

Costs

APEC 3001

Summer 2007

Readings: Chapter 10 & Appendix in Frank

Objectives

- Short Run Production Costs
- Long Run Production Costs
- Long Run Production Costs & Industry Structure

Short Run Production Costs Definitions

- Total Cost (TC):
 - All costs of production in the short run.
- Fixed Cost (FC):
 - Cost that does not vary with the level of output in the short run.
- Variable Cost (VC):
 - Cost that varies with the level of output in the short run.

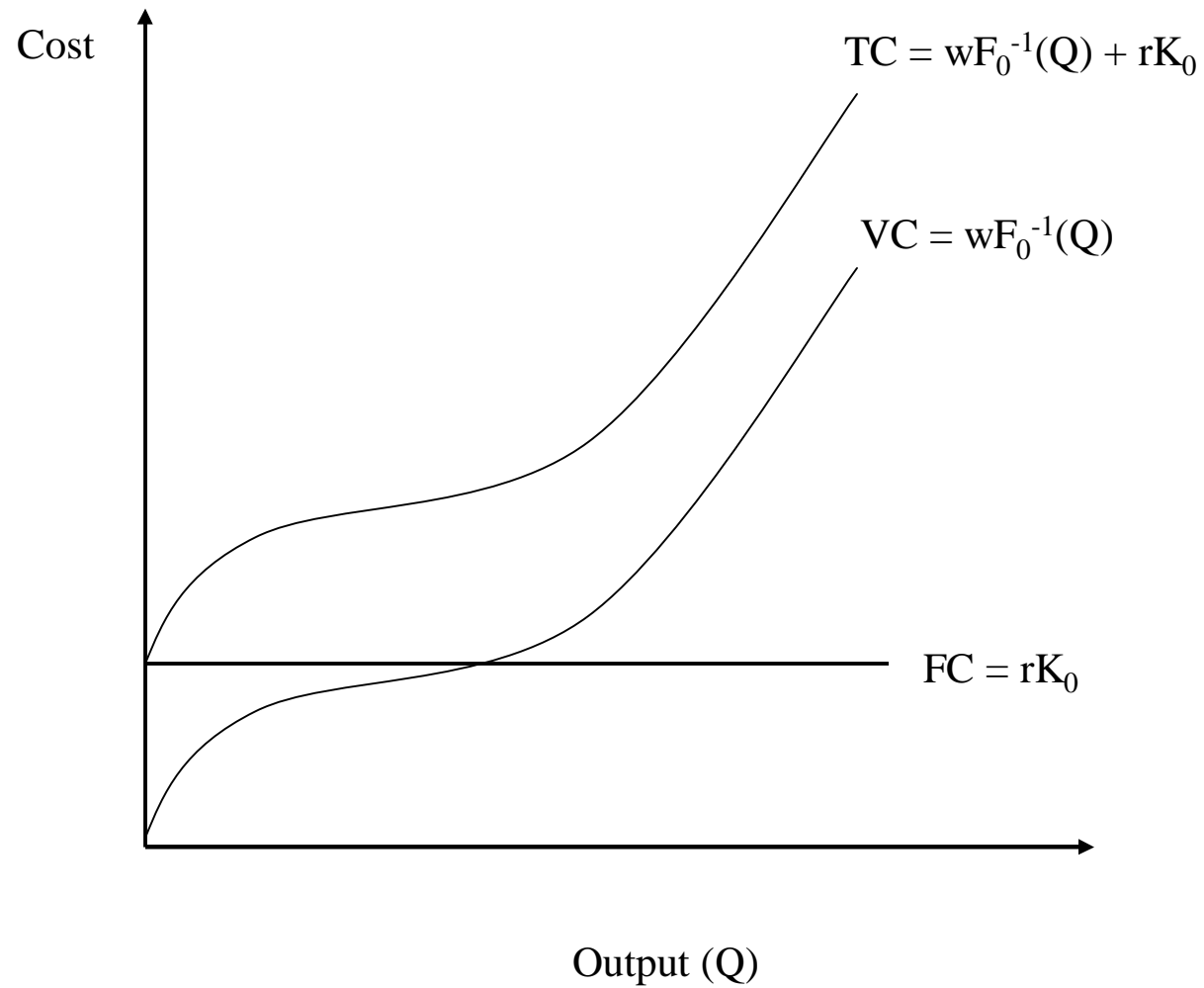
Important Relationship: $TC = VC + FC$

Derivation of Short Run Production Costs

- Suppose
 - we have only two inputs labor (L) & capital (K) such that $Q = F(K,L)$.
 - the price of labor is w & the price of capital is r .
- In the short run, some inputs are fixed, say $K = K_0$ such that $Q = F(K_0,L) = F_0(L)$.
 - $VC = wL = wF_0^{-1}(Q)$
 - $FC = rK_0$
 - $TC = wL + rK_0 = wF_0^{-1}(Q) + rK_0$

Important To Remember: TC & VC are functions of Q, not L or K! FC is not a function of Q, L, or K!

Example of Variable, Fixed, & Total Cost Curve



A Numeric Example of TC, VC, & FC

- Suppose
 - we have only two inputs labor (L) & capital (K) such that $Q = KL^{0.5}$.
 - the price of labor is $w = \$15$ & the price of capital is $r = \$25$.
- In the short run, some inputs are fixed, say $K = K_0 = 10$ such that $Q = 10L^{0.5}$ and $L = Q^2/10^2 = Q^2/100$.
 - $VC = wL = 15 \times Q^2/100 = 0.15Q^2$
 - $FC = rK = 25 \times 10 = 250$
 - $TC = wL + rK_0 = 0.15Q^2 + 250$

Important To Remember: TC & VC are functions of Q, not L or K! FC is not a function of Q, L, or K!

Short Run Production Costs

Some More Definitions

- Average Total Cost (ATC):
 - Total cost divided by the quantity of output, $ATC = TC/Q$.
- Average Fixed Cost (AFC):
 - Fixed cost divided by the quantity of output, $AFC = FC/Q$.
- Average Variable Cost (AVC):
 - Variable cost divided by the quantity of output, $AVC = VC/Q$.
- Marginal Cost (MC):
 - Change in total cost resulting from a one unit increase in output, $MC = \Delta TC/\Delta Q = \Delta VC/\Delta Q = wF_0^{-1}'(Q)$ where ΔTC , ΔVC , & ΔQ are the change in TC, VC, & Q.

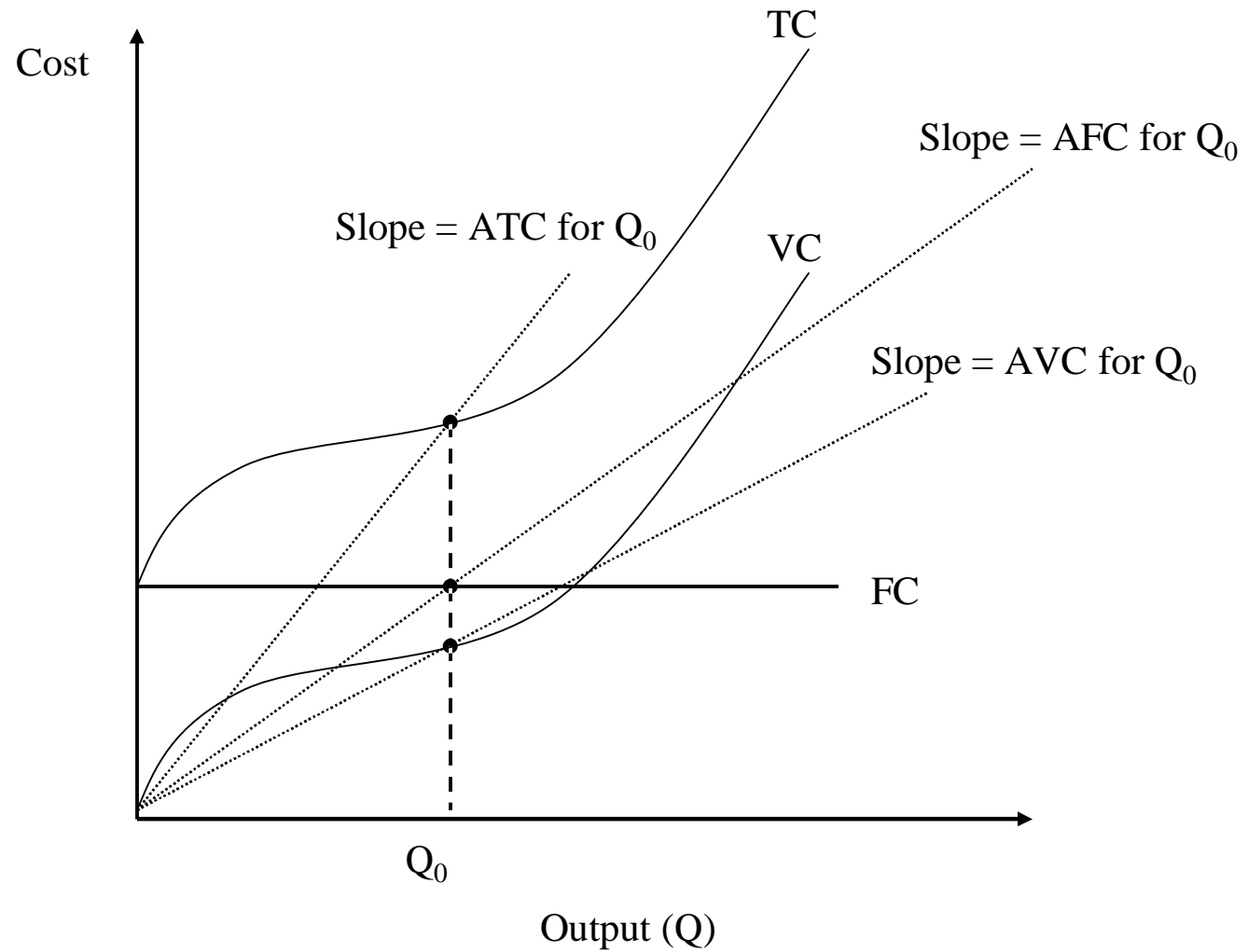
Important Relationship: $ATC = AVC + AFC$

**Important To Remember: ATC, AVC, & AFC
are functions of Q, not L or K!**

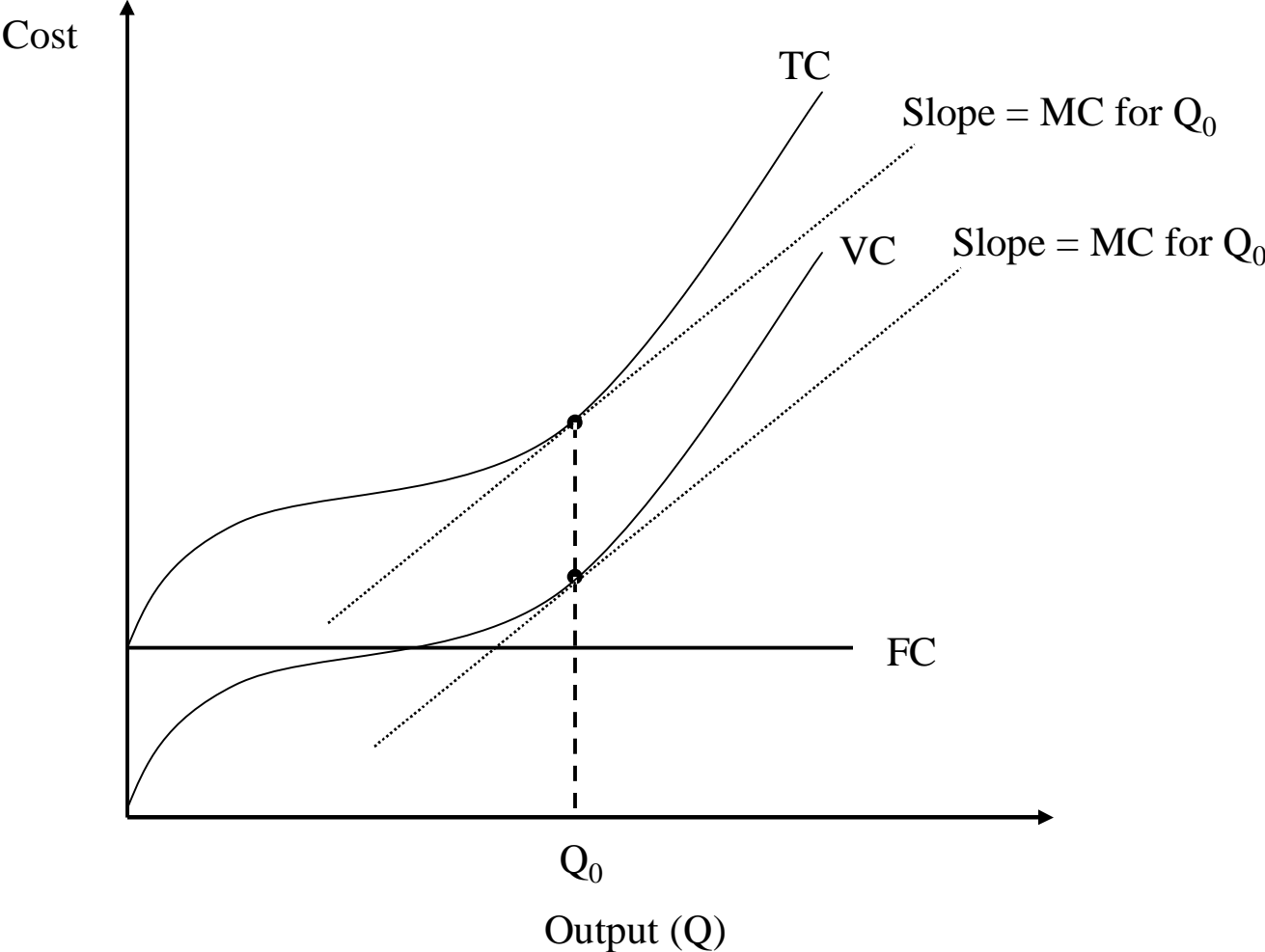
Graphical Interpretation of ATC, AVC, AFC & MC

- ATC is the slope of a line from origin to total cost curve.
- AVC is the slope of a line from origin to variable cost curve.
- AFC is the slope of a line from origin to fixed cost curve.
- MC is the slope of a line tangent to the total & variable cost curves.

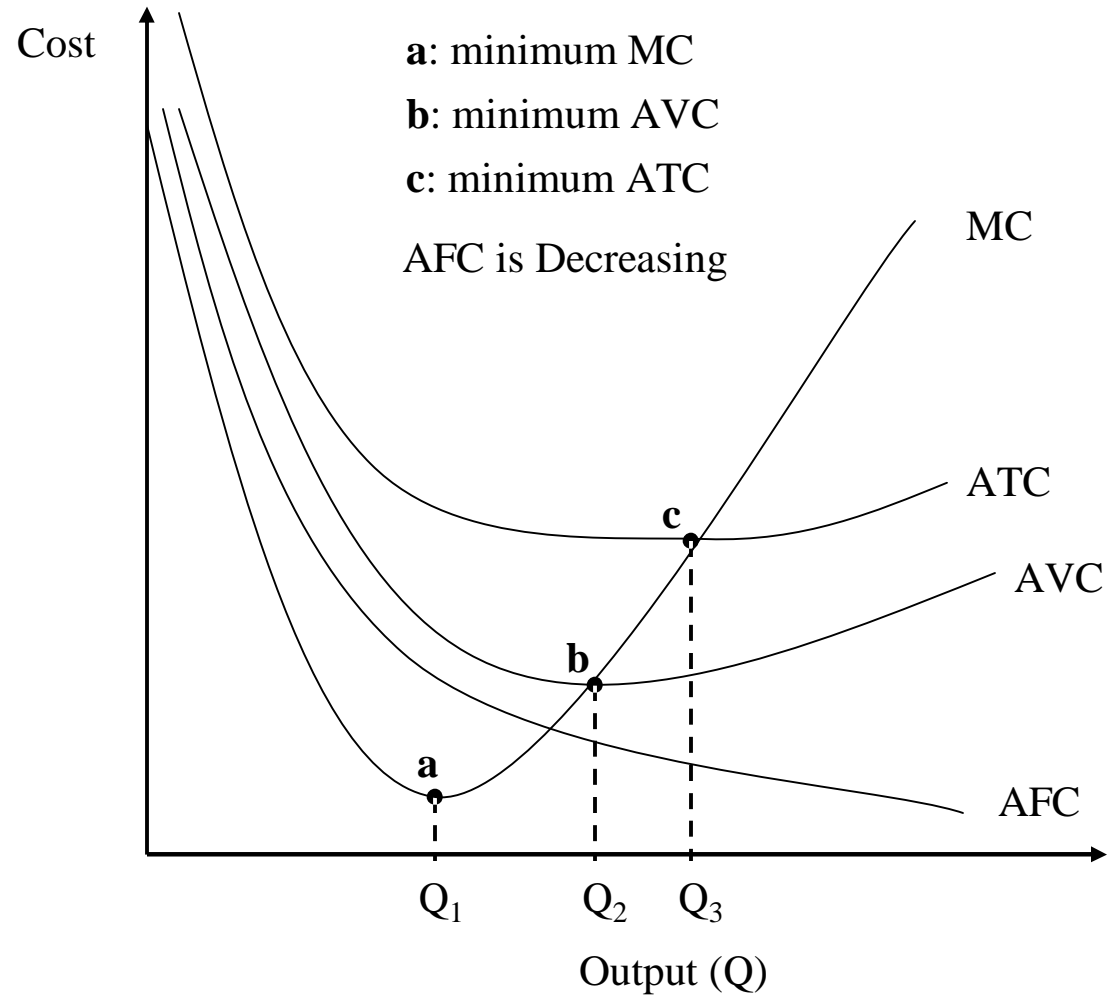
Graphical Relationships Costs & Average Costs



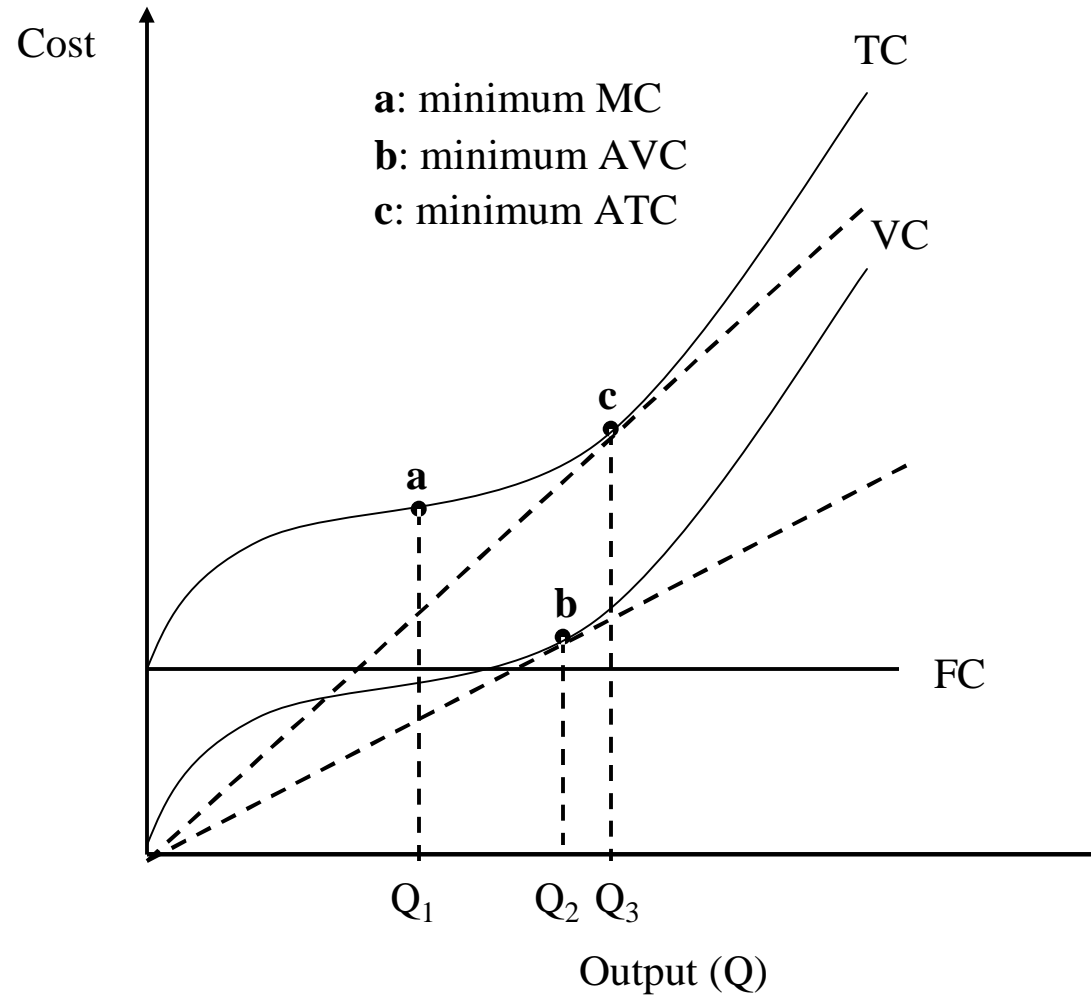
Graphical Relationships Costs & Marginal Costs



Relationship Between ATC, AVC, AFC, & MC



Relationship Between ATC, AVC, AFC, & MC



A Numeric Example of ATC, AVC, AFC, & MC

- Suppose
 - we have only two inputs labor (L) & capital (K) such that $Q = KL^{0.5}$.
 - the price of labor is $w = \$15$ & the price of capital is $r = \$25$.
- In the short run, some inputs are fixed, say $K = K_0 = 10$ such that $Q = 10L^{0.5}$ and $L = Q^2/10^2 = Q^2/100$.
 - $VC = wL = 15 \times Q^2/100 = 0.15Q^2 \Rightarrow AVC = VC/Q = 0.15Q^2/Q = 0.15Q$
 - $FC = rK = 25 \times 10 = 250 \Rightarrow AFC = FC/Q = 250/Q$
 - $TC = wL + rK_0 = 0.15Q^2 + 250 \Rightarrow ATC = TC/Q = (0.15Q^2 + 250)/Q = 0.15Q + 250/Q$
 - $MC = TC' = VC' = 2 \times 0.15Q^{2-1} = 0.3Q$

**Important To Remember: ATC, AVC, & AFC
are functions of Q, not L or K!**

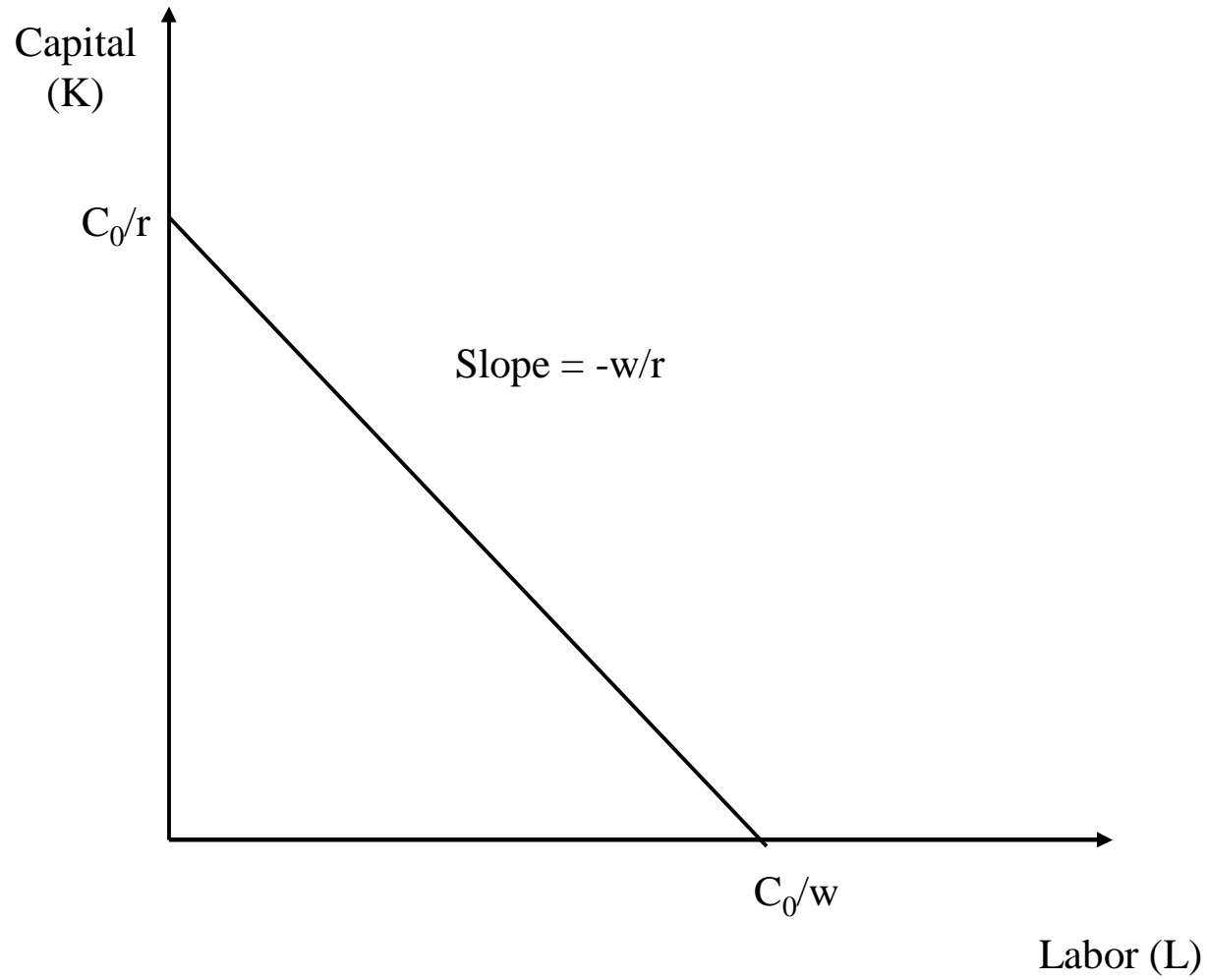
A Few More Important Relationships

- $MC = w/MP_L$
 - When MC is at a minimum, MP_L is at a maximum.
- $AVC = wL/Q$ & $AP_L = Q/L$, so $AVC = w/AP_L$
 - When AVC is at a minimum, AP_L is at a maximum.

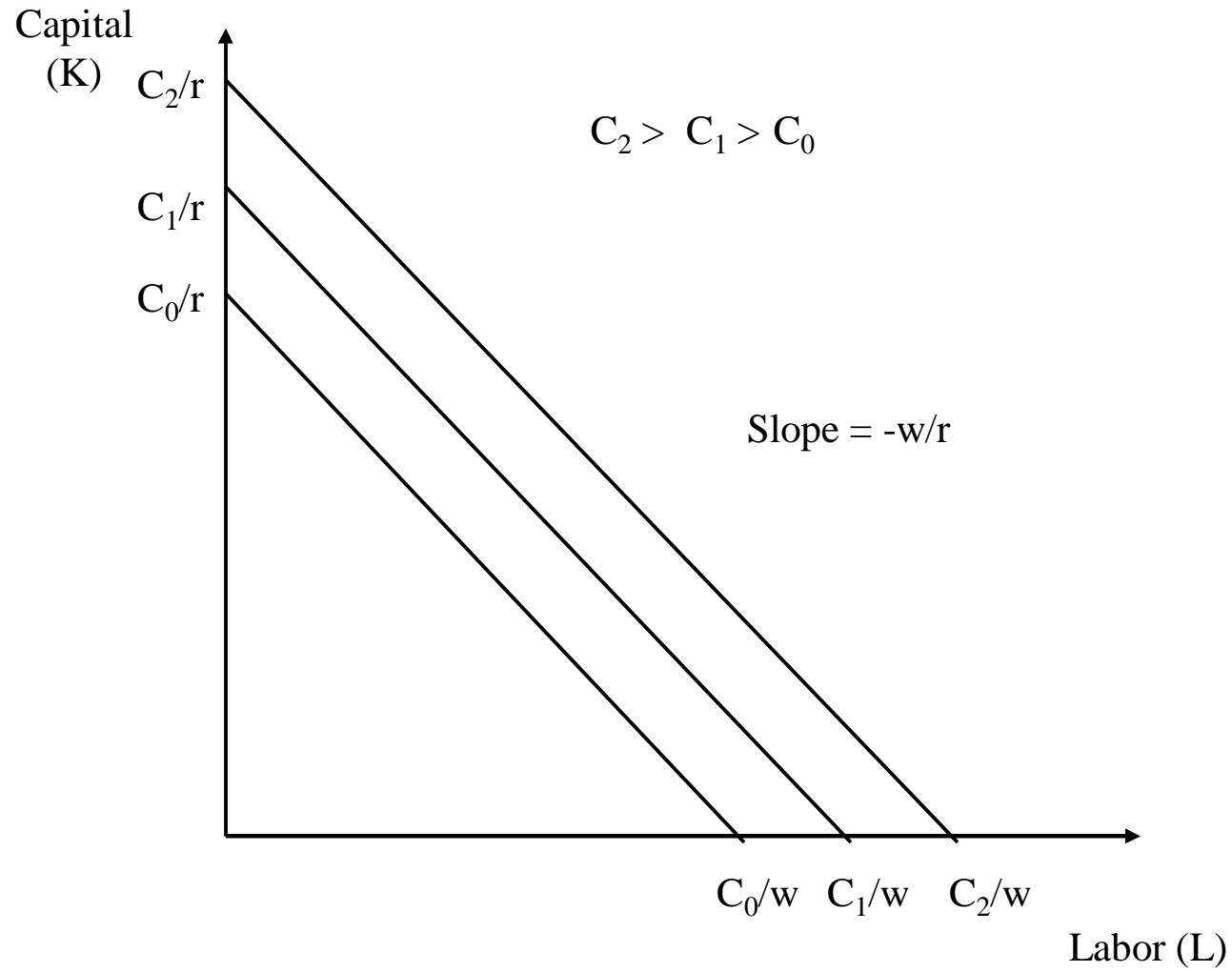
Long Run Production Costs

- In the long run, there are no fixed costs!
- Suppose
 - there are only two inputs labor (L) & capital (K).
 - the price of labor is w and the price of capital is r .
 - Long Run Total Costs (LTC) then equals total expenditures on labor & capital: $LTC = wL + rK$.
- Definition
 - Isocost Curve:
 - All combinations of inputs that result in the same cost of production.
 - If LTC is set constant to say C_0 , $C_0 = wL + rK \Rightarrow K = C_0/r - wL/r$.
 - Equation of a Line
 - Intercept = C_0/r
 - Slope = $-w/r$

Graphical Example of Isocost Curve



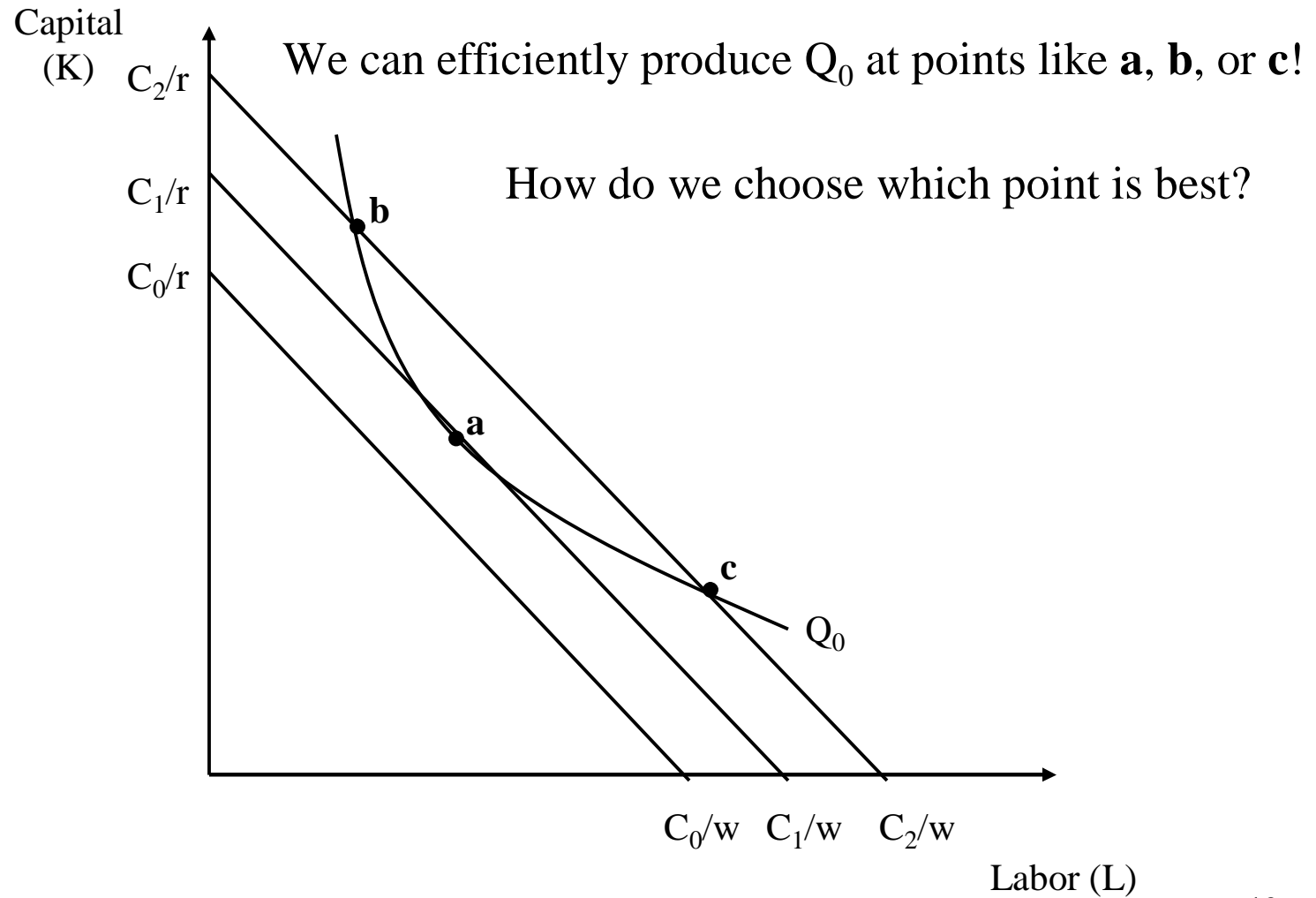
Graphical Example of Isocost Map



Long Run Production Costs

- Question: If a firm wants to produce some level of output, say Q_0 , how much L & K should it use?
- Remember
 - Isoquants tells us the most efficient combinations of inputs for producing some level of output.
- So, why not look at our isoquant with our isocost curves?

Example Isocost Map and Isoquant



Important Assumption for Long Run Production Costs

- Choose a combination of inputs that minimize costs!
 - Point **a** is better than point **b** or **c** because costs are lower!
 - But is there a point that is better than **a**?
 - No! To decrease costs below C_1 we must reduce L , K , or both, but monotonicity implies that reducing L , K , or both must reduce output below Q_0 .
 - What condition holds at point **a**?
 - The isocost curve for C_1 is just tangent to the isoquant for Q_0 .
 - The slope of the isocost curve for C_1 equals the slope of the isoquant for Q_0 :
 $MRTS = w/r$.

Intuitive Interpretation of $MRTS = w/r$

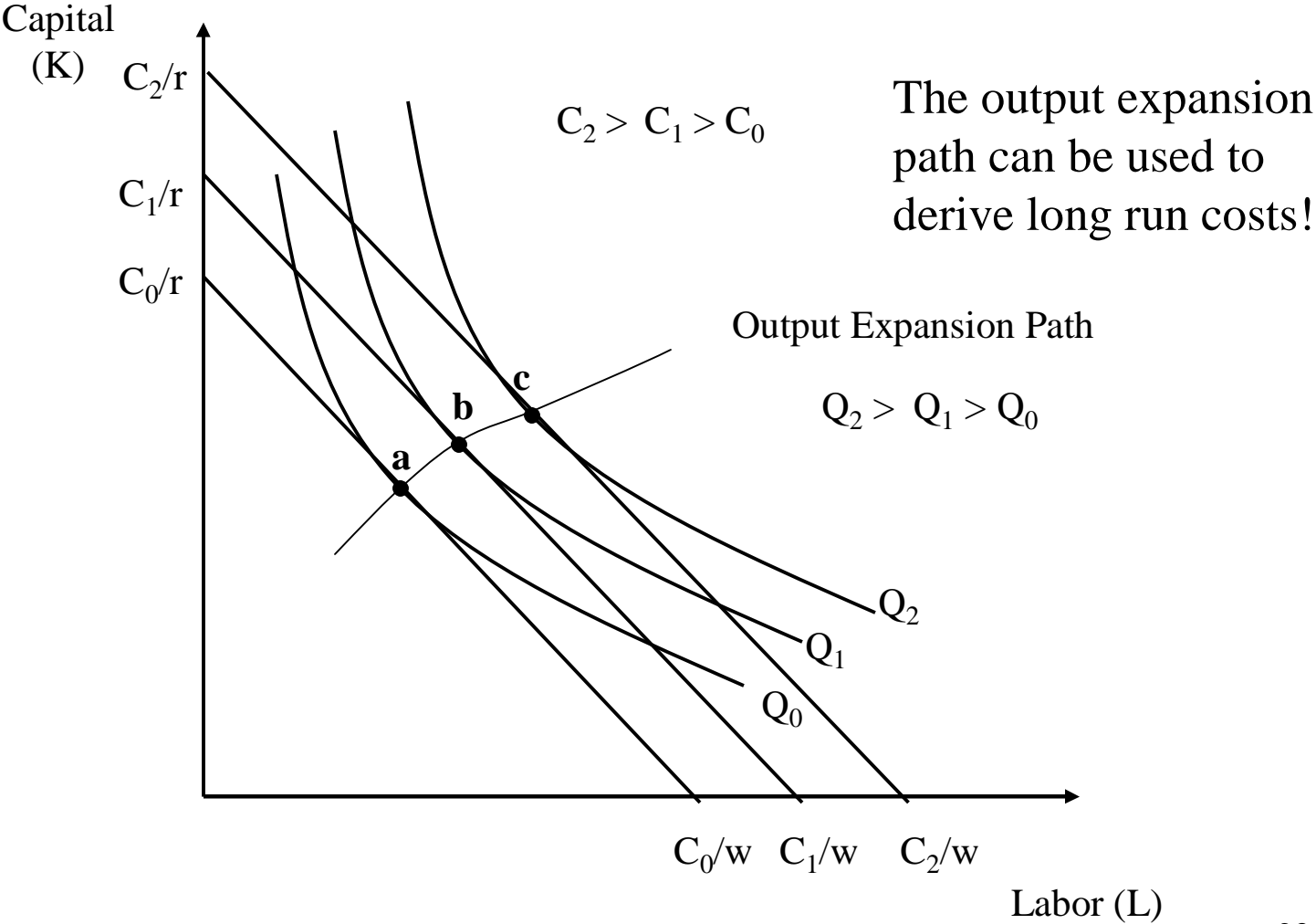
- $MRTS = MP_L/MP_K$
- $MRTS = w/r \Rightarrow MP_L/MP_K = w/r \Rightarrow MP_L/w = MP_K/r$
 - MP_L/w is the increase in output for an extra \$1 spent on L.
 - MP_K/r is the increase in output for an extra \$1 spent on K.
 - Cost are minimized when the increase in output for a \$1 spent on L just equals the increase in output for a \$1 spent on K.
- $MRTS > w/r \Rightarrow MP_L/w > MP_K/r$
 - We can reduce costs & produce the same level of output by using more L & less K.
- $MRTS < w/r \Rightarrow MP_K/r > MP_L/w$
 - We can reduce costs & produce the same level of output by using more K & less L.

Long Run Production Costs

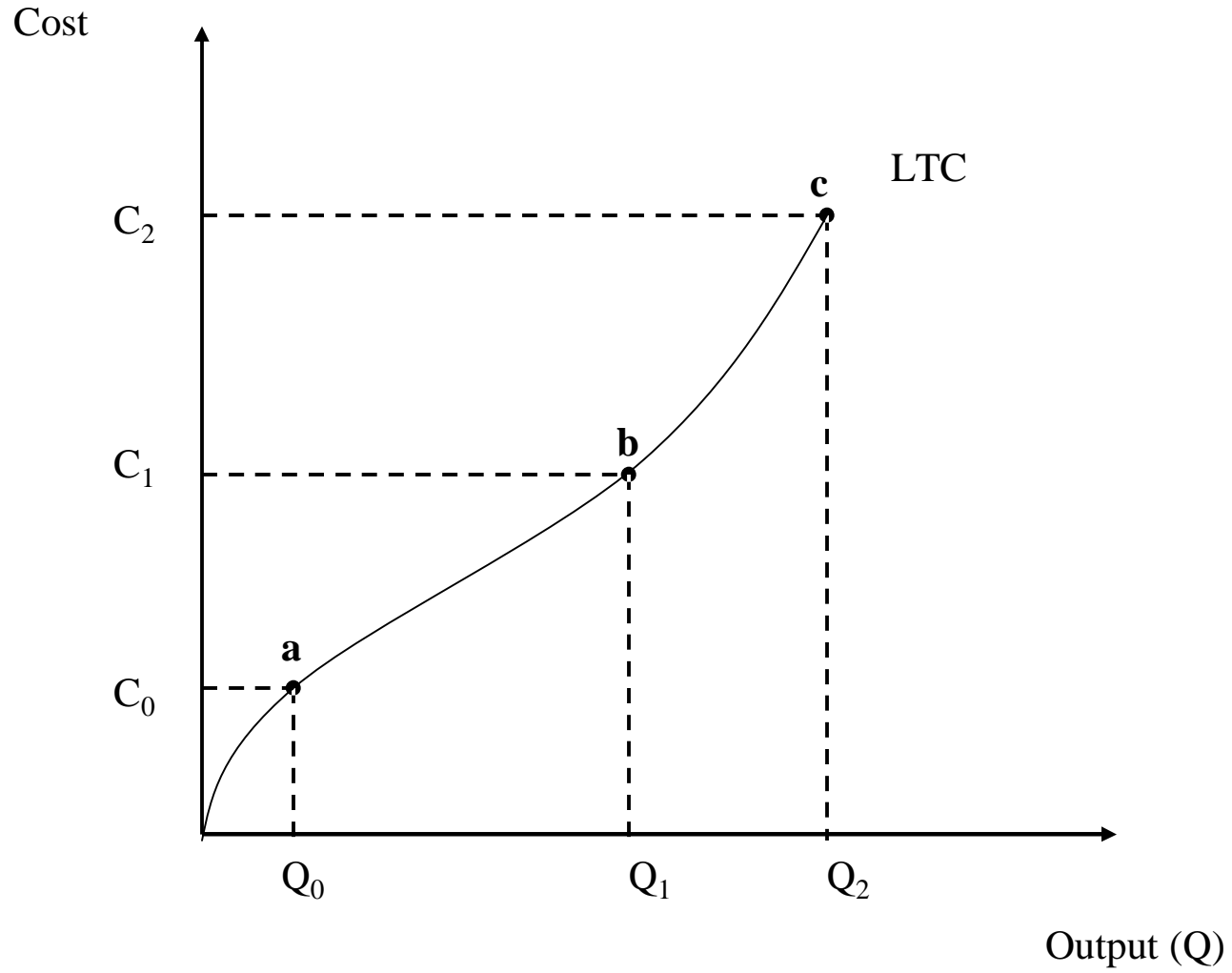
Another Definition

- Output Expansion Path:
 - The locus of tangencies (minimum cost input combinations) traced out by an isocost line of a given slope as it shifts outward into the isoquant map for the production process.

Output Expansion Path



Long Run Total Cost Curve

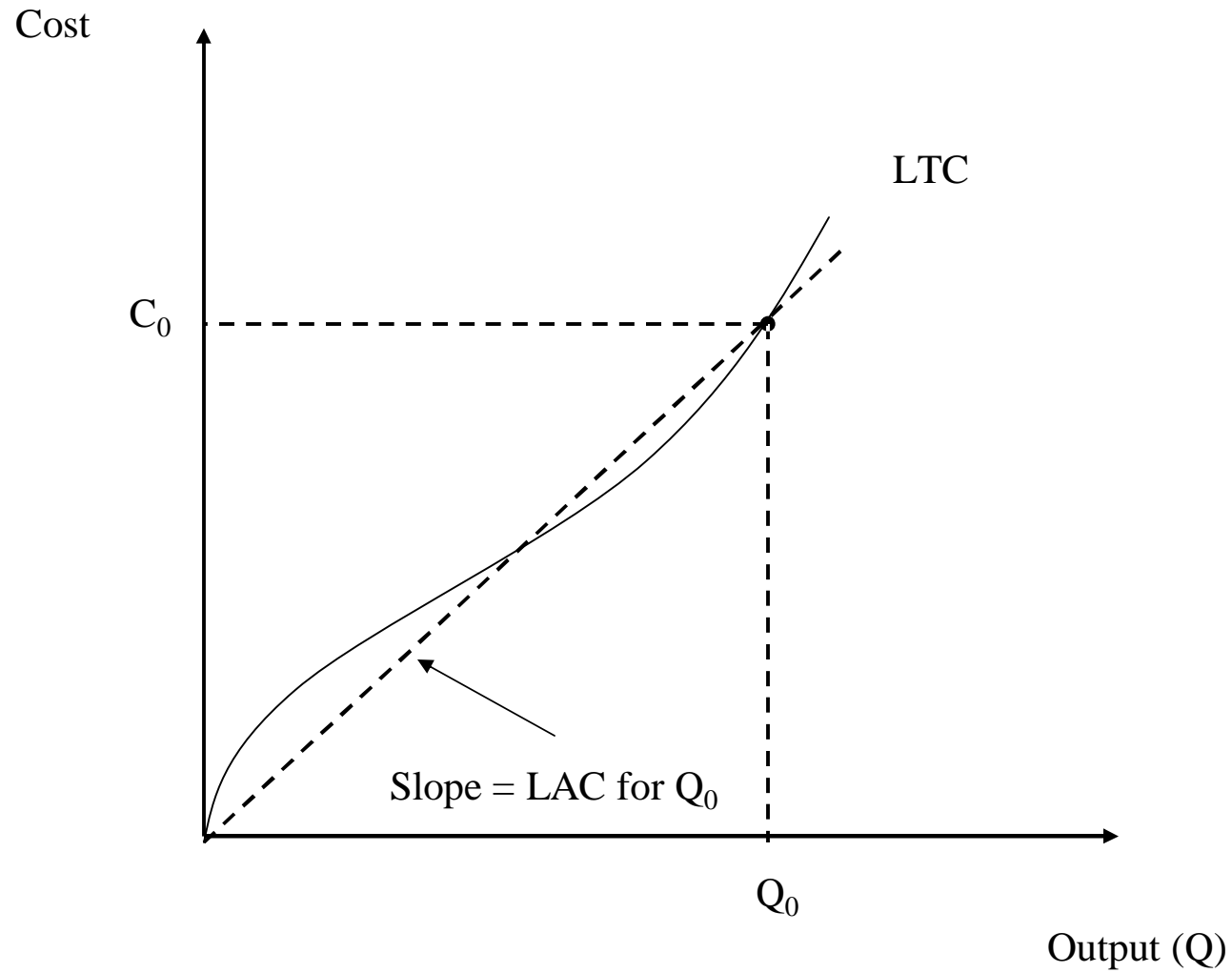


Long Run Production Costs

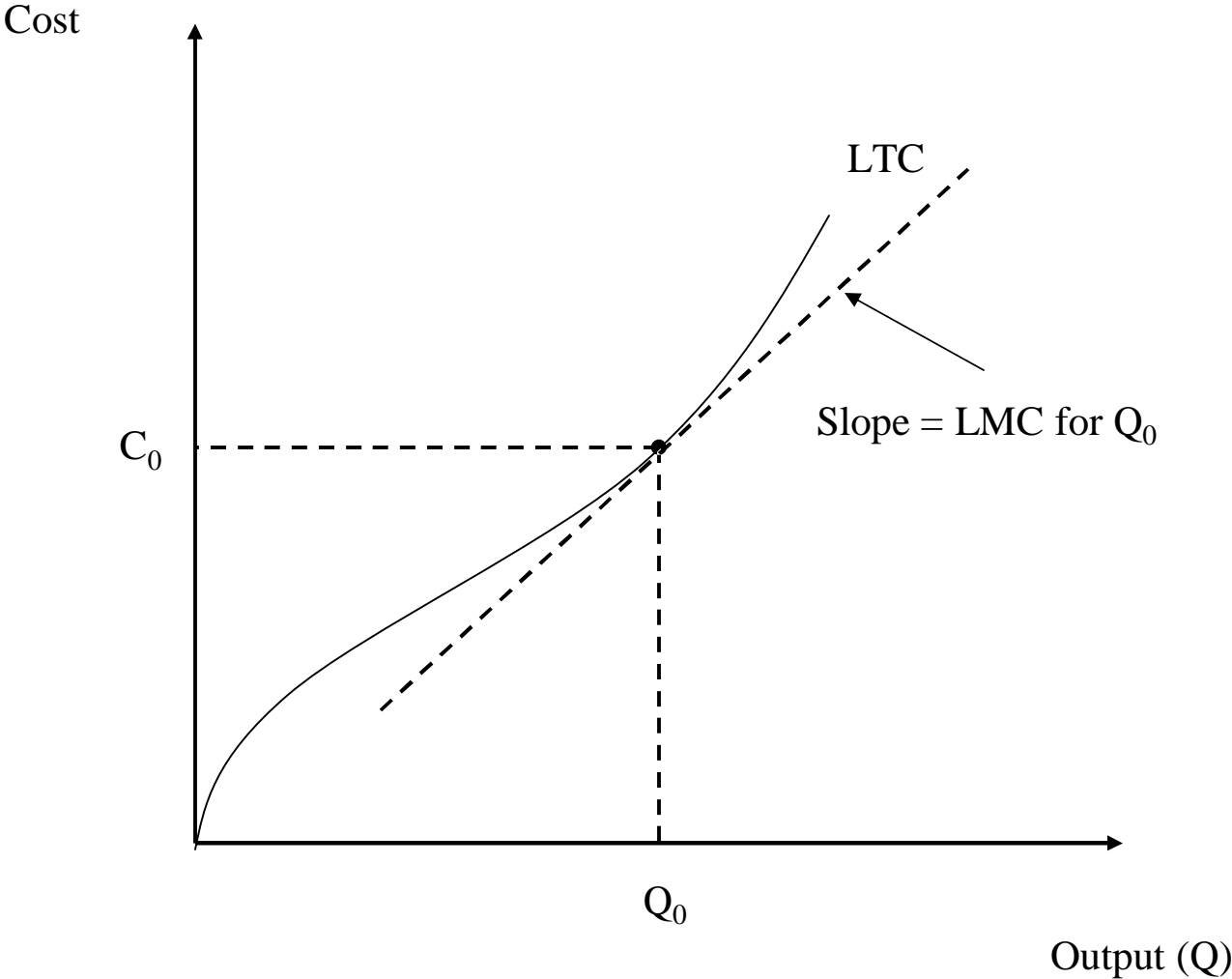
Some More Definitions

- Long Run Average Cost (LAC):
 - Long run cost divided by the quantity of output, $LAC = LTC/Q$.
- Long Run Marginal Cost (LMC):
 - Change in long run cost resulting from a one unit increase in output, $LMC = \Delta LTC/\Delta Q = LTC'$.

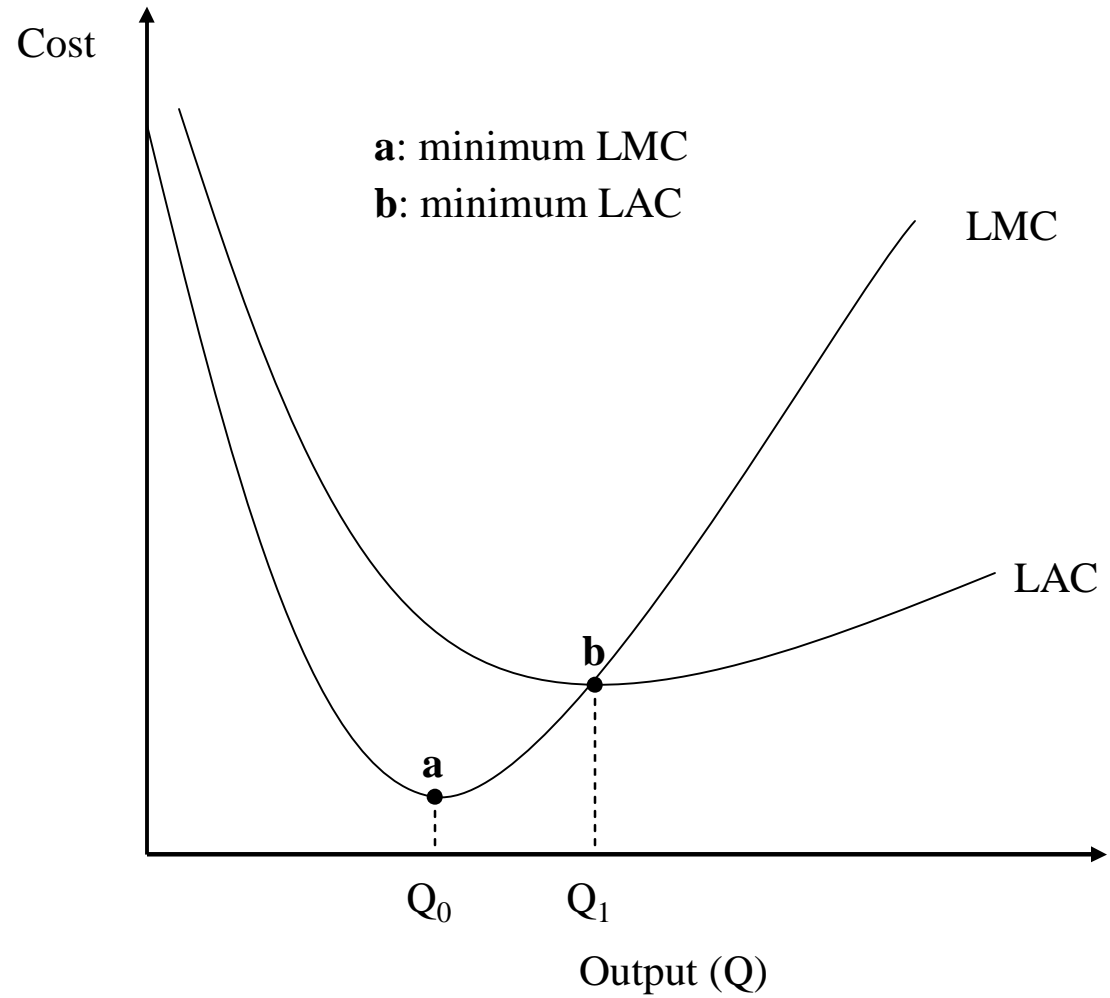
Graphical Interpretation of Long Run Average Cost



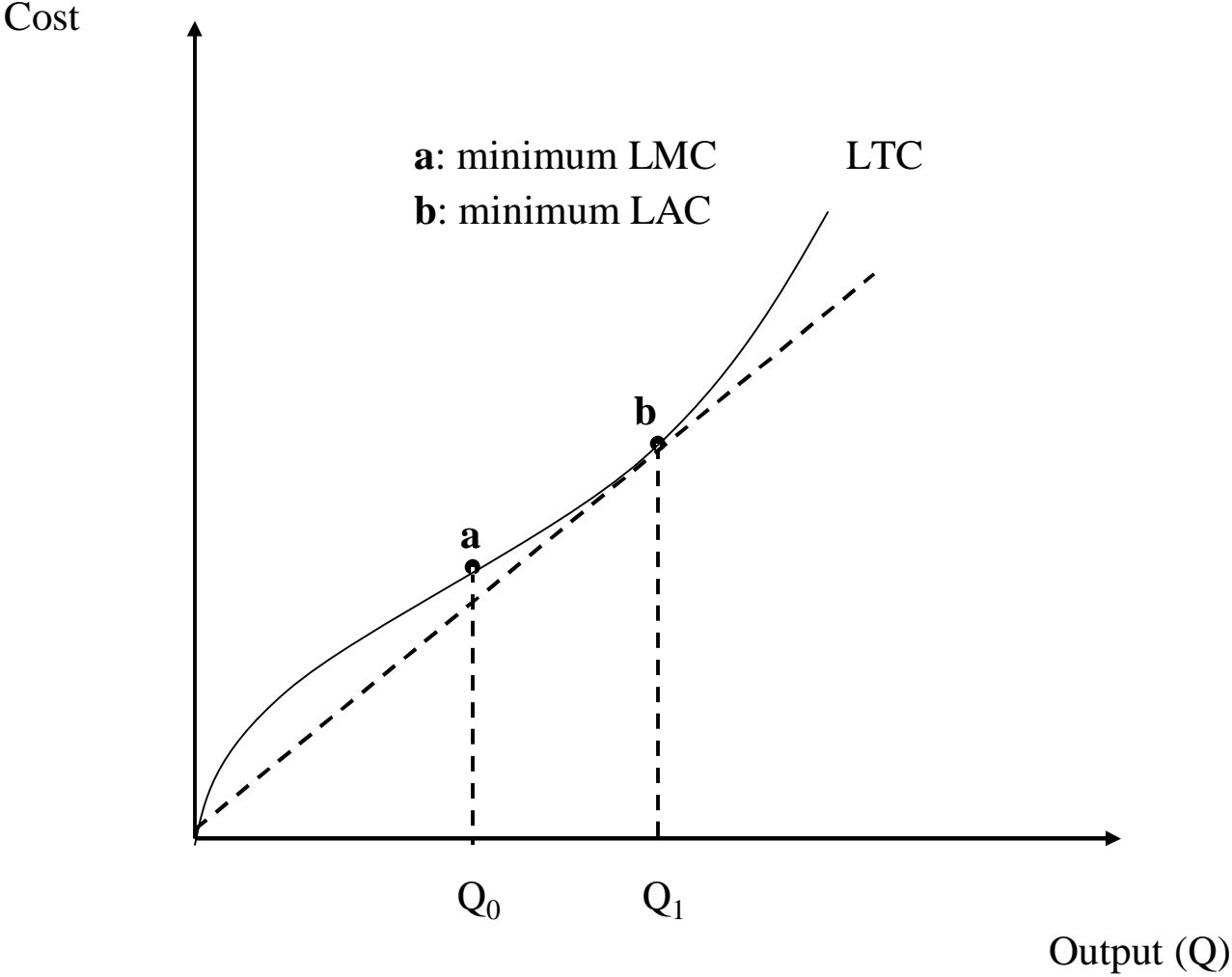
Graphical Interpretation of Long Run Marginal Cost



Long Run Average and Marginal Cost



Graphical Interpretation of Long Run Marginal Cost



A Numeric Example of ATC, AVC, AFC, & MC

- Suppose
 - we have only two inputs labor (L) & capital (K) such that $Q = KL$.
 - the price of labor is $w = \$10$ & the price of capital is $r = \$40$.

$$\text{MRTS} = \frac{\text{MP}_L}{\text{MP}_K} = \frac{K}{L} \qquad \text{MRTS} = \frac{w}{r} \Rightarrow \frac{K}{L} = \frac{10}{40} \Rightarrow K = 0.25L$$

$$Q = KL = 0.25LL = 0.25L^2 \Rightarrow L = (4Q)^{0.5} = 2Q^{0.5} \Rightarrow K = 0.25 \times 2Q^{0.5} = 0.5Q^{0.5}$$

$$\text{LTC} = wL + rK = 10 \times 2Q^{0.5} + 40 \times 0.5Q^{0.5} = 40Q^{0.5}$$

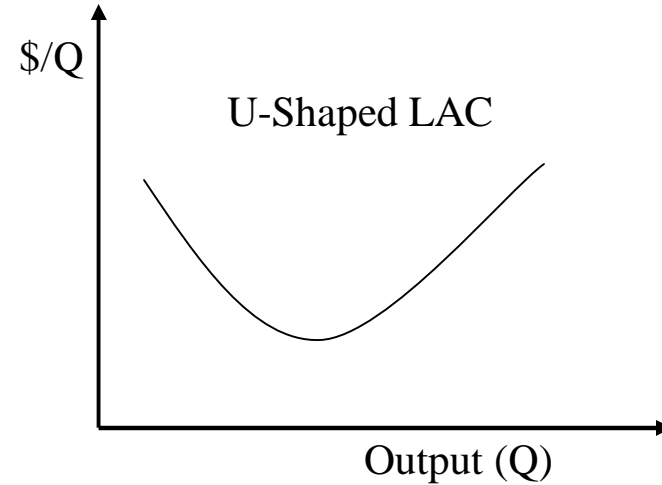
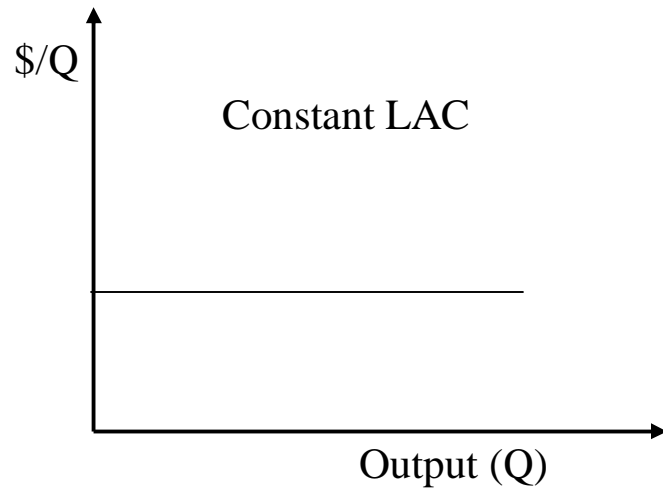
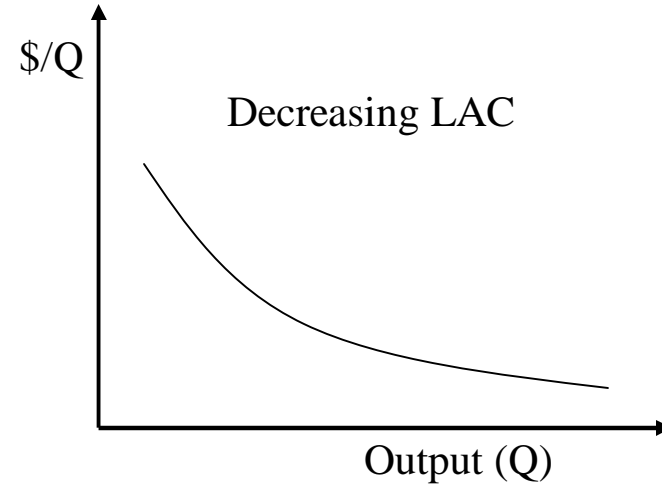
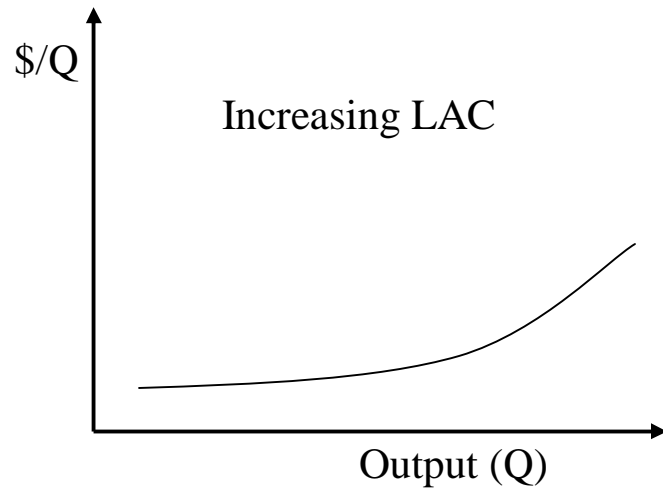
$$\text{LAC} = \frac{\text{LTC}}{Q} = \frac{40Q^{0.5}}{Q} = \frac{40}{Q^{0.5}}$$

$$\text{LMC} = \text{LTC}' = 0.5 \times 40Q^{0.5-1} = \frac{20}{Q^{0.5}}$$

Long Run Production Costs & Industry Structure

- Long Run Average Costs Can Take A Variety of Shapes
 - Increasing
 - Decreasing
 - Constant
 - U
- The shape of the LAC tells us something about industry structure.

Types of Long Run Average Cost



Industry Structure & LAC

- Increasing LAC
 - Small Firms Produce At Lowest Average Cost \Rightarrow Industry With Lots of Small Firms
- Decreasing LAC
 - A Single Firm Can Produce At Lowest Average Cost \Rightarrow Industry With Only One Firm
 - Natural Monopoly: An industry whose market output is produced at lowest cost when production is concentrated in the hands of a single firm.
- Constant LAC
 - Firm Size Doesn't Matter At Lowest Average Cost \Rightarrow Industry With Lots of Different Firm Sizes
- U LAC
 - A Specific Firm Size Can Produce At Lowest Average Cost \Rightarrow Industry With Several Same Size Firms

So, what type of industry is it?

- Suppose
 - we have only two inputs labor (L) & capital (K) such that $Q = KL$.
 - the price of labor is $w = \$10$ & the price of capital is $r = \$40$.
- Recall From Before: $LAC = 40Q^{-0.5}$
- $LAC' = -0.5 \times 40Q^{-0.5-1} = -20Q^{-1.5} < 0$
 - Decreasing Cost

What You Need To Know

- Short Run Production Costs
 - Variable, Fixed, & Total Costs
 - Average Variable, Fixed, & Total Costs
 - Marginal Costs
- Long Run Production Costs
 - Long Run Cost, Long Run Average Cost, & Long Run Marginal Cost
- Long Run Production Costs & Industry Structure
 - Increasing, Decreasing, Constant, & U Shaped Average Cost Industries