Individual and Market Demand
Readings: Ch. 4

Objective: Understand how to derive an individual's demand curve.

Now that we have a better understanding of how an economist views individual choice, we are ready to see precisely where a market demand curve comes from. But to start, we must find an individual's demand curve.

Definition
Price Consumption Curve: Holding income and the prices of other goods constant, the price consumption curve for a good is the set of optimal bundles as the price of the good varies.

The price consumption curve shows the optimal consumption bundles as we increase the price of one of the goods. Figure 1 illustrates. Note that the price consumption curve may be increasing or decreasing depending on whether goods are substitutes or complements.

What is nice about the price consumption curve is that it tells us everything we need to know about an individual's demand. If we use the price consumption curve to plot the price of housing on the vertical axis and the optimal quantity of housing on the horizontal axis, the relationship will be an individual demand curve (See Figure 2). That is all there is to it. Note that this demand curve is also dependent of the level of income, price of food, and individual preferences. If we change any or all of these factors, the price consumption curve will change as will the demand curve.

Objective: Understand Engle curves and what they tell us about a product.

The price consumption curve is useful for finding an individual's demand for a product. Another important and interesting relationship is between the optimal consumption bundle and income.

Definition
Income Consumption Curve: Holding the price of all goods constant, the income consumption curve for a good is the set of optimal bundles as income varies.

Figure 3 provides an illustration of an income consumption curve. We can plot the optimal quantity of housing against income holding prices constant, which results in the Engel curve drawn in Figure 4.

Definition
Engel Curve: The curve that plots the relationship between the quantity of a good consumed and income.

In Figure 3 and 4, the quantity of a good consumed is increasing with income. It is important to note that this need not be the case. It is possible that the quantity of a good consumed decreases with income: hotdogs and hamburgers are a good example. In fact, it is exactly this type of distinction that makes the Engel curve useful.

Definition
Normal Good: a good whose quantity demanded rises as income rises.

Definition
Inferior Good: a good whose quantity demanded falls as income rises.

Both these definitions assume prices do not change.

Objective: Understand how a change in the price of a good affects the optimal level of consumption for that good.

Increasing the price of a good makes it more expensive relative to other goods, so it makes sense to consume less of it. But increasing the price of a good also makes it so that we have relatively less income to spend on both goods, so it makes sense to consume less of it if the good is normal and more of it if it is inferior. This suggests two distinct and important effects of a change in price on the demand for a good.

Definition
Substitution Effect: The component of the total effect of a price change that results from the associated change in the relative attractiveness of other goods.

Definition
Income Effect: The component of the total effect of a price change that results from the associated change in real purchasing power.

To better understand these two distinct effects, let us consider an example. Suppose Bob is consuming his optimal bundle of food, F_0, and housing, H_0, on the budget constraint labeled B_0 in Figure 5. At this optimal consumption bundle his level of satisfaction is equal to I_0. Now suppose the price of housing increases resulting in the new budget constraint B_1, the new optimal consumption bundle, F_1 and H_1, and the new reduced level of satisfaction I_1.

Question: How much of the change in the consumption of housing from H_0 to H_1 is due to a change in relative prices and how much is due to a change in real purchasing power?

First, suppose we had enough income given our new prices to still achieve a level of satisfaction equal to I_0. How much housing would we consume? We can find this quantity by shifting our new budget constraint out to B' where it is just tangent to our original indifference curve, I_0. The resulting quantity of housing is H'.

The difference between H' and H_0 tells us how the change in the relative price of housing would have decreased our demand for housing even if we could still achieve the same level of satisfaction. This difference is what we call the *substitution effect*. Since the relative price of housing increased, we are better off spending more of our income on food.

The difference between H_1 and H' tells us how the increase in the price of housing effectively reduces the real purchasing power of our income. This difference is what we call the *income effect*. Since our relative income is lower, we are better off spending less on housing if it is a normal good and more on housing if it is an inferior good. In Figure 5, the income effect decreases the amount of housing we consume even further, $H_1 < H'$, so housing is a normal good in this example. If housing was an inferior good, $H_1 > H'$.

If a good is normal, then both the income and substitution effect of an increase in price reduce the consumption of the good, so there will be an unequivocal inverse relationship between the price and quantity demanded. The law of demand will hold.

If a good is inferior, then the income effect of a price increase increases the quantity demanded, while the substitution effect decreases the quantity demanded. Therefore, the total effect is indeterminate. If the substitution effect is bigger in magnitude than the income effect, the optimal quantity of housing will decrease and the law of demand will hold true. But, if the income effect is bigger in magnitude than the substitution effect, the optimal quantity of housing increases, violating the law of demand. Goods that violate the law of demand are called *Giffen goods*. All Giffen goods are inferior. Furthermore, their income effect must be larger in magnitude than their substitution effect, an occurrence that is rare, but not unheard of.

Objective: Understand how income and substitution effects determine whether a good is a complement or substitute.

Increasing the price of another good makes a good relatively cheaper, so you should consume more of it. But it also decreases relative income, so you should consume less of it if it is normal and more of it if it is inferior. Again, we can use substitution and income effects to better understand how an increase in the price of another good affects consumption.

Figure 6 shows the decomposition of an increase in the price of food on the optimal quantity of housing consumed. Starting at the budget line B_0, the optimal amount of housing to consume is H_0 with a level of satisfaction equal to I_0. An increase in the price of food rotates the budget constraint to B_1. Now the optimal amount of housing is H_1, with a level of satisfaction equal to I_1. To see how much housing we would consume at the new prices if we had enough income to be as satisfied as we were originally, we can shift B_1 up to B' where it is just tangent to I_0. The resulting quantity is H'. The substitution effect is then $H' - H_0$, which is always positive for an increase in the price of another good when there are only two goods (***note that this need not be the case with more than two goods***). The income effect is $H_1 - H'$, which will be negative for a price increase if housing is a normal good or positive for a price increase if housing is an inferior good. In Figure 6, $H_1 - H' < 0$, so housing is a normal good.

In Figure 6, the net effect of an increase in the price of food is to increase the amount of housing. When an increase in the price of a good increases the demand for another good, it is called a ***substitute good***. Therefore, food is a substitute for housing in Figure 6.

Alternatively, when an increase in the price of a good decreases the demand for another good, it is called a *complement good*.

When a good is normal, the increase in the price of another good will produce a positive substitution effect and a negative income effect. Which effect is larger in magnitude tells you whether goods are substitutes or complements. When the substitution effect is bigger in magnitude than the income effect, it is a substitute. When the income effect is bigger in magnitude than the substitution effect, it is a complement.

When a good is inferior, the increase in the price of another good will produce a positive substitution effect and a positive income effect. Therefore, the other good must be a substitute.

Objective: Understand how to derive market demand from individual demand.

Now that we have an understanding of individual demand and the factors determining that demand, we can turn to how to derive the market demand. We accomplish this by horizontally summing the individual market demands. What I mean by the horizontal sum, is that we need to sum the quantity demanded by each individual for a particular price and then repeat this process for all possible prices.

This process is best illustrated with an example. Suppose individual A's demand is given by the function $Q_A = 50 – 5P$ and that individual B's demand is given by the function $Q_B = 30 – 2P$.

Question: What is the market demand for these two individuals?

Table1, Figure 7a, and Figure 7b illustrate. What is important to note in this example is that A does not enter the market until the price falls below 10, while B enters the market when the price is below 15. This produces the kink in the demand curve seen in Figure 7b. Therefore, we must describe the market demand function in pieces: for $P \geq 15$, $Q_M = 0$; for $15 > P \geq 10$, $Q_M = Q_B = 30 – 2P$; and for $P < 10$, $Q_M = Q_A + Q_B = 50 – 5P + 30 – 2P = 80 – 7P$.

Table 1: Derivation of Market Demand

Price	A's Quantity Demanded	B's Quantity Demanded	Market Demand
15	0	0	0
14	0	2	2
13	0	4	4
12	0	6	6
10	0	10	10
9	5	12	17
8	10	14	24
7	15	16	31
6	20	18	38
5	25	20	45
4	30	22	52
3	35	24	59
2	40	26	66
1	45	28	73
0	50	30	80

Objective: Understand the price elasticity of demand and what it tells us about the relationship between price, quantity demanded, and total revenue.

For most of what we have done with demand up until now, we have focused on using the slope of the demand curve as the basis for describing the relationship between the price and quantity demanded. While this has certainly been useful, there is an inherent problem in solely relying on the slope. The slope of the demand curve depends on the units of measurement. If we talk about the slope of the demand curve for individual bagels, it will differ from the slope of the demand curve for a dozen bagels, even if the same increase in price results in the same decline in the number of bagels demanded. Furthermore, it is hard to compare the slope of demand curve for two products with different units of measurement: gallons of milk and dozens of bagels.

To get around the sensitivity of the slope to the unit of measurement, economists use another quantity to measure the responsiveness of the quantity demanded to price. This quantity is called the price elasticity of demand.

Definition

Price elasticity of Demand: The percentage change in the quantity of a good demanded that results from a one percent change in price:

$$h = \frac{\frac{\Delta Q_D}{Q_D}}{\frac{\Delta P}{P}} = \frac{\Delta Q_D}{\Delta P}\frac{P}{Q_D}.$$

The elasticity of demand is unit free—its value does not depend on how we measure price or quantity.

If we have a general demand curve $Q_D = D(P)$, the price elasticity of demand is $h = \frac{P}{Q_D} D'(P)$.

For the linear demand curve $Q_D = a_D - b_D P$, the price elasticity of demand is $h = -\frac{P}{Q_D} b_D$. If demand is described as $P = c_D - k_D Q_D$, $h = -\frac{P}{Q_D}\frac{1}{k_D}$.

Note that the price elasticity of demand depends on the price and quantity as well as the slope of the demand curve. Therefore, even though the slope of a linear demand curve does not change for any price and quantity, the price elasticity of demand will. Also note that since the slope of a demand curve is typically negative, the price elasticity of demand will also be negative.

The price elasticity of demand can be categorized based on how a decrease in price changes the total revenue generated from the sale of a good, i.e. $TR = PQ_D = PD(P)$ where TR represents total revenue. To see how, we can take the derivative of total revenue with respect to price to get $\frac{\partial TR}{\partial P} = D(P) + PD'(P)$ or with a little algebra $\frac{\partial TR}{\partial P} = D(P)(1 + h)$

Inelastic Demand ($0 > \eta > -1$): If the price elasticity of demand is between 0 and –1, a one percent increase in price decreases the quantity demanded by less than one percent. Therefore, the revenue gained from increasing the price will be greater than the revenue lost from selling less product, resulting in a net gain in total revenue.

Unit Elastic Demand ($\eta = -1$): If the price elasticity of demand equals –1, a one percent increase in price decreases the quantity demanded by exactly one percent. Therefore, the revenue gain from increasing the price will just equal the revenue lost from selling more product, resulting in no net change in total revenue.

Elastic Demand ($\eta < -1$): If the price elasticity of demand is less than –1, a one percent increase in price decreases the quantity demanded by more than one percent. Therefore, the revenue gained from increasing the price will be lower than the revenue lost from selling less product, resulting in a net loss in total revenue.

Word of Caution: Since the slope of the demand curve is usually negative, the price elasticity of demand is often reported as a positive number by simply taking the absolute value. Therefore, if you see a positive price elasticity of demand, don't assume the good violates the law of demand.

For our general linear demand curve, $Q_D = a_D - b_D P$, it is easy to divide it up into elastic, unit elastic, and inelastic regions (See Figure 8). When $Q_D < a_D/2$, demand is elastic. When $Q_D = a_D/2$, demand is unit elastic. When $Q_D > a_D/2$, demand is inelastic. Figure 9 shows the relationship between the quantity demanded and total revenue for this linear demand curve.

When demand is elastic, increasing Q_D (or alternatively lowering P) increases total revenue. When demand is inelastic, increasing Q_D (or alternatively lowering P) decreases total revenue.

There are two other special cases for the price elasticity of demand: perfectly elastic ($\eta = -\infty$) and perfectly inelastic ($\eta = 0$). Perfectly elastic demand can be represented by a horizontal line (See Figure 10), while perfectly inelastic demand can be represented by a vertical line (See Figure 11).

Determinants of the Price Elasticity of Demand:

1) Substitution Possibilities: If there are lots of substitutes available, the demand for a good is more elastic.
2) Budget Share: If more of your total income is spend on a good, the demand for that good is more elastic.
3) Direction of the Income effect: Normal goods tend to be more elastic than inferior goods because the income effect reinforces the substitution effect.
4) Time: When there is more time available for individuals to respond to price changes, demand is more elastic. In the near term, we are not likely to buy an economy car and use less gas just because the price of gas has increased. But given enough time and a prolonged increase in the price of gas, we are more likely to replace our gas hog with an economy car and substantially reduce our demand for gas.

Objective: Understand the income elasticity of demand and what it tells us about a good.

The price elasticity of demand tells us useful information about how responsive the quantity demanded is to price. Similarly, the Income Elasticity of Demand tells us how responsive the quantity demanded is to income.

Definition
Income Elasticity of Demand: The percentage change in the quantity of a good demanded that results from a one percent increase in income:

$$e = \frac{\frac{\Delta Q_D}{Q_D}}{\frac{\Delta M}{M}} = \frac{\Delta Q_D}{\Delta M}\frac{M}{Q_D}.$$

Since an increase in income may increase or decrease the quantity demanded of a good depending on whether or not the good is normal or inferior, the income elasticity of demand will be positive for normal goods ($\varepsilon > 0$) and negative for inferior goods ($\varepsilon < 0$). But the income elasticity of demand allows us to categorize normal goods even further.

When a one percent increase in income increases the quantity demanded by less than one percent ($1 > \varepsilon > 0$), the good is called a *necessity*.

When a one percent increase in income increases the quantity demanded by more than one percent ($\varepsilon > 1$), the good is called a *luxury*.

Objective: Understand the cross-price elasticity of demand and what it tells us about a good.

The cross-price elasticity of demand tells us how responsive the quantity demanded of one good is with respect to the price of some other good. It provides a useful way for measuring the degree of substitutability or complementarity between two goods.

Defintion

Cross-price Elasticity of Demand: The percentage change in the quantity of one good demanded that results from a one percent change in the price of another good:

$$h_{XZ} = \frac{\frac{\Delta Q_X}{Q_X}}{\frac{\Delta P_Z}{P_Z}} = \frac{\Delta Q_X}{\Delta P_Z}\frac{P_Z}{Q_X}.$$

When two goods are substitutes, the cross-price elasticity will be positive ($\eta_{XZ} > 0$). For complements, it will be negative ($\eta_{XZ} < 0$).

Figure 1: Price Consumption Curve

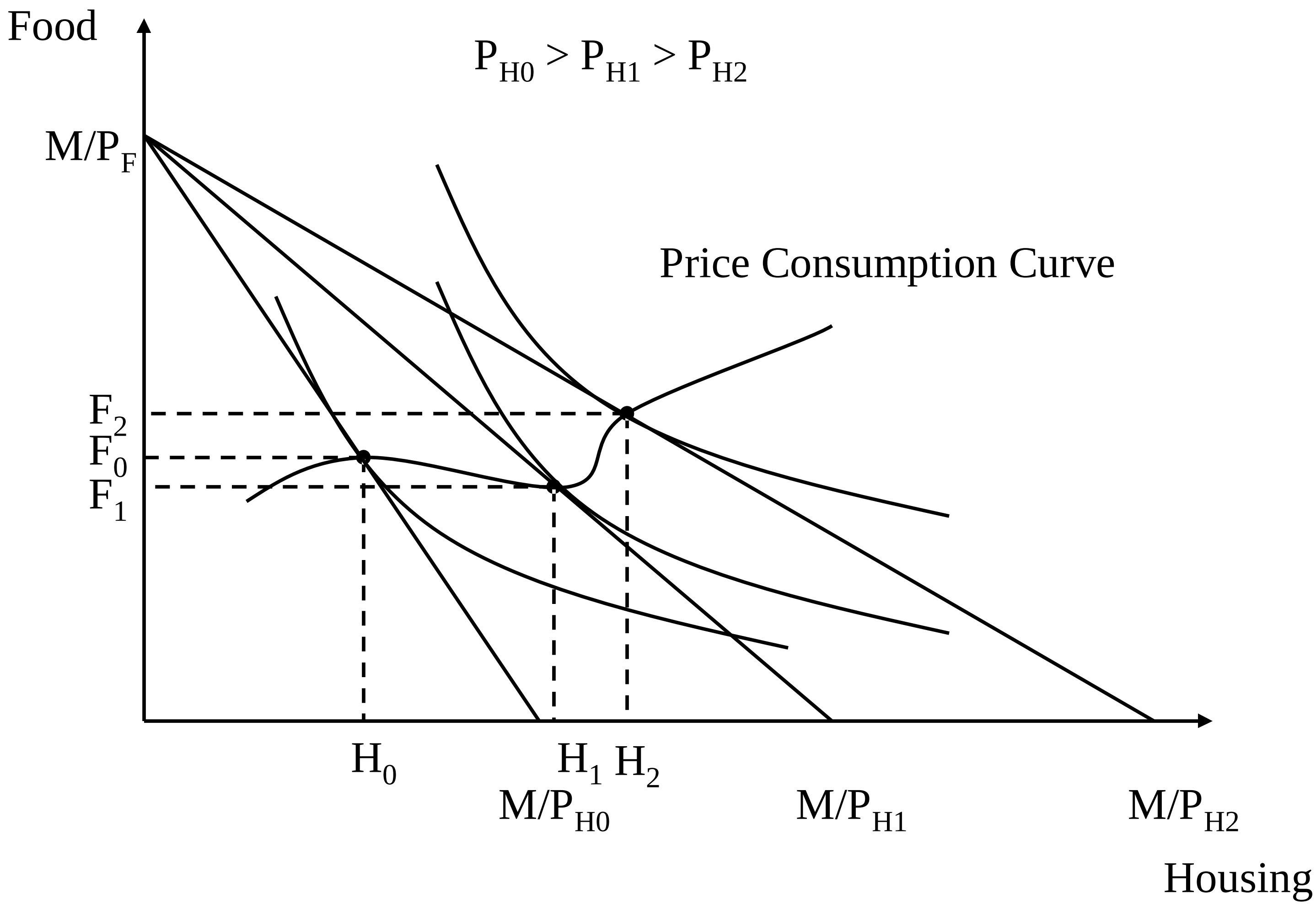

Figure 2: Individual Demand Curve

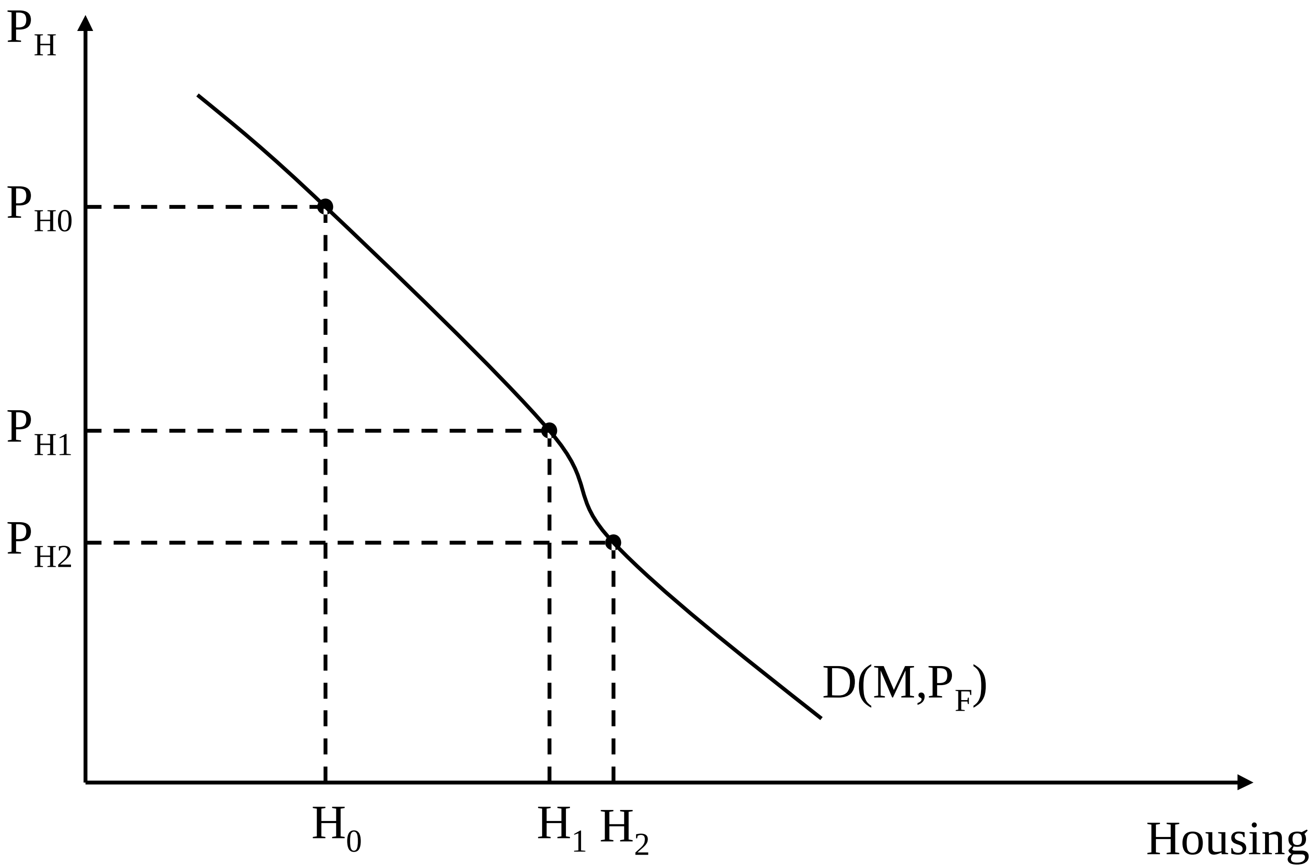

Figure 3: Income Consumption Curve

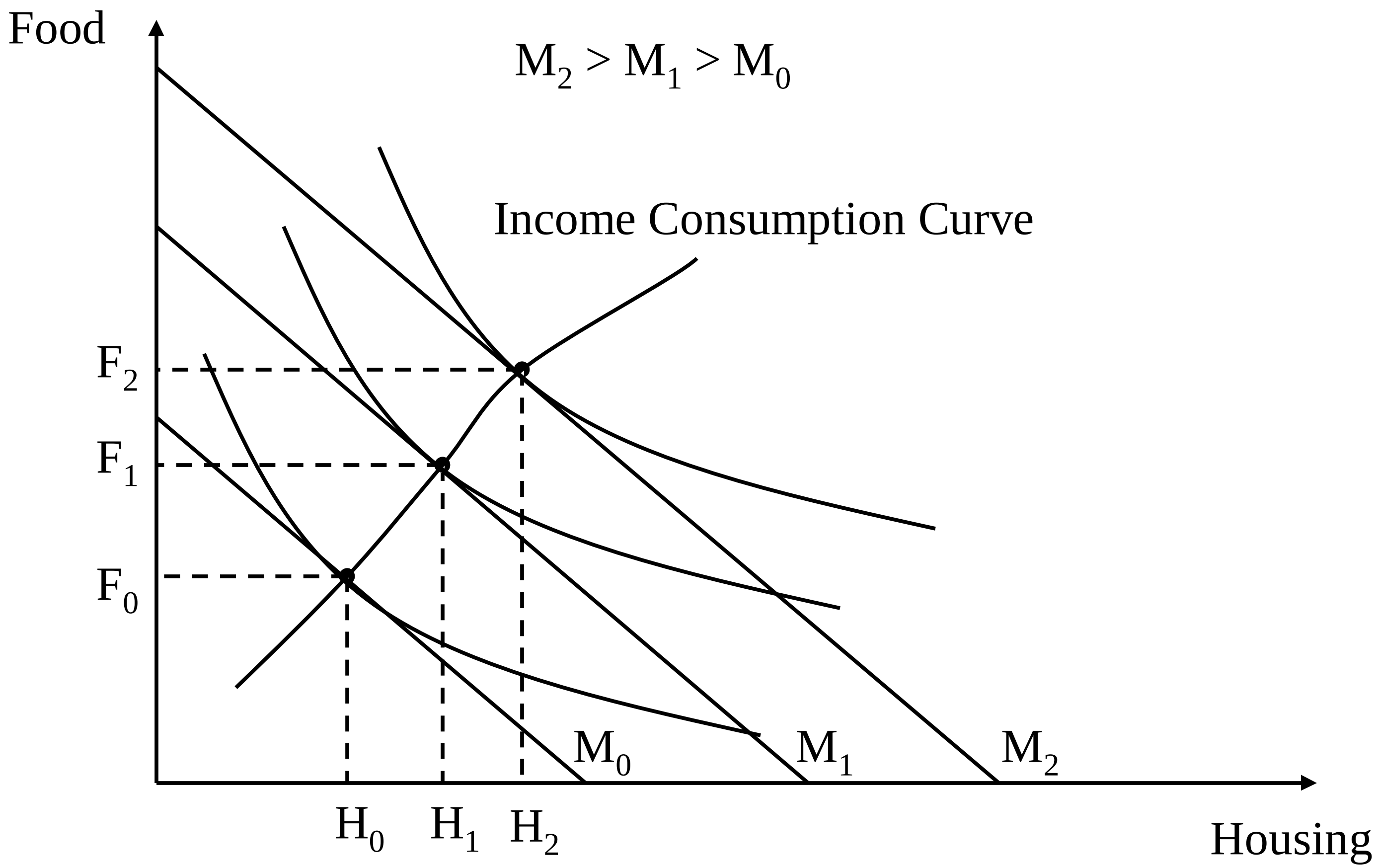

Figure 4: Engel Curve

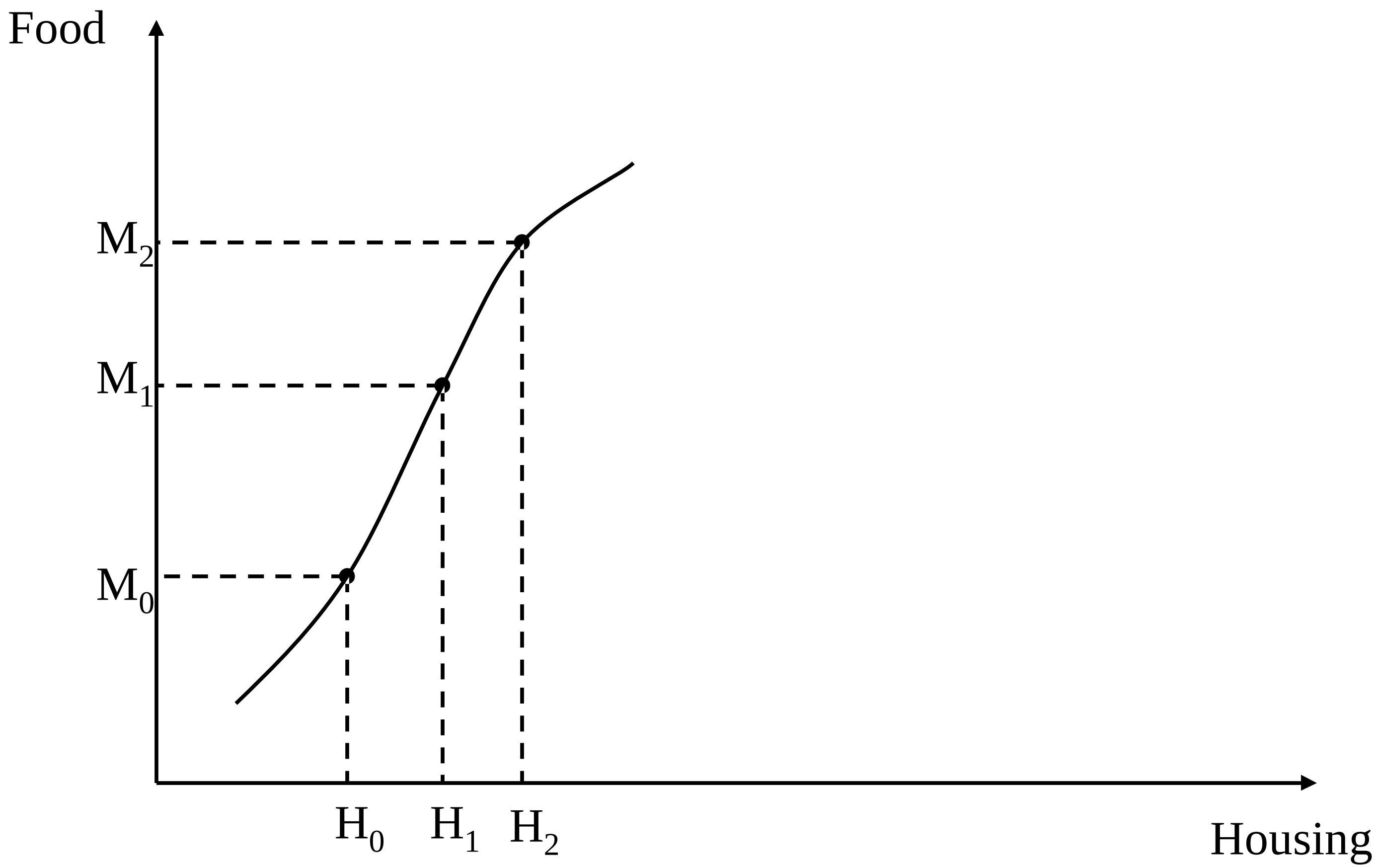

Figure 5: Substitution and Income Effects for an Increase in the Price of Housing

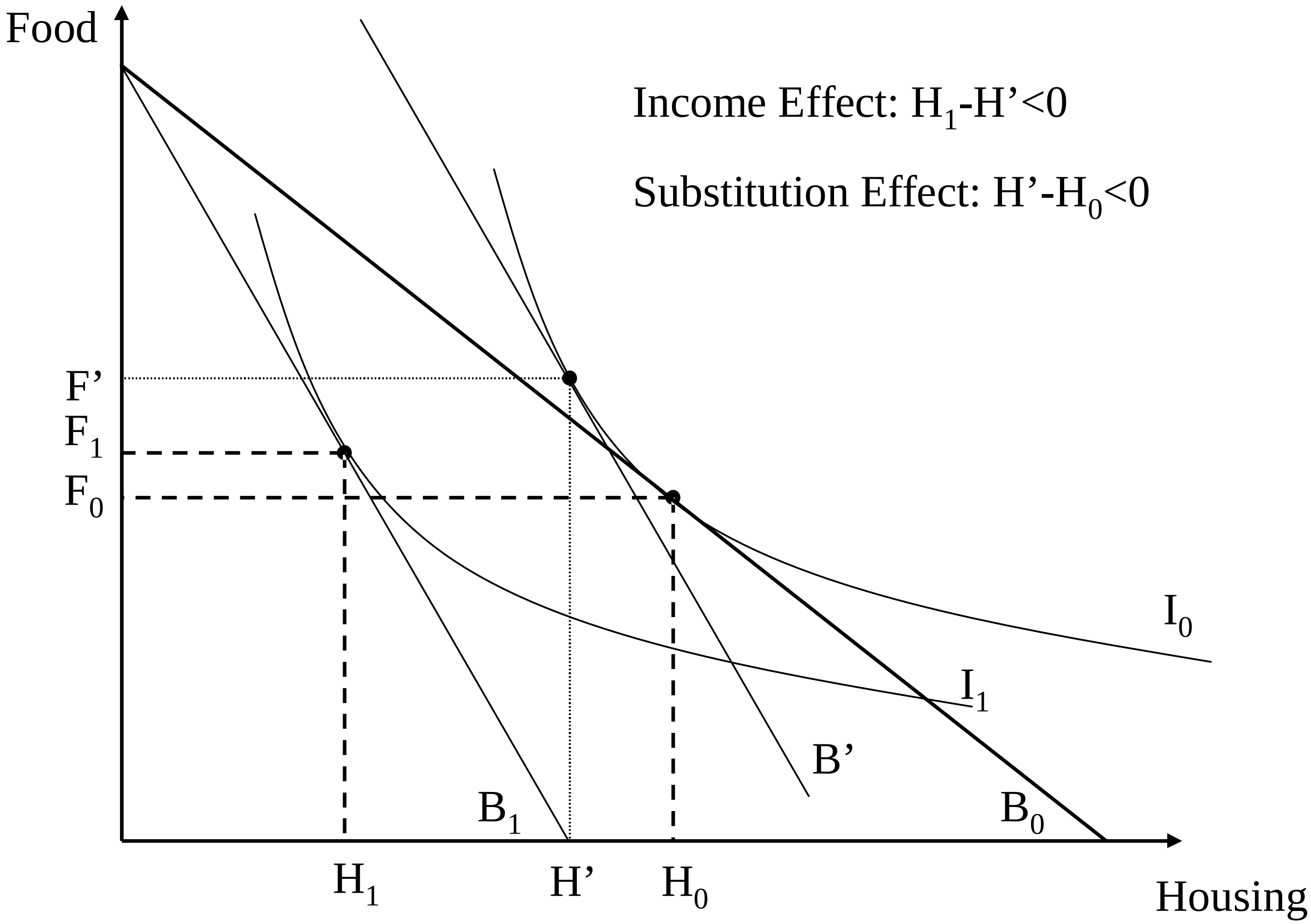

Figure 6: Substitution and Income Effects for a Change in the Price of Another Good: An Increase in the Price of Food

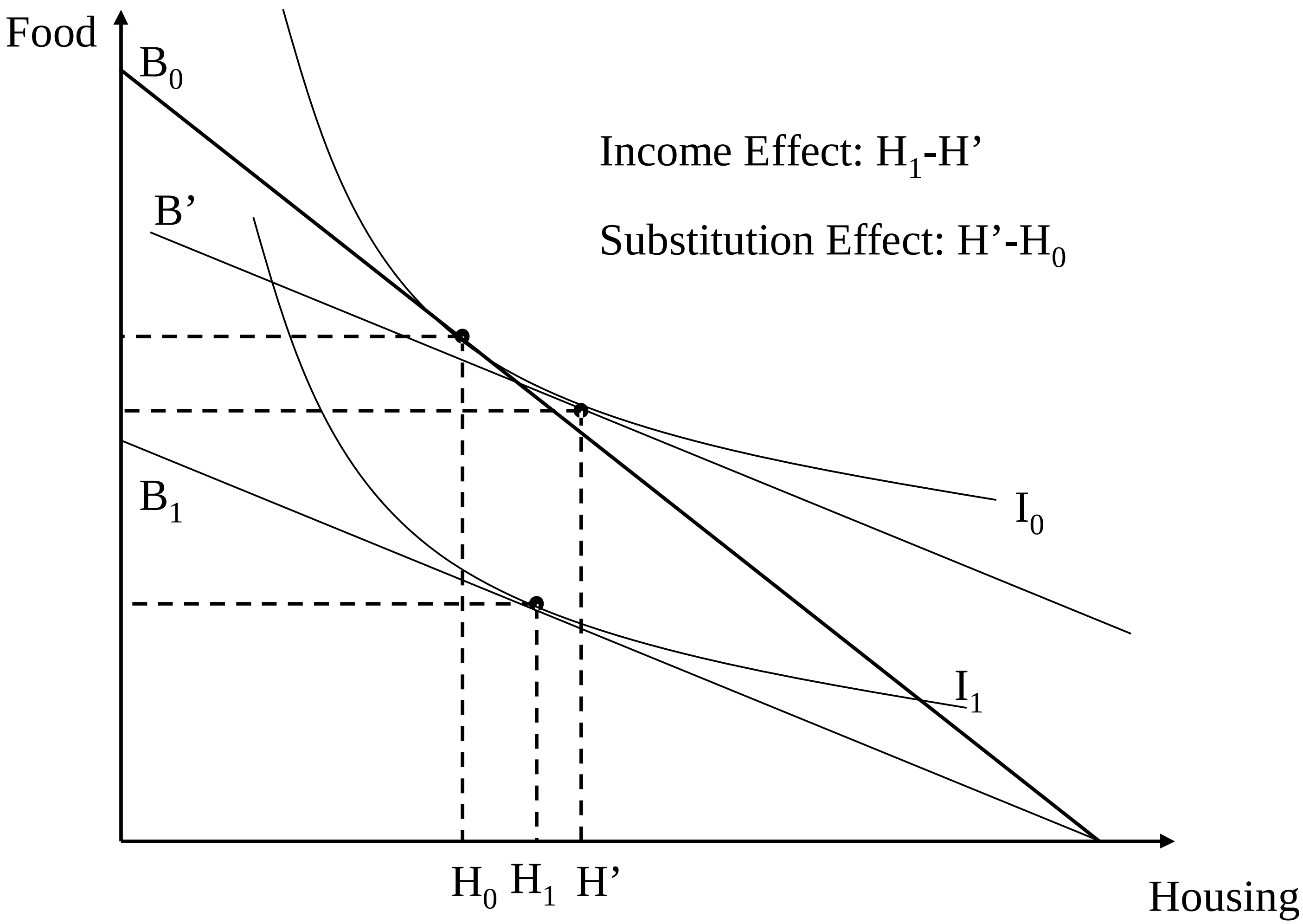

Figure 7a: Derivation of Market Demand

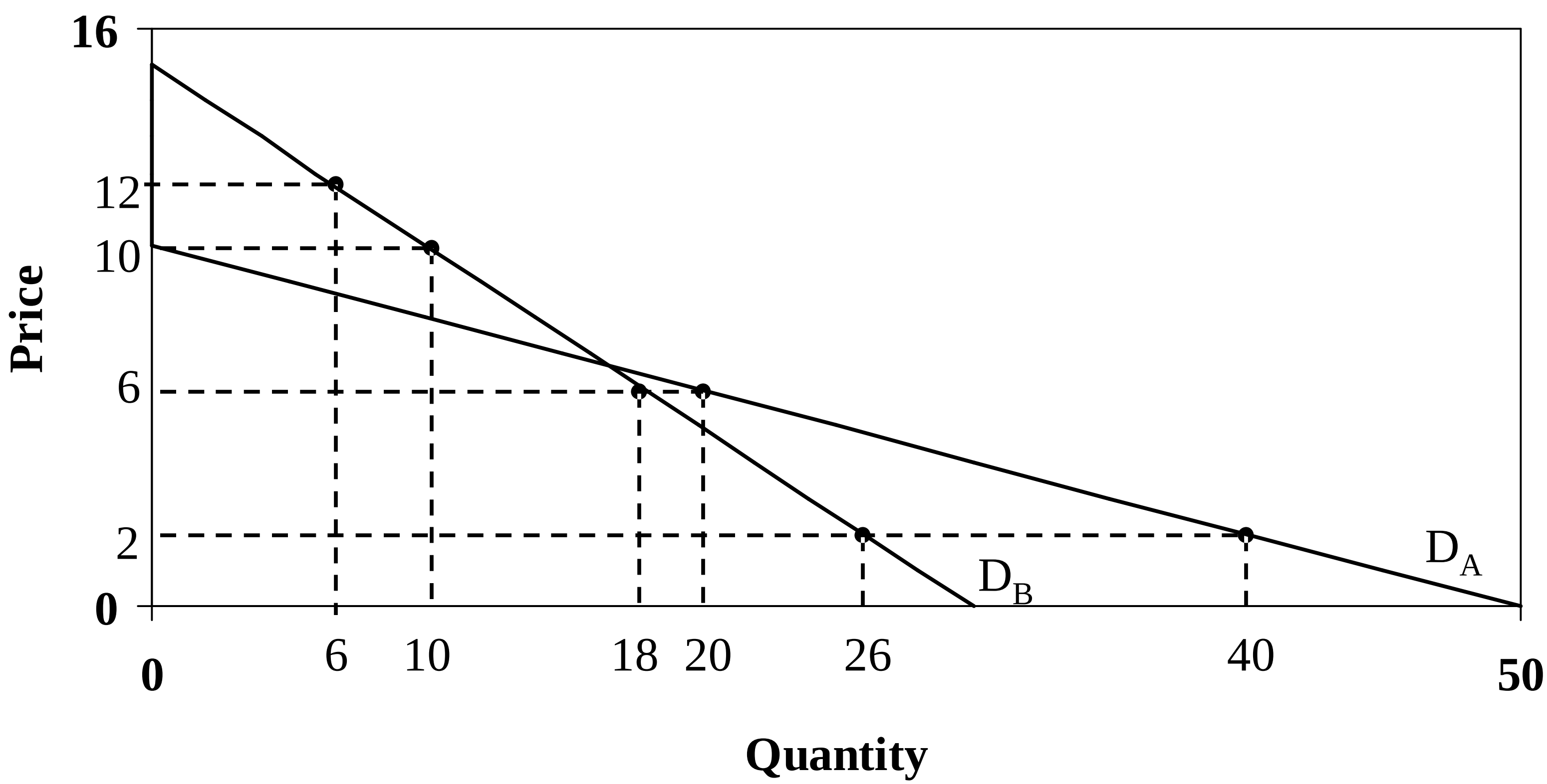

Figure 7b: Derivation of Market Demand

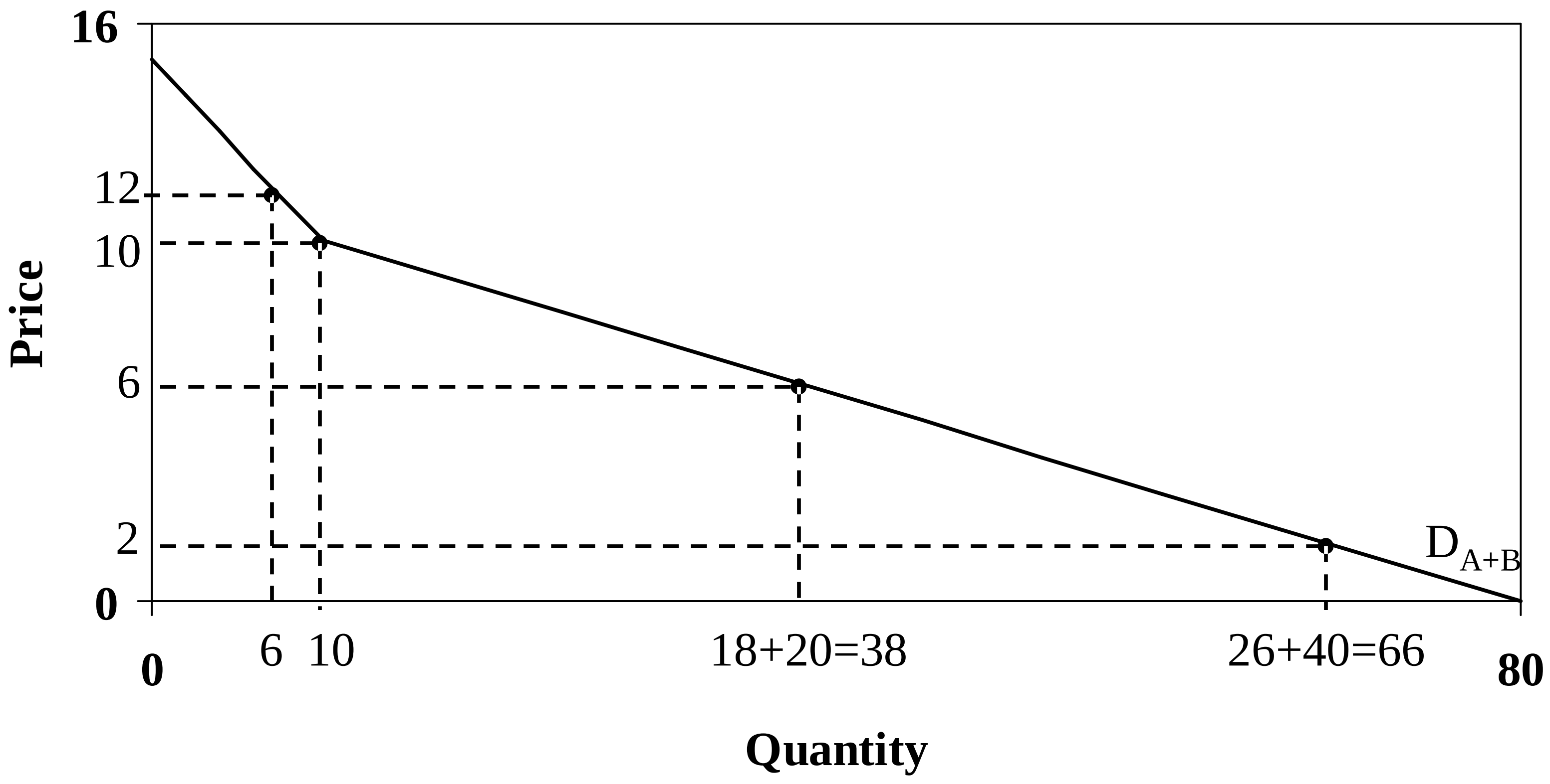

Figure 8: Elastic, Unit Elastic, and Inelastic Regions of a Linear Demand Curve

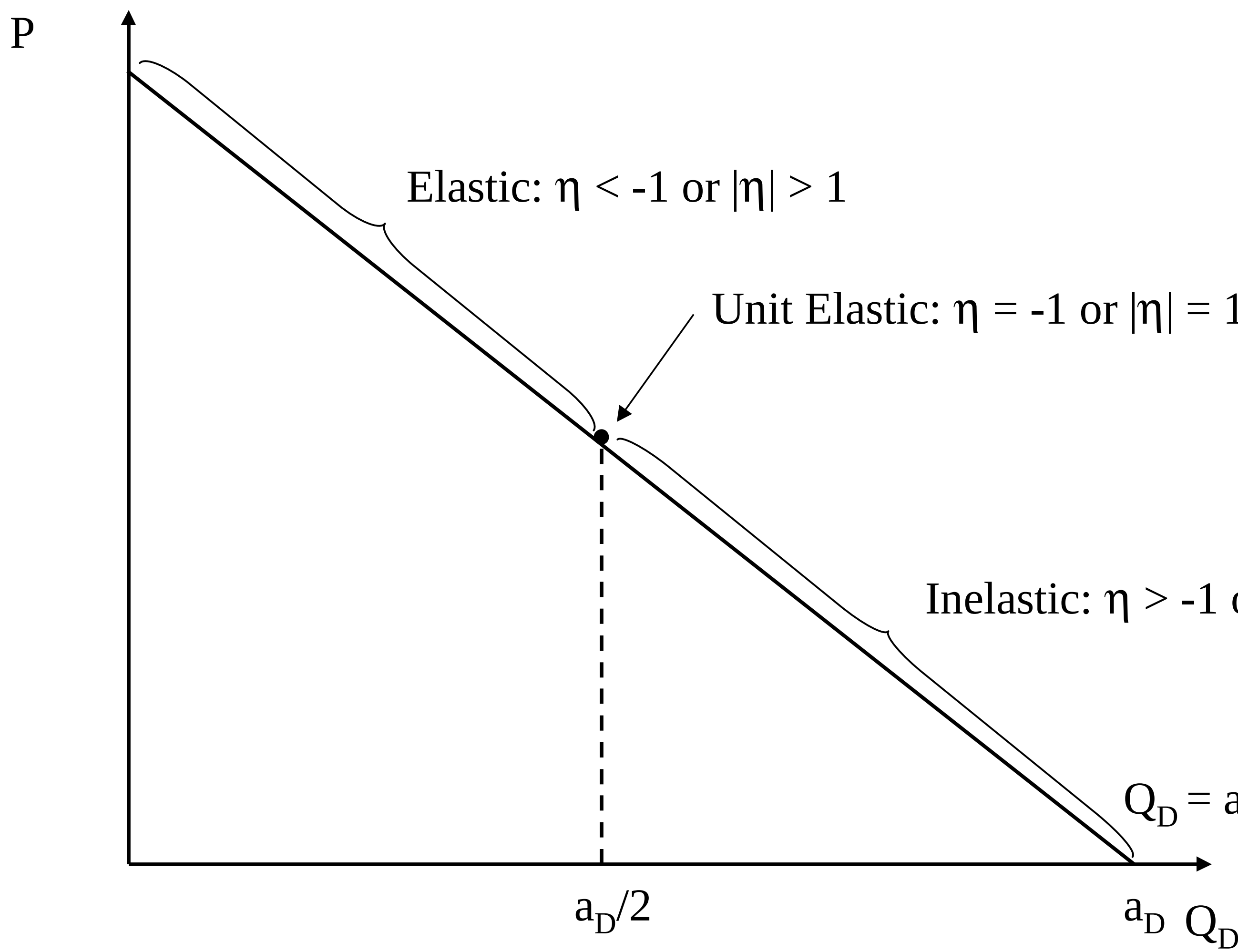

Figure 9: Relationship Between Total Revenue and the Elasticity of Demand with a Linear Demand Curve: $Q_D = a_D - b_D P$

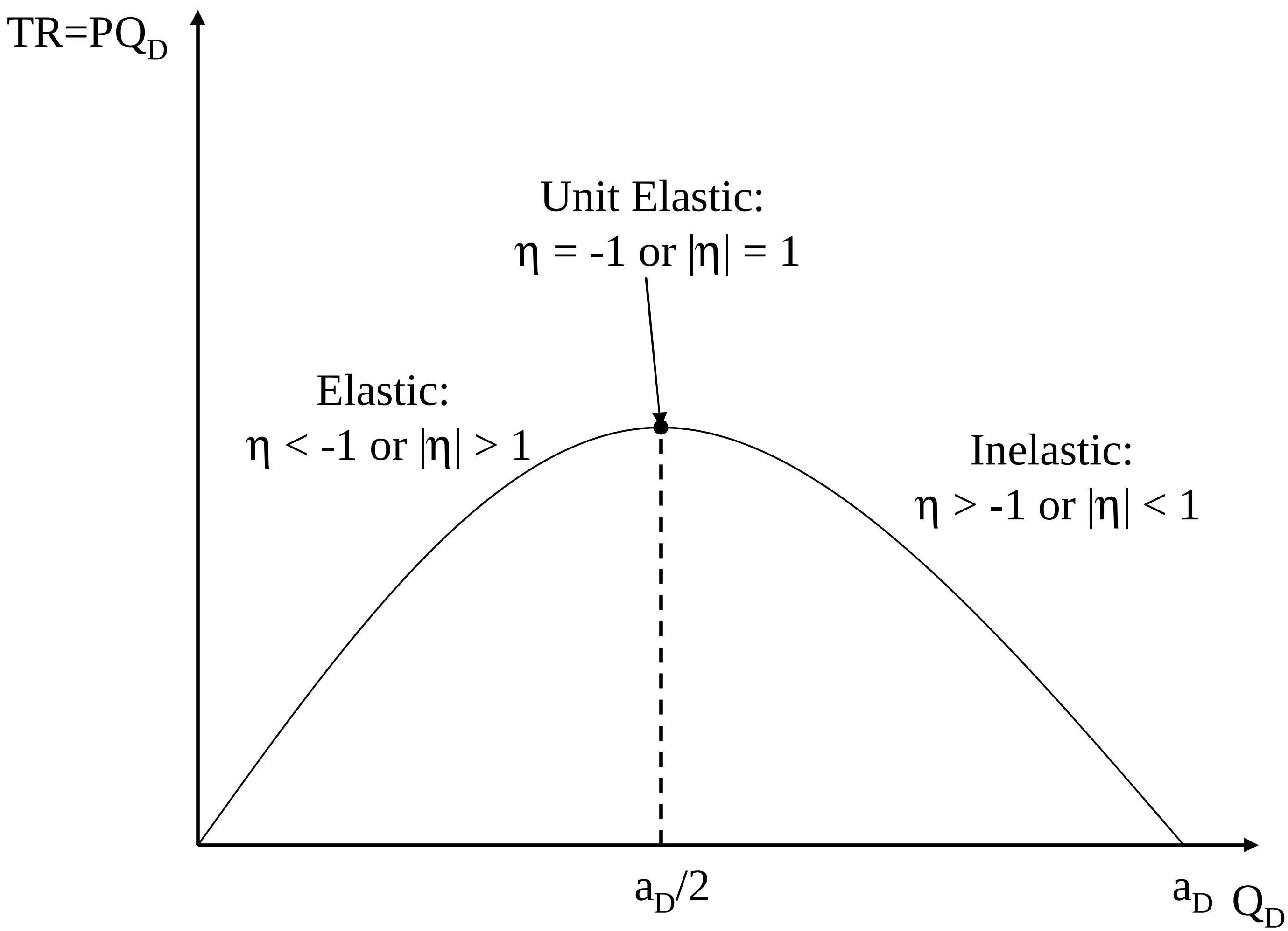

Figure 10: Perfectly Elastic Demand Curve

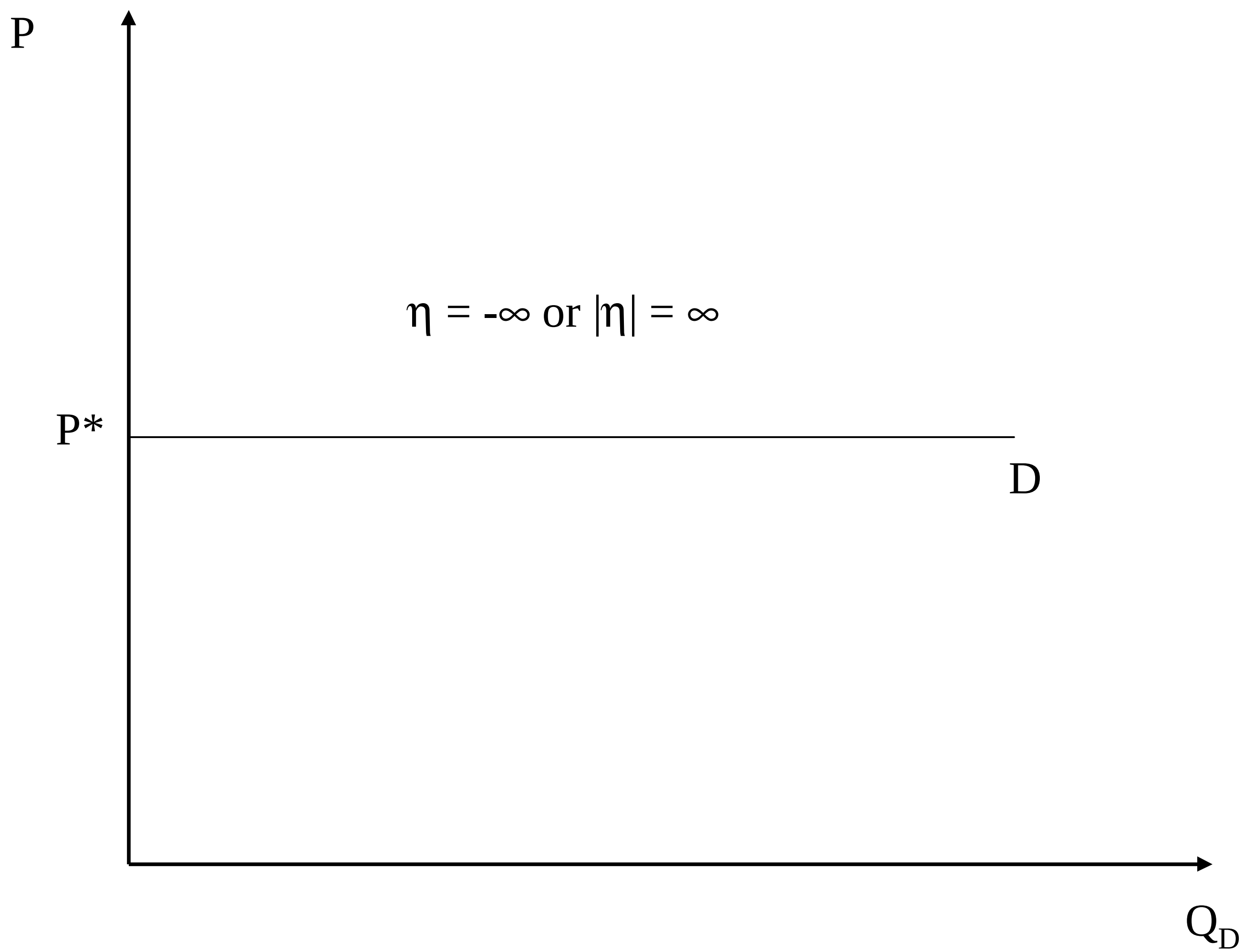

Figure 11: Perfectly Inelastic Demand Curve