Q_1^* = a/(2b)$

What is Firm 2's profit maximizing output, the price, & profits?

- $Q_2{}^* = R_2(Q_1{}^*) = (a – ab/(2b))/(2b) = (a – a/2)/(2b) = a/(4b)$
- $P^* = a – b(Q_1{}^* + Q_2{}^*) = a – b(a/(2b) + a/(4b)) = a – (a/2 + a/4) = a/4$
- $\Pi_1{}^* = P^*Q_1{}^* = (a/4) (a/(2b)) = a^2/(8b)$
- $\Pi_2{}^* = P^*Q_2{}^* = (a/4) (a/(4b)) = a^2/(16b)$
- $\Pi^* = \Pi_1{}^* + \Pi_2{}^* = a^2/(8b) + a^2/(16b) = 3a^2/(16b)$

For a = 100 & b = 5

- $Q_1^* = a/(2b) = 100/(2 \times 5) = 10$
- $Q_2^* = a/(4b) = 100/(4 \times 5) = 5$
- $P^* = a/4 = 100/4 = 25$
- $\Pi_1^* = a^2/(8b) = 100^2/(8 \times 5) = 250$
- $\Pi_2^* = a^2/(16b) = 100^2/(16 \times 5) = 125$
- $\Pi^* = \Pi_1^* + \Pi_2^* = 250 + 125 = 375$

How do the models compare?

Model	Industry Output (Q^*)	Market Price (P^*)	Industry Profit (Π^*)
Monopoly	$Q_M^* = a/(2b)$	$P_M^* = a/(2b)$	$\Pi_M^* = a^2/(4b)$
Cournot	$(4/3)Q_M^*$	$(2/3)P_M^*$	$(8/9)\Pi_M^*$
Stackelberg	$(3/2)Q_M^*$	$(1/2)P_M^*$	$(3/4)\Pi_M^*$
Bertand	$2Q_M^*$	0	0
Perfect Competition	$2Q_M^*$	0	0

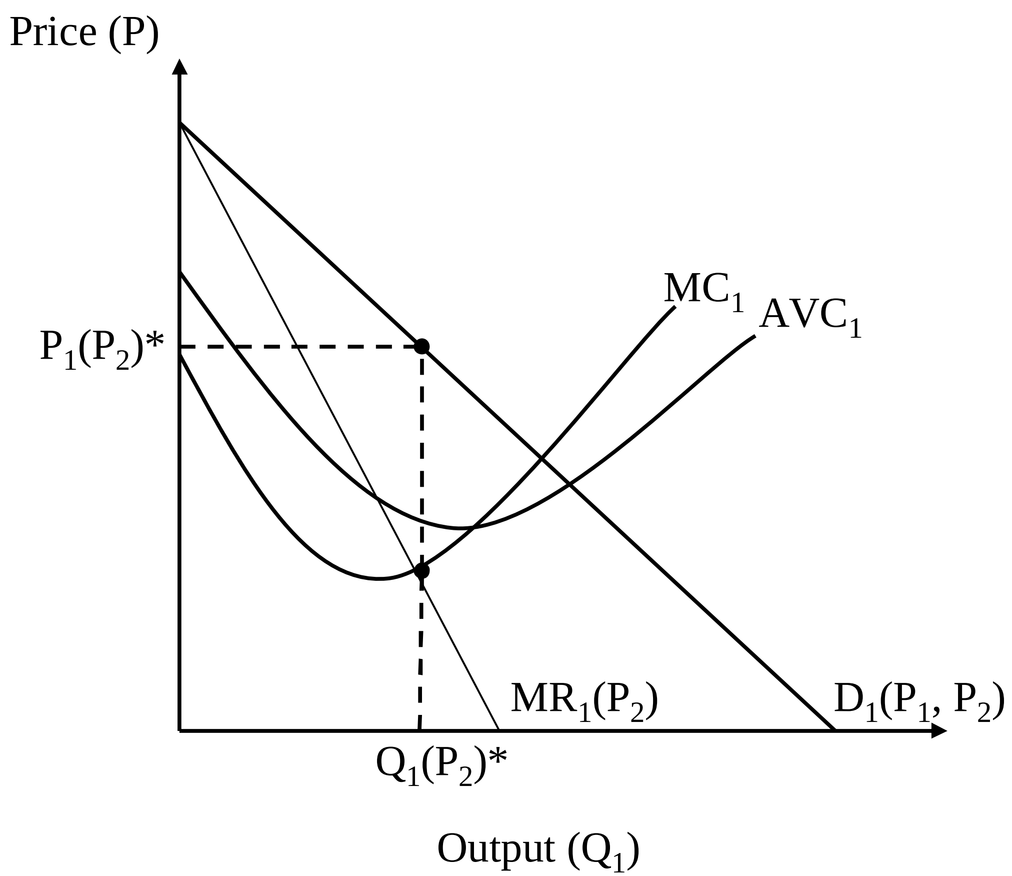
Monopolistic Competition Model

- Recall that for monopolistic competitors
  - Products are distinct, but close substitutes.
  - There is free entry & exit.
- Implications
  - Demand for one firm's product will fall when a competitor decreases price.
  - There can be no economic profits in the long run.
- Assumptions
  - Two Firms
  - Firm 1's Demand: $Q_1 = D_1(P_1,P_2)$
  - Firm 2's Demand: $Q_2 = D_2(P_2,P_1)$

Short Run Profit Maximization With Monopolistic Competition

- Firm 1
  - $MC_1 = MR_1(P_1, P_2)$
  - $MC_1' > MR_1'(P_1, P_2)$
  - $P_1 > AVC_1$
- Firm 2
  - $MC_2 = MR_2(P_2, P_1)$
  - $MC_2' > MR_2'(P_2, P_1)$
  - $P_2 > AVC_2$

Monopolistic Competitor In the Short Run



Problem

- Firm 1’s profit maximizing price & output depends on Firm 2’s profit maximizing price.
- Firm 2’s profit maximizing price & output depends on Firm 1’s profit maximizing price.

What do we do now?

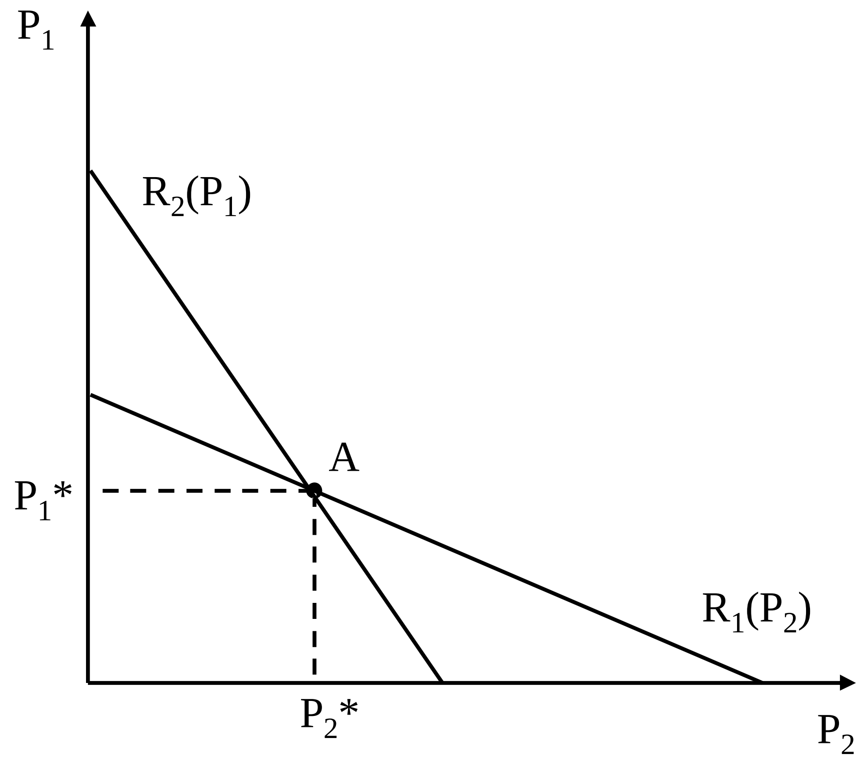
Look For Nash Equilibrium

- $MC_1 = MR_1(P_2)$

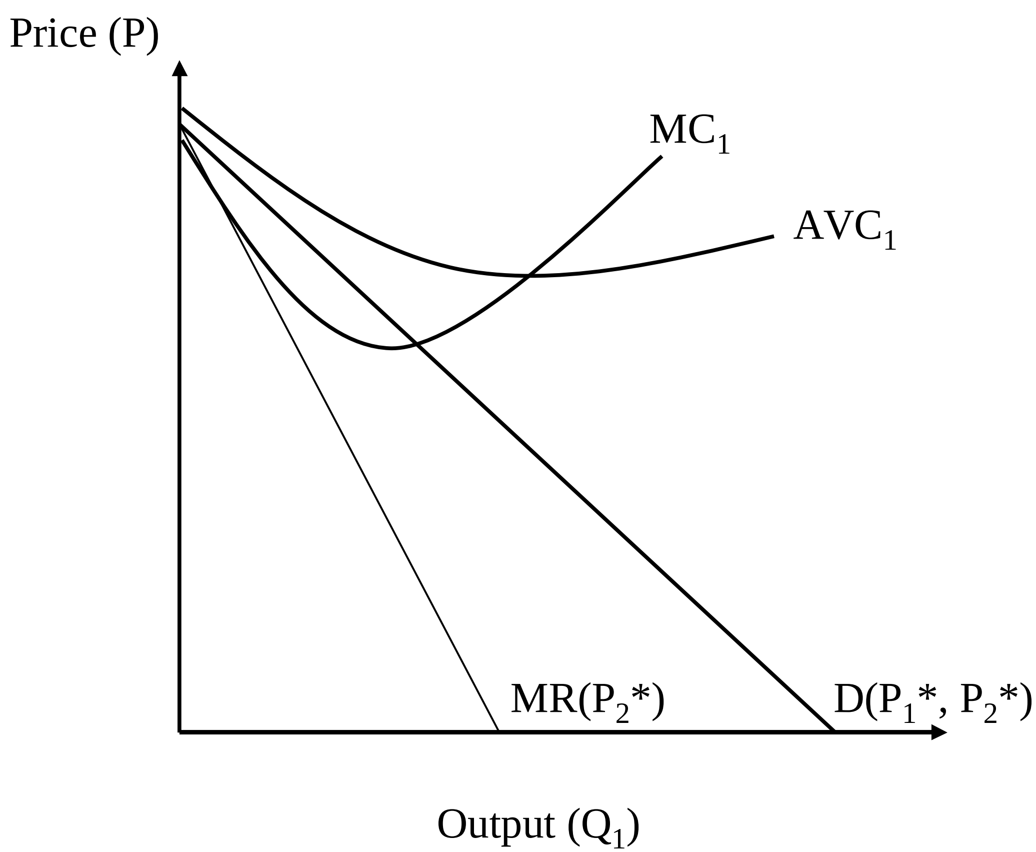
  $\Rightarrow P_1 = R_1(P_2)$
- $MC_2 = MR_2(P_1)$

  $\Rightarrow P_2 = R_2(P_1)$

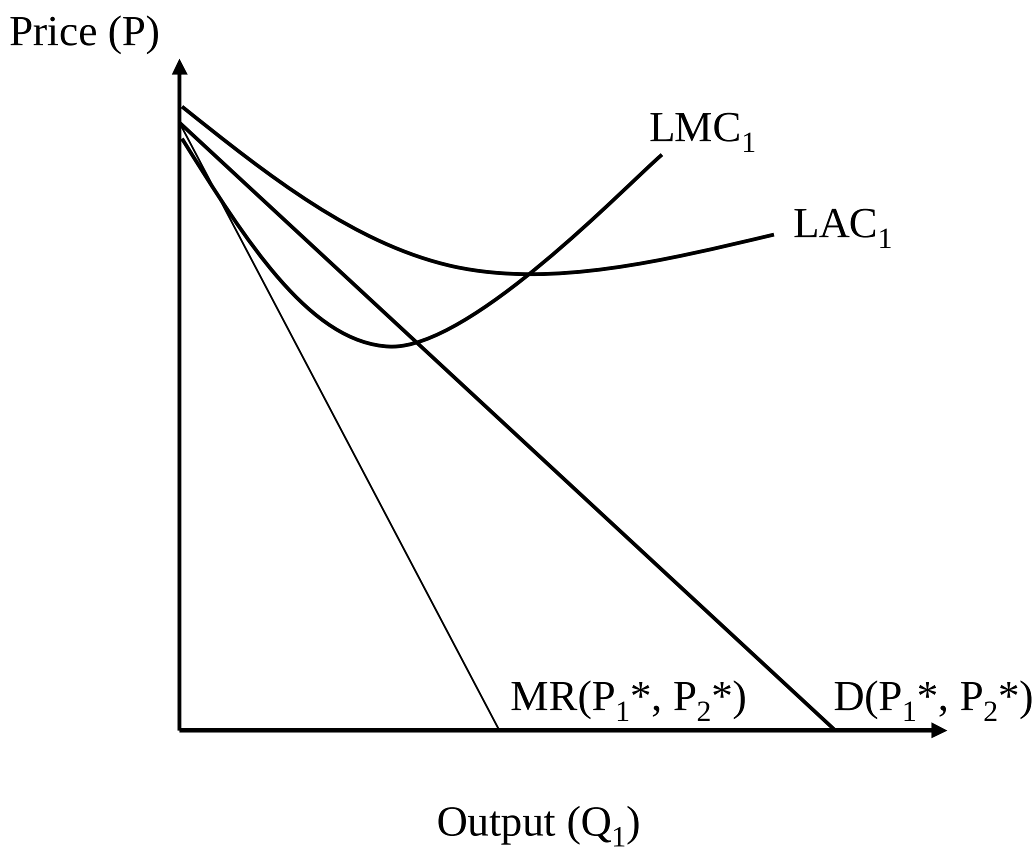
Example Reaction Functions For Monopolistic Competitors



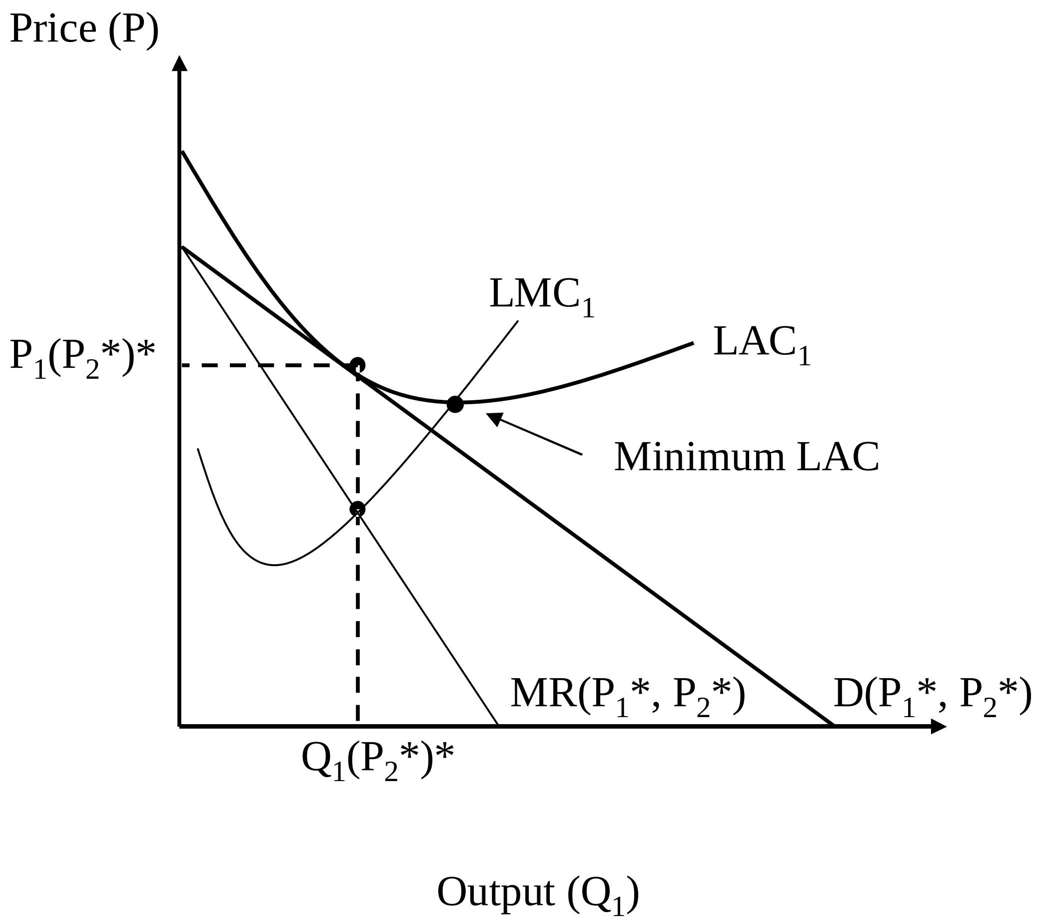
Example of When a Monopolistic Competitor Will Not Operate In the Short Run



Example of When a Monopolistic Competitor Will Not Operate In the Long Run



Monopolistic Competitor Long Run Equilibrium



Things to Remember About Monopolistic Competition

- Produce above minimum average costs in the long run!
- Never produce where demand is inelastic!
- Have no supply curve!

Basic Concepts of Economic Games & Their Solutions

- What is a game?
  - Players
  - Rules
    - Who does what & when?
    - Who knows what & when?
  - Rewards
- Simultaneous Game:
  - Players learn nothing new during the play of the game (e.g. Cournot & Bertrand Duopoly).
- Sequential Game:
  - Some players learn something new during the play of the game (e.g. Stackelberg Duopoly).
- Strategy:
  - A complete description of what a player does given what it knows.

Example of Simultaneous Game: Rock/Paper/Scissors

- Players:
  - Mason & Spencer
- Rules
  - Players choose either Rock (R), Paper (P), or Scissors (S).
  - Players make choice at the same time.
  - Rock Beats Scissors
  - Paper Beats Rock
  - Scissors Beats Paper
- Rewards
  - Winner gets $10 & Loser Pays $10.
  - For ties everyone gets $0.
- Strategies:
  - R, P, & S

Example of Sequential Move Game: Rock/Paper/Scissors Spencer's Preferred Version

- Players:
  - Mason & Spencer
- Rules
  - Players choose either Rock (R), Paper (P), or Scissors (S).
  - Tall player makes choice first.
  - Rock Beats Scissors
  - Paper Beats Rock
  - Scissors Beats Paper
- Rewards
  - Winner gets $10 & Loser Pays $10.
  - For ties everyone gets $0.

Strategies for sequential games must specify contingency plans.

- Tall Player Strategies:
  - R, P, & S
- Short Player Strategies
  - (If Tall Player Chooses R, If Tall Player Chooses P, If Tall Player Chooses S)
  - Total of number of strategies = $3\times3\times3 = 27$
  - Examples
    - (R,R,R)
    - (S,S,S)
    - (P,S,R)

Describing Simultaneous Move Games

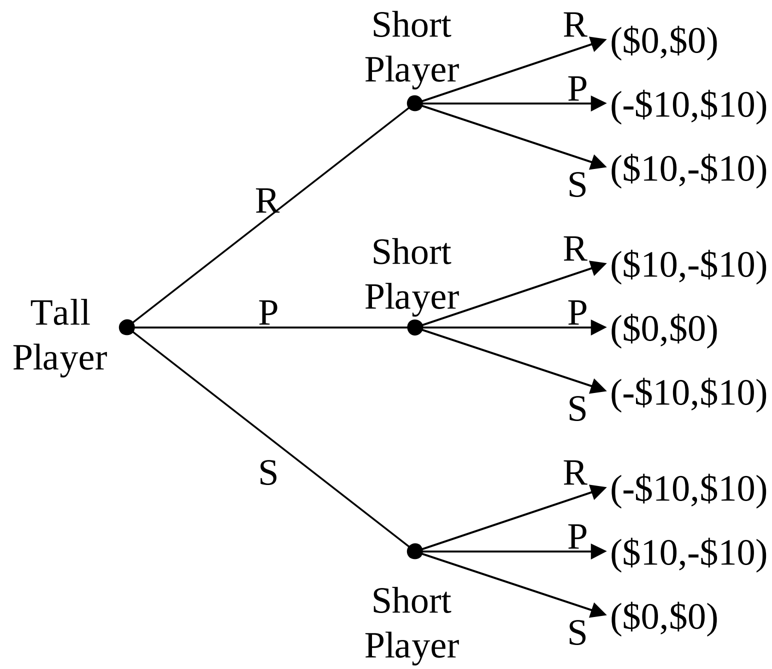
		Your Partner's Choice	
		A	*B*
Your Choice	**A**	*100* **100**	*200* **0**
	B	*0* **200**	*50* ***50***

You get to choose the row, while your opponent gets to choose the column.

The rewards for the game are determined by the row & column that is chosen.

Your reward is in **bold**, your opponent's reward is in *italics*.

Describing Sequential Move Games



Solving Games
Equilibrium

- Dominant Strategy:
  - The strategy in a game that produces the best results irrespective of the strategy chosen by an opponent.

		Your Partner's Choice	
		A	*B*
Your Choice	**A**	**100**, *100*	**0**, *200*
	B	**200**, *0*	***50***, *50*

- Your dominant strategy is to play **B**.
- Your Opponent's dominant strategy is to also play *B*.
- This is the dominant strategy equilibrium.

There is not always a dominant strategy equilibrium!

		Your Partner's Choice	
		A	*B*
Your Choice	**A**	**100** / *100*	**0** / *75*
	B	**200** / *0*	***50*** / *50*

- Here you still will always want to play **B**.
- But your opponent will want to play *A* if you choose **A** and *B* if you choose **B**.
- There is no dominant strategy equilibrium!

Nash Equilibrium

- General Definition:
  - A combination of strategies such that each player maximizes its reward given the strategy chosen by other players.

		Your Partner's Choice	
		A	*B*
Your Choice	**A**	**100** / *100*	**0** / *75*
	B	**200** / *0*	***50*** / *50*

- For **B** & *B*, neither player can do better by changing their strategy unless another player changes his.
- So **B** & *B* is a Nash equilibrium.
- We can always find at least one Nash equilibrium.

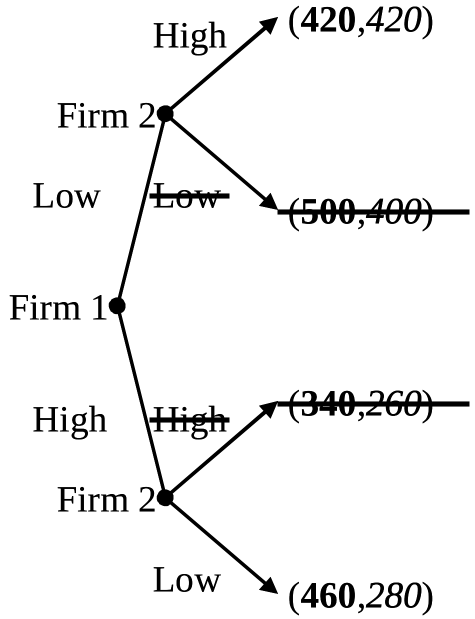
Multiplicity of Nash Equilibrium

		Your Partner's Choice	
		A	*B*
Your Choice	**A**	**100**, *100*	**0**, *75*
	B	**75**, *0*	***50***, *50*

- **B** & *B* is a Nash equilibrium.
- But so is **A** & *A*.
- How do we choose?
  - Everyone is better off for **A** & *A*.
  - But this is only one possibility.

Note: A dominant strategy equilibrium is a Nash equilibrium!

Solving Sequential Games



- Work Backwards
  - If Firm 1 chooses Low, Firm 2 should choose High.
  - If Firm 1 choose High, Firm 2 should choose Low.
  - Now Firm 1 knows it should choose High!
- Equilibrium Strategy
  - Firm 1: High
  - Firm 2:
    - High if Firm 1 chooses Low
    - Low if Firm 1 choose High

This is more than a Nash equilibrium!

- Firm 1 Strategies:
  - High
  - Low
- Firm 2's Strategies:
  - (i) Choose High if Firm 1 chooses Low & High if Firm 1 Chooses High,
  - (ii) Choose High if Firm 1 chooses Low & Low if Firm 1 Chooses High,
  - (iii) Choose Low if Firm 1 chooses Low & High if Firm 1 Chooses High,
  - (iv) Choose Low if Firm 1 chooses Low & Low if Firm 1 Chooses High.

This is more than a Nash equilibrium!

			Firm 2		
		(i)	*(ii)*	*(iii)*	*(iv)*
Firm 1	**Low**	**420**, *420*	**420**, *420*	**500**, *400*	**500**, *400*
	High	**340**, *260*	**460**, *280*	**340**, *260*	**460**, *280*

- The Nash equilibrium for this game are: (1) **Low** & *(i)* and (2) **High** & *(ii)*.
- (1) **Low** & *(i)* depends on an incredible threat!
  - Working backward eliminates incredible threats.

What You Should Know

- Characteristics of Oligopoly & Monopolistic Competition
- Cournot, Bertrand, & Stackelberg Duopoly Models
  - Differences in Assumptions
  - Differences in Predicted Behavior
- Reaction Functions & Nash Equilibrium
- Monopolistic Competition Model
  - Assumptions
  - Characteristics
    - No Long Run Economic Profit
    - No Supply Curve
    - Produce Where Demand is Elastic
- Simultaneous & Sequential games and how they are solved.