

## Costs

Readings: Ch. 10

Our description and discussion of the production process was analogous to our description and discussion of indifference curves and maps, and utility functions. We will now bring this together with the producer's analogy of the budget constraint and use this analogue with our production function to understand a producer's cost of production.

### **Objective: Understand the short run cost of production.**

As with production, we will focus on distinguishing cost in the short run from cost in the long run. Short run cost can be broken down into two components, variable (VC) and fixed (FC) cost, such that total cost is  $TC = VC + FC$ .

#### *Definition*

Total cost: All costs of production in the short run.

#### *Definition*

Fixed cost: Cost that does not vary with the level of output in the short run.

#### *Definition*

Variable cost: Cost that varies with the level of output in the short run.

Recall our short run production function when capital was fixed at  $K_0$ . If the cost of capital is  $r$  per unit of capital, fixed cost is  $FC = rK_0$ . For this short run production function, we can vary labor all we wanted. If the cost of labor is  $w$  per unit, variable cost is  $VC = wL$ . Total cost is then just the sum of variable and fixed costs:  $TC = wL + rK_0$ .

Since  $K_0$  doesn't vary with the level of output, FC will not vary with the level of output. Variable cost however depends on how much labor we use and in the short run how much labor we use solely determines how much output we produce. Therefore, we can find the inverse of  $Q = F_0(L)$ ,  $L = F_0^{-1}(Q)$ , and substitute it into variable cost to get  $VC = wF_0^{-1}(Q)$ , cost as a function of the amount of output we produce. Similarly, total cost will also depend on the amount of output we produce:  $TC = wF_0^{-1}(Q) + rK_0$ .

All this allows us to analyze and graph costs as a function of output. Figure 1 provides an example.

What is important to note from this figure is that fixed cost is just a horizontal line at  $rK_0$ . Variable cost intersects the origin because if we don't use labor it doesn't cost us anything. Total cost is just the variable cost curve shifted up to intersect the vertical axis at  $FC = rK_0$ .

In addition to variable, fixed, and total costs, we will also find a use for the average variable, average fixed, and average total costs. To obtain these quantities, all we need to do is divide them by the level of output.

*Definition*

Average Variable Cost (AVC): Variable cost divided by the quantity of output,  $AVC = VC/Q$ .

*Definition*

Average Fixed Cost (AFC): Fixed cost divided by the quantity of output,  $AFC = FC/Q$ .

*Definition*

Average Total Cost (ATC): Total cost divided by the quantity of output,  $ATC = TC/Q$ .

The final quantity relating to cost that will be of interest is the marginal cost.

*Definition*

Marginal Cost (MC): Change in total cost resulting from a one unit increase in output,  $MC = \Delta TC/\Delta Q = \Delta VC/\Delta Q = wF_0^{-1}(Q)$ .

Graphically, the average cost will be the slope of the line from the origin to the cost curve at the point of interest. The marginal cost will be the slope of a line tangent to the cost curve at the point of interest.

Figure 2 illustrates the average variable, fixed, and total cost curves along with the marginal cost curve. There are a number of important relationships to note when drawing these curves.

The first is that average fixed cost continuously declines with output because fixed cost does not increase with output.

Second, average total cost will equal the sum of average variable and average fixed cost.

Third, when the marginal cost is below the average variable cost, the average variable cost is decreasing. When marginal cost is above the average variable cost, the average variable cost is increasing. Therefore, average variable cost will be at a minimum when it is just equal to the marginal cost (Point **b** in Figure 2).

Fourth, when the marginal cost is below the average total cost, the average total cost is decreasing. When marginal cost is above the average total cost, the average total cost is increasing. Therefore, average total cost will be at a minimum when it is just equal to the marginal cost (Point **c** in Figure 2).

Finally, marginal cost reaches a minimum when the law of diminishing returns becomes relevant (Point **a** in Figure 2).

At this point, it is also useful to note the connections between the short run cost functions and the short run total, average, and marginal product curves.

Marginal cost is equal to the change in variable cost divided by the change in output. The change in variable cost for a one unit increase in labor is equal to  $w$ , while the change in output for a one unit increase in labor is just the marginal product. Therefore,  $MC = w/MP_L$ . Since  $w$

does not depend on output, when the marginal product is at a maximum, the marginal cost will be at a minimum.

Similarly, average variable cost is equal to variable cost divided by output:  $AVC = wL/Q$ . Average product is equal to output divided by the quantity of variable input:  $AP_L = Q/L$ . Therefore, we can also write the average variable cost as  $w$  divided by the average product:  $AVC = w/AP_L$ . Since  $w$  does not depend on output, when average product is at a maximum, average variable cost will be at a minimum.

### **Objective: Understand the long run cost of production.**

In the long run, all inputs are variable, so there will be no fixed cost. Therefore, there is no longer a need to distinguish between total, variable, and fixed cost. Cost will simply equal the sum of the quantity of inputs used in production multiplied by the price or cost of those inputs. For our example of production with capital and labor, cost will equal  $C = wL + rK$ , which should look very familiar to the budget constraint for the consumer's decision problem.

If we hold cost constant at  $C_0$ , we can ask which combinations of labor and capital are just affordable. These combinations make up what economist call the isocost curve.

#### *Definition*

Isocost Curve: All combinations of inputs that result in the same cost of production.

Figure 3 illustrates. The isocost curve intersects the vertical (K) axis at  $C_0/r$ , which represents the most we could spend on capital if we only used capital. It intersects the horizontal (L) axis at  $C_0/w$ , which is the most we can spend on labor if we only use labor. The slope of the isocost curve tells us how much we must reduce capital in order to increase labor without increasing cost,  $\Delta K/\Delta L = -w/r$ .

To find the long run total cost function, we will find it useful to look at a set of isocost curves each representing different levels of cost:  $C_2 > C_1 > C_0$  (see Figure 4). As we move to isocost curves further from the origin, cost increases, but the price of labor and capital do not change, so the slope of the isocost curves do not change.

To find the long run cost function, we need to ask how much it costs to produce various levels of output. Here it is easier to start by picking some particular level of output, say  $Q_0$ , and then ask the question: What is the cost of producing  $Q_0$  units of output?

To answer this question, we need to bring in our isoquant for  $Q_0$ , see Figure 4. In Figure 4, can we produce  $Q_0$  with cost equal to  $C_0$ ? Can we produce  $Q_0$  with cost equal to  $C_1$  or  $C_2$ ?

We cannot produce  $Q_0$  by only expending  $C_0$  because all the input combinations on  $Q_0$  lie above this isocost curve. We can however produce  $Q_0$  for a cost of  $C_1$  or  $C_2$ . For example, point **a** for  $C_1$  and point **b** or **c** for  $C_2$ . But here we reach a bit of a dilemma. There is not a unique cost that allows us to produce  $Q_0$ . In particular, we have identified two different costs for producing  $Q_0$  depending on how much of each input we use. Recall that for any of these three combinations

we are technically efficient. That is we are getting the most output for each of those particular combination of inputs.

So, how can we resolve this dilemma?

Economists resolve this dilemma by assuming the firm will prefer the input combination with the lowest cost. If we compare points **a**, **b**, and **c**, we see that point **a** corresponds to the lower cost.

But, can we do better than point **a**?

For the input combination at point **a**, the isocost curve is just tangent to the isoquant  $Q_0$ . To lower cost below  $C_1$ , we must reduce our use of labor or capital or both, but any of these three options puts us below the isoquant  $Q_0$ , meaning we are no longer able to produce  $Q_0$ . Therefore, we cannot find a combination of inputs that allows us to produce  $Q_0$ , but costs less than the input combination at point **a**.

At this point of tangency, the marginal rate of technical substitution of capital for labor is equal to the absolute value of the slope of the isocost curve,  $MRTS = w/r$ , which is the general rule we can use to find the minimum cost combination of inputs for producing any particular level of output. Recall that  $MRTS = MP_L/MP_K$ . Therefore, this minimum cost combination of inputs will satisfy the condition  $MP_L/w = MP_K/r$ .  $MP_L/w$  is the marginal product of labor per dollar spent on labor and  $MP_K/r$  is the marginal product of capital per dollar spent on capital. The minimum cost bundle sets the marginal product of labor per dollar spent on labor equal to the marginal product of capital per dollar spent on capital. If  $MP_L/w > MP_K/r$  we should increase our use of labor and decrease our use of capital because each dollar spent on labor is more productive. If  $MP_L/w < MP_K/r$  we should increase our use of capital and decrease our use of labor because each dollar spent on capital is more productive.

Again, we have a direct analogy to the consumer's problem. The difference between the two is that for the consumer's problem we find the bundle of goods that maximizes our satisfaction. For the producer's problem, we find the combination of inputs that achieves a particular level of output while minimizing our cost.

We are now ready to use what we have learned to derive long run cost. For each level of output, we can find the minimum cost combination of inputs using an output expansion path.

#### *Definition*

**Output Expansion Path:** The locus of tangencies (minimum cost input combinations) traced out by an isocost line of a given slope as it shifts outward into the isoquant map for the production process.

Figure 6 illustrates an output expansion path where the cost minimizing combinations of output for  $Q_0$ ,  $Q_1$ , and  $Q_2$  are at point **a**, **b**, and **c**. Point **a**, **b**, and **c** correspond to the costs  $C_0$ ,  $C_1$ , and  $C_2$ . Using  $Q_0$ ,  $Q_1$ , and  $Q_2$  and  $C_0$ ,  $C_1$ , and  $C_2$ , we can plot the long run total cost curve (LTC) seen in Figure 7.

As with the short run total cost curve, we will also find the long run average and marginal cost to be useful quantities.

*Definition*

Long Run Average Cost (LAC): Long run cost divided by the quantity of output,  $LAC = LTC/Q$ .

Graphically, the long run average cost is equal to the slope of a line from the origin to the point of interest on the long run total cost curve (See Figure 8). The minimum long run average cost is equal to the slope of the line that passes through the origin and is just tangent to the long run total cost curve.

*Definition*

Long Run Marginal Cost (LMC): Change in long run cost resulting from a one unit increase in output,  $LMC = \Delta LTC/\Delta Q$ .

Graphically, the long run marginal cost is equal to the slope of a line tangent to the long run total cost curve at the point of interest (See Figure 9). Note that at the minimum LAC, the slope of the tangent will equal the slope of a line through the origin. The bottom line is that the LAC reaches a minimum where  $LAC = LMC$ . Figure 10 illustrates this relationship between the long run average and marginal cost curves.

**Objective: Understand the relationship between long-run average cost and industry structure.**

Long run average cost curves are typically characterized as increasing, decreasing, constant, or U shaped. Figure 11 shows examples of each.

Each of these different types of long run average cost has a different implication for production and the structure of industries.

When LAC is increasing, small firms are able to produce output cheaper than larger firms on average, so we are likely to see lots of small firms dominate a market.

When LAC is decreasing, larger firms are able to produce output cheaper on average than smaller firms. In fact, a single firm is capable of producing all the output the market demands at the lowest possible cost. This result is what is called a natural monopoly.

*Definition*

Natural Monopoly: An industry whose market output is produced at lowest cost when production is concentrated in the hands of a single firm.

When LAC is constant, the size of firm does not matter to cost. We can see large and small firms coexist in an industry.

When LAC is U shaped, there is a specific firm size that can produce at the lowest long run average cost. This is a case where we may see a few moderately sized firms form the industry.

Figure 1: Variable, Fixed, & Total Cost Curve

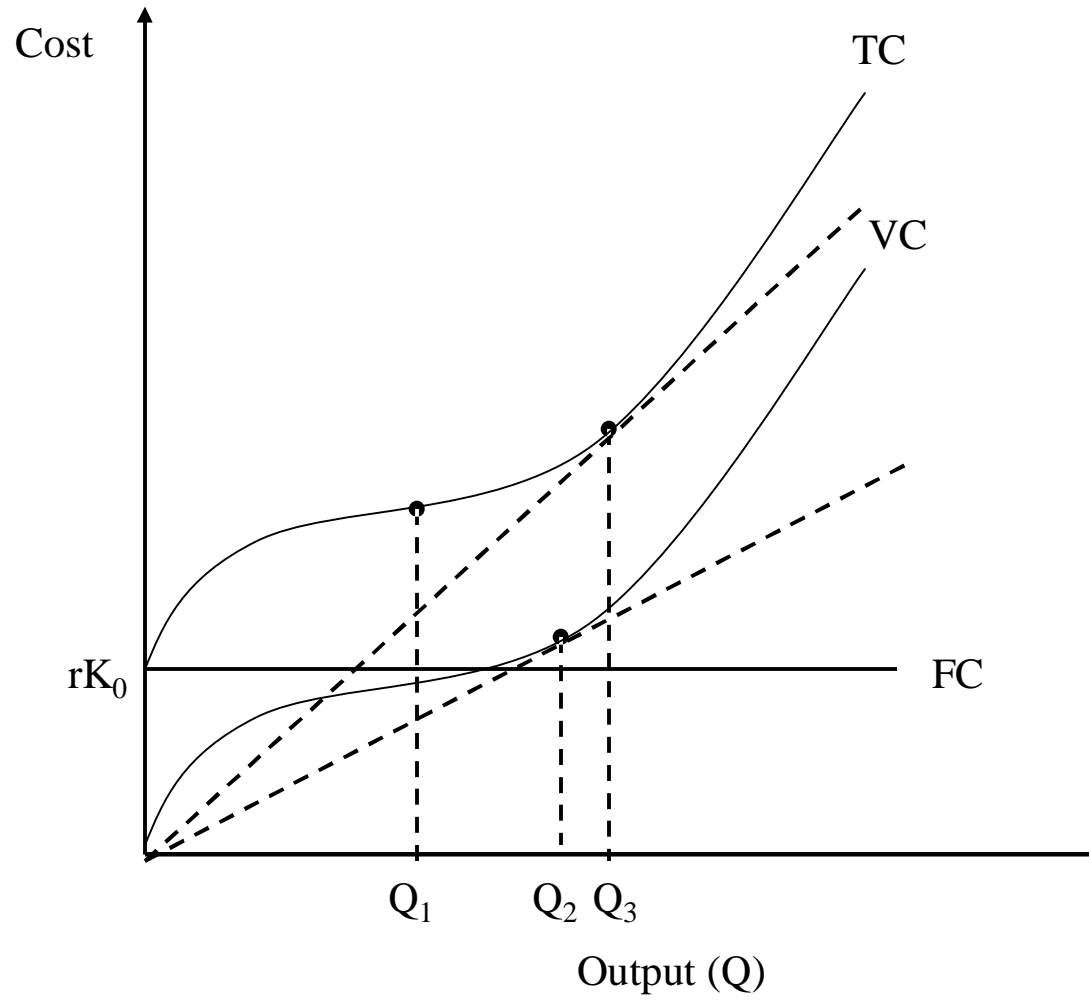


Figure 2: Average Variable, Average Fixed, Average Total, & Marginal Cost

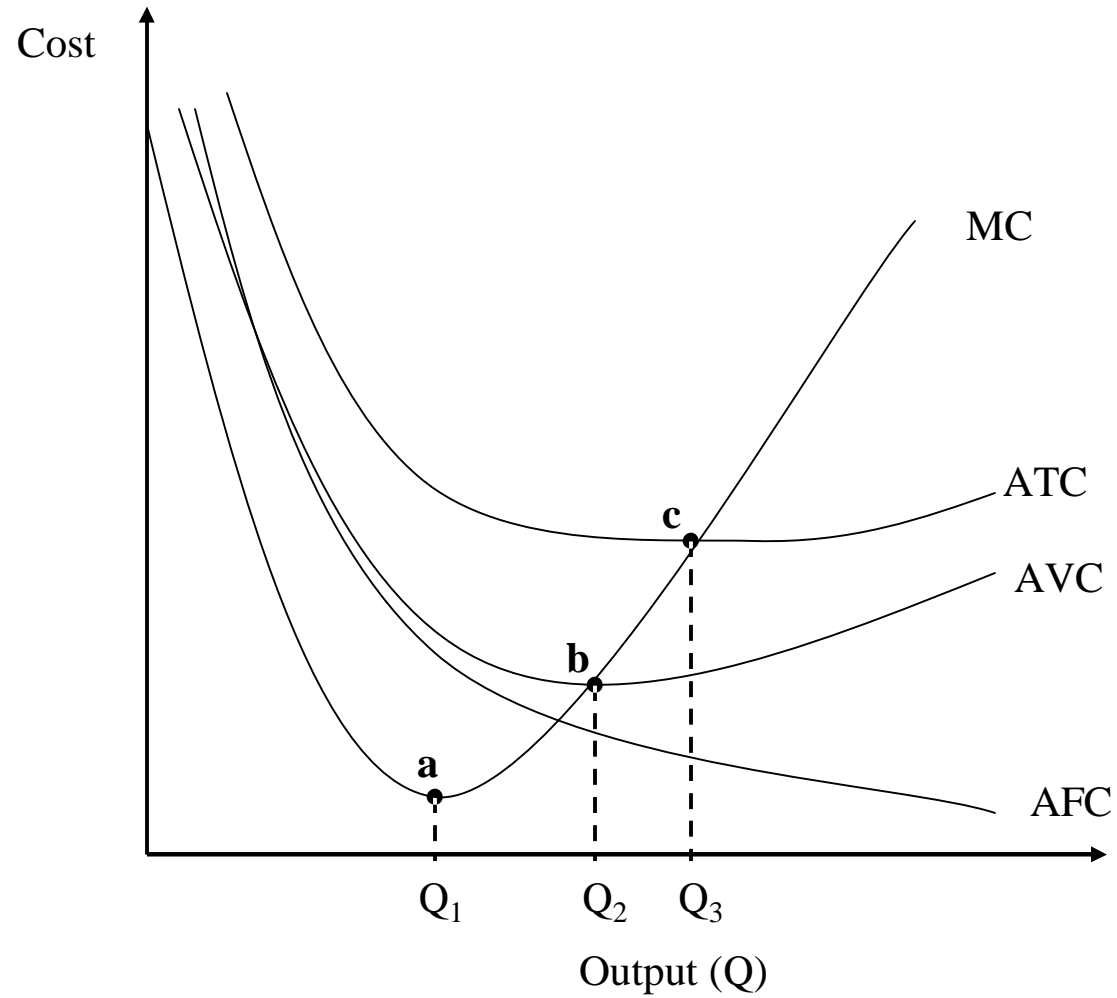


Figure 3: Example Isocost Curve

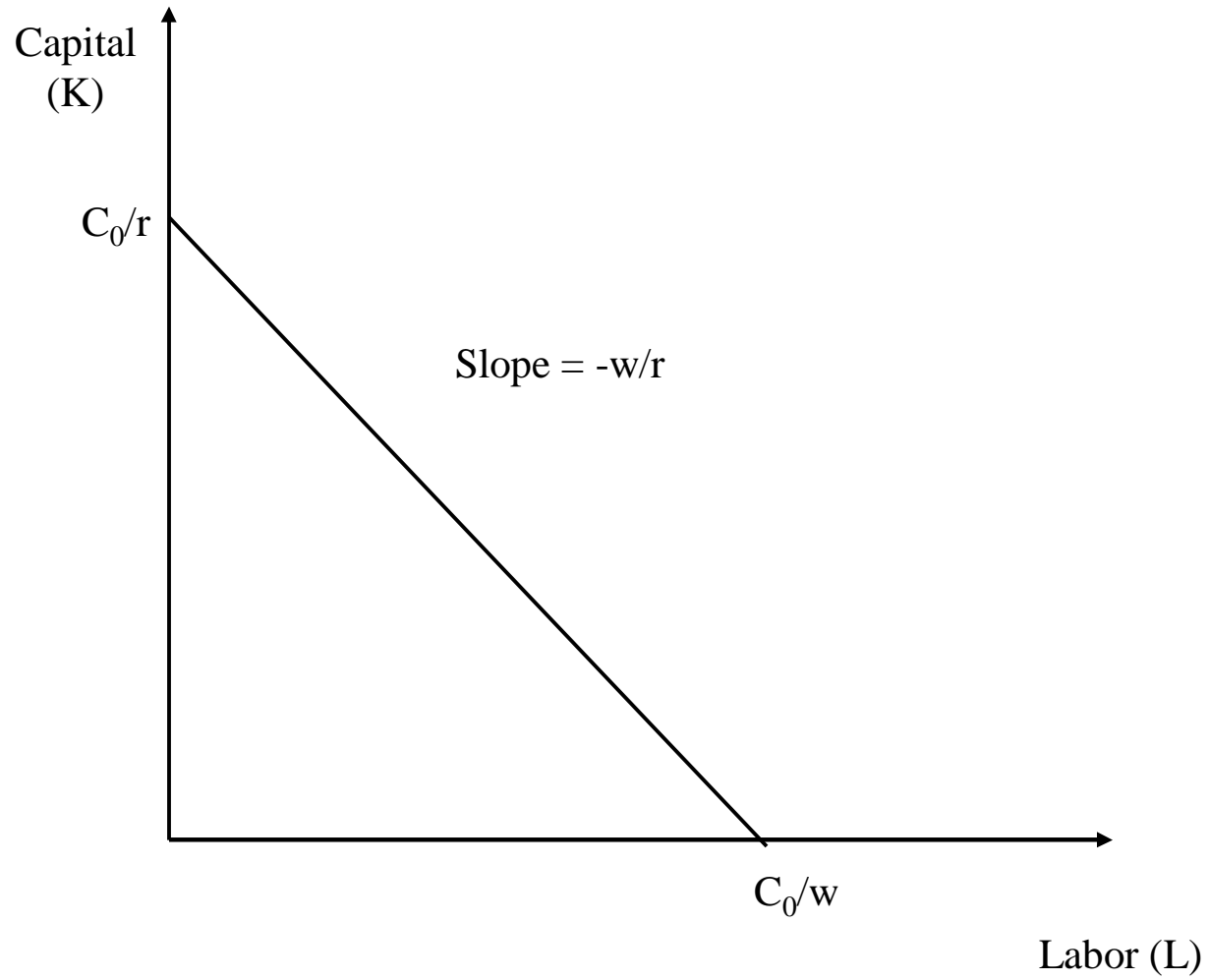


Figure 4: Example Isocost Map

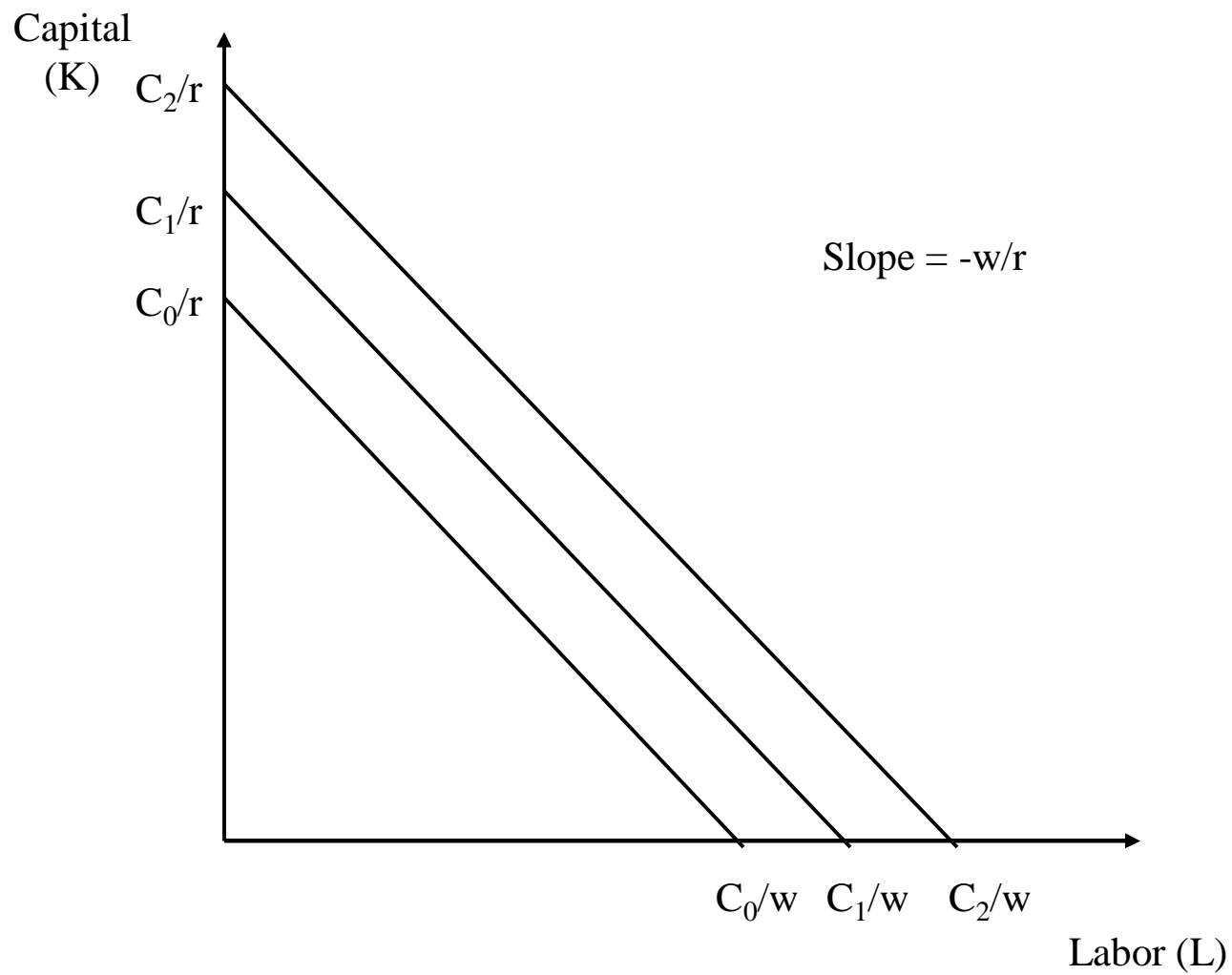


Figure 5: Example Isocost Map and Isoquant

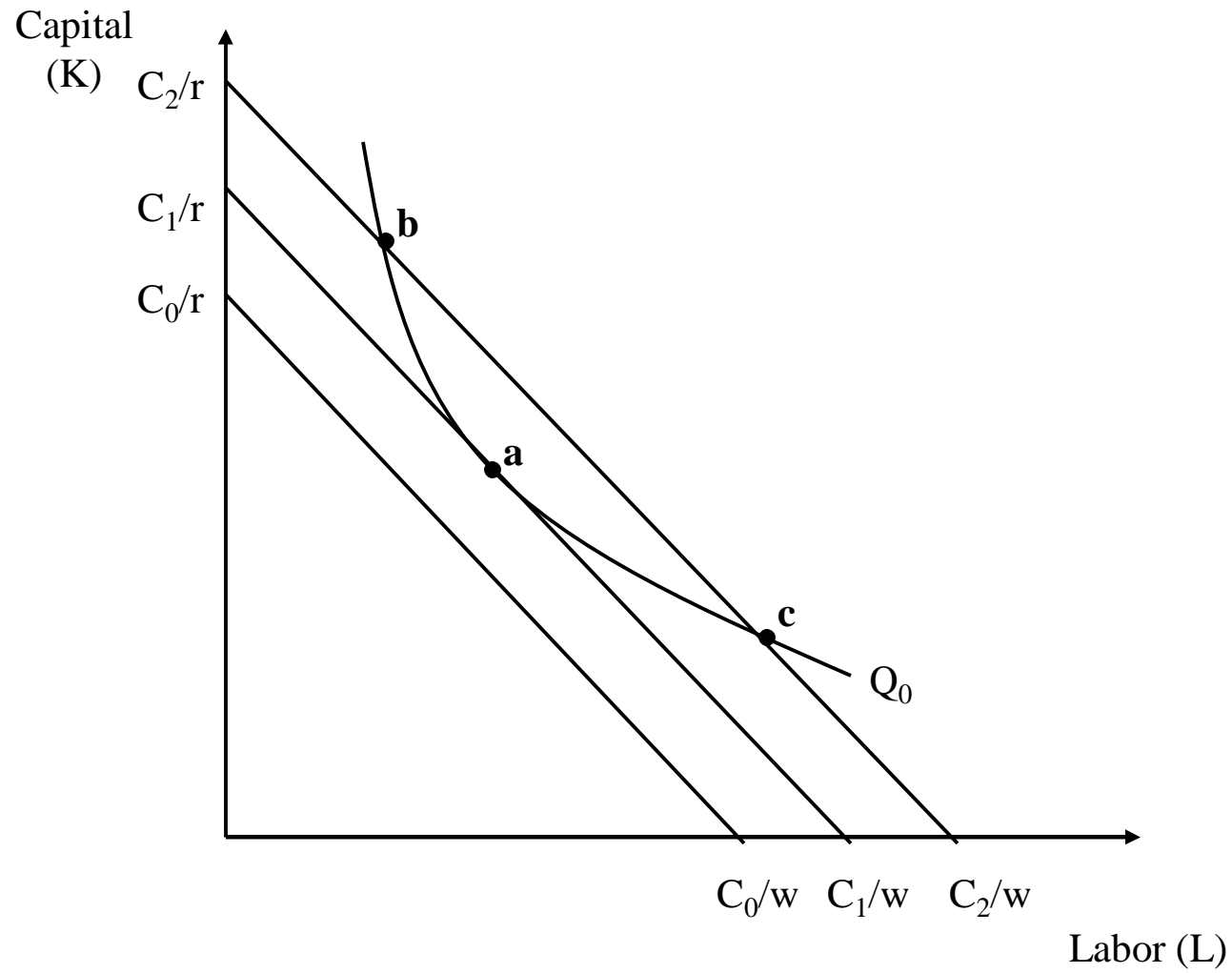


Figure 6: Output Expansion Path

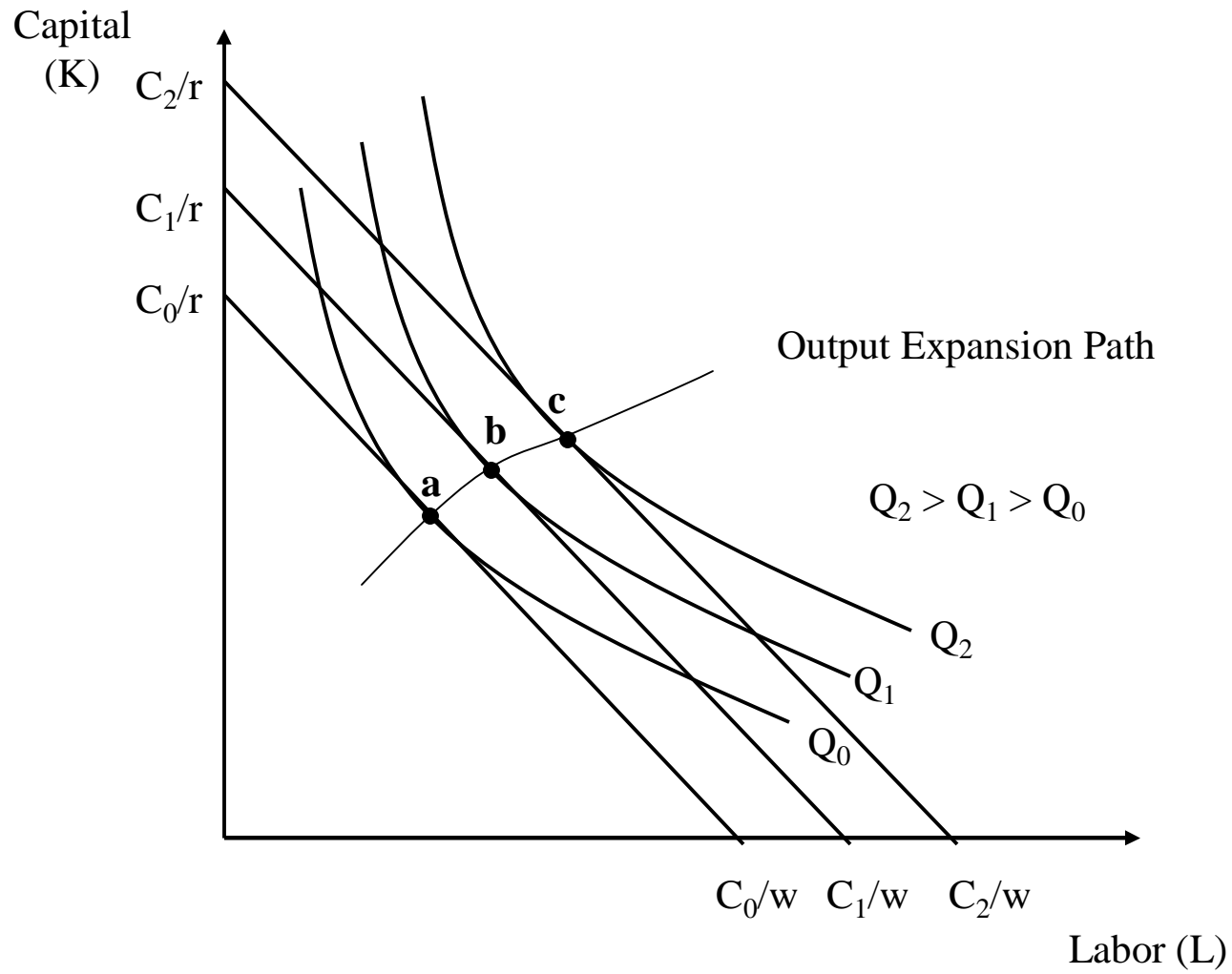


Figure 7: Long Run Total Cost Curve

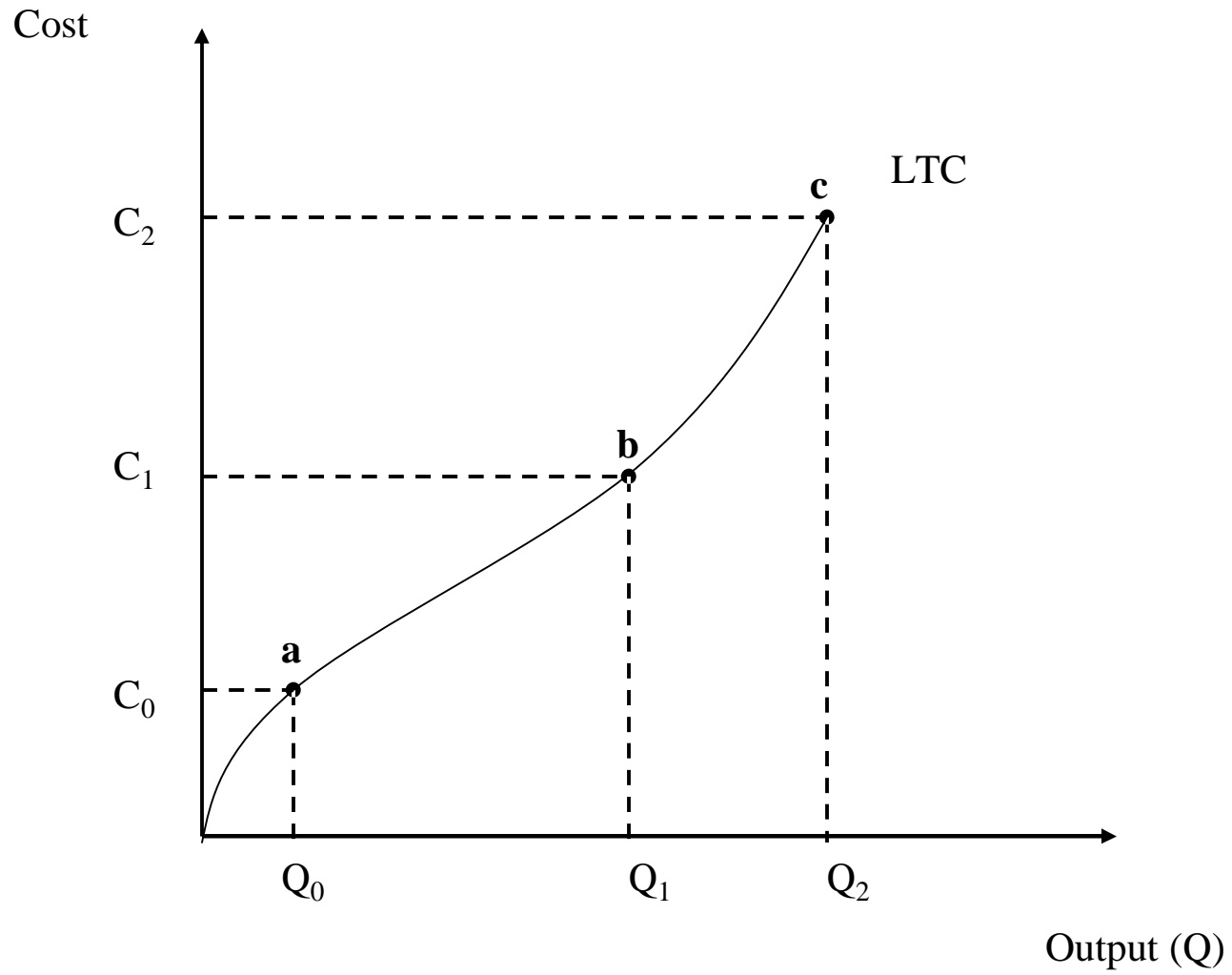


Figure 8: Long Run Average Cost

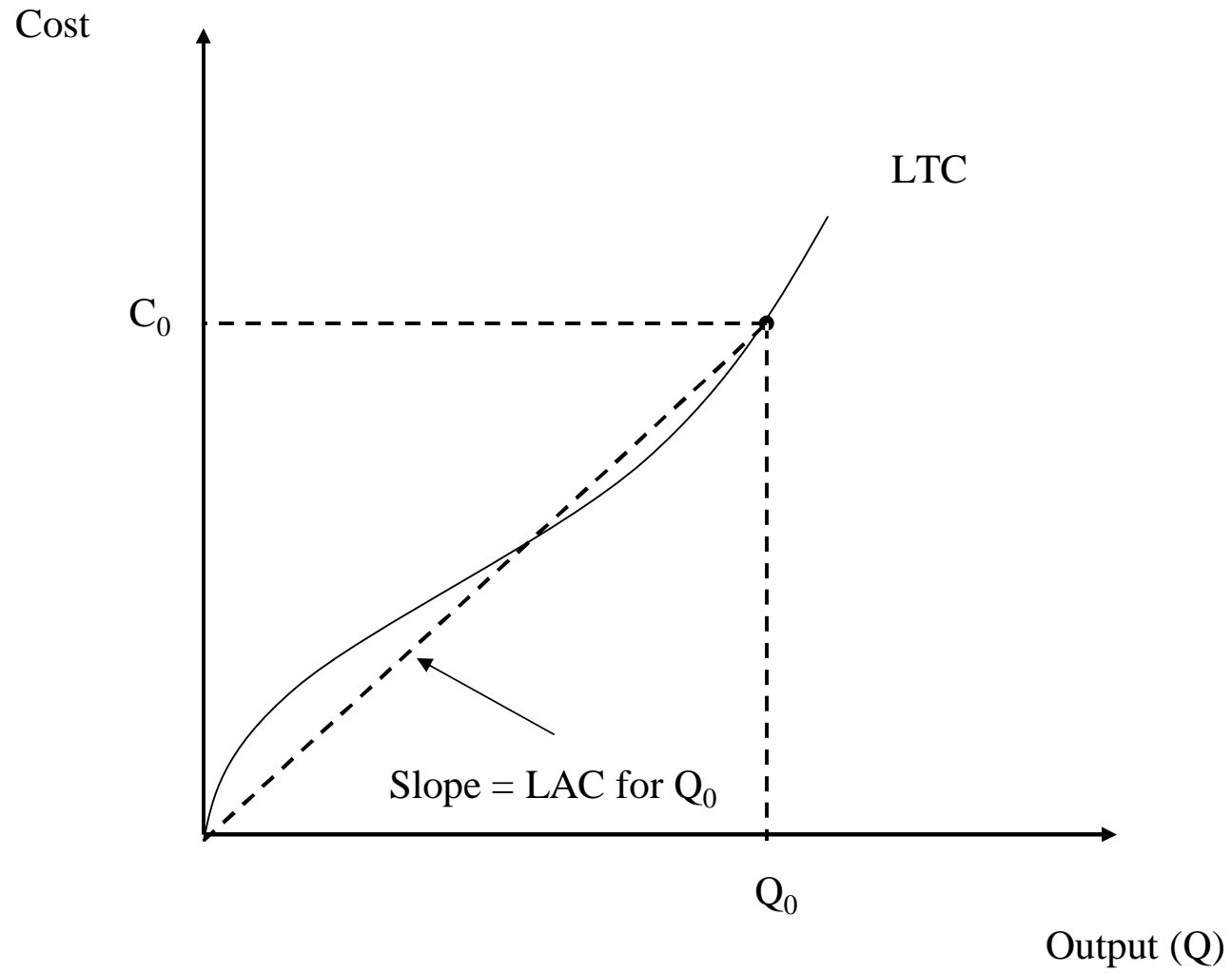


Figure 9: Long Run Marginal Cost

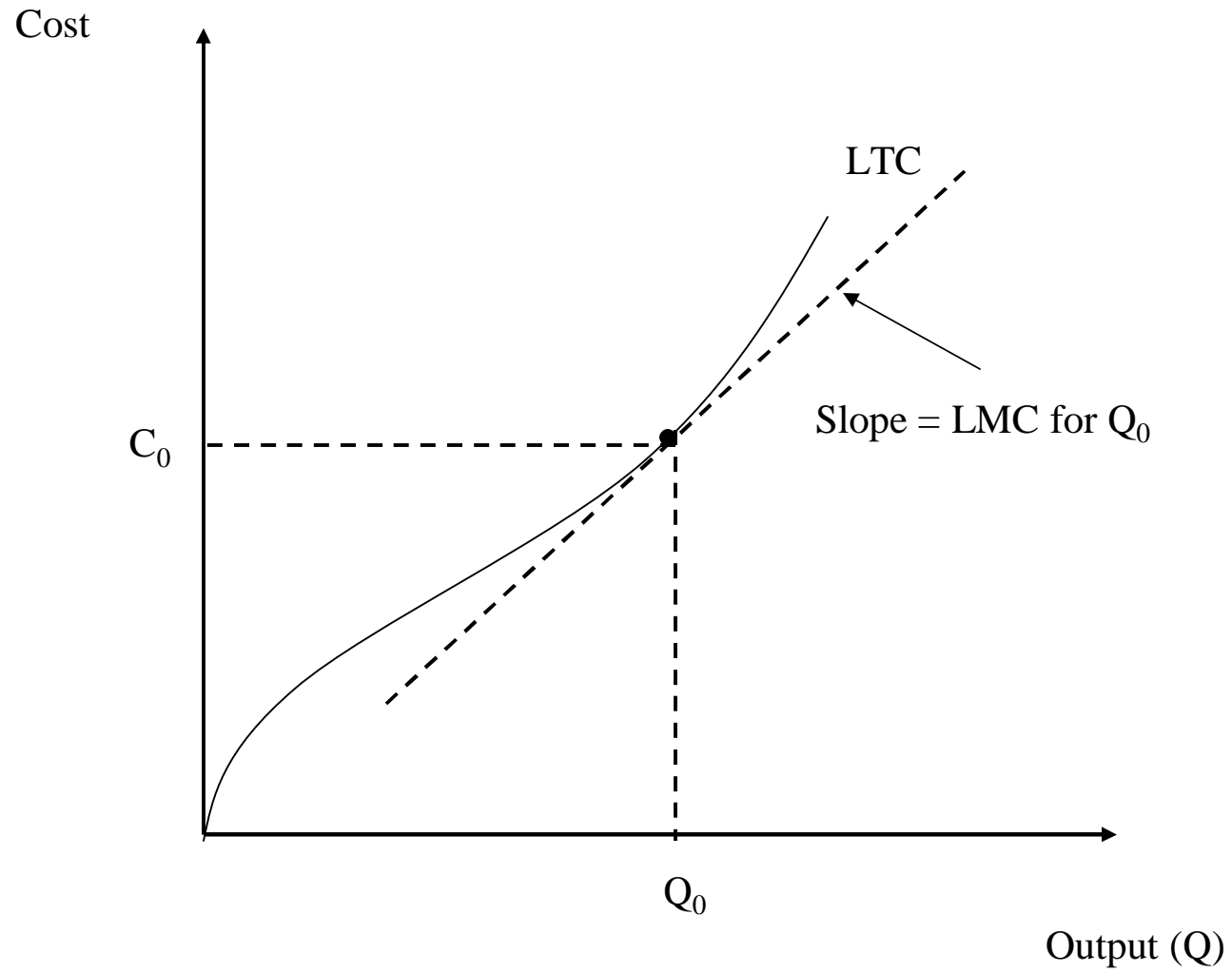


Figure 10: Long Run Average and Marginal Cost

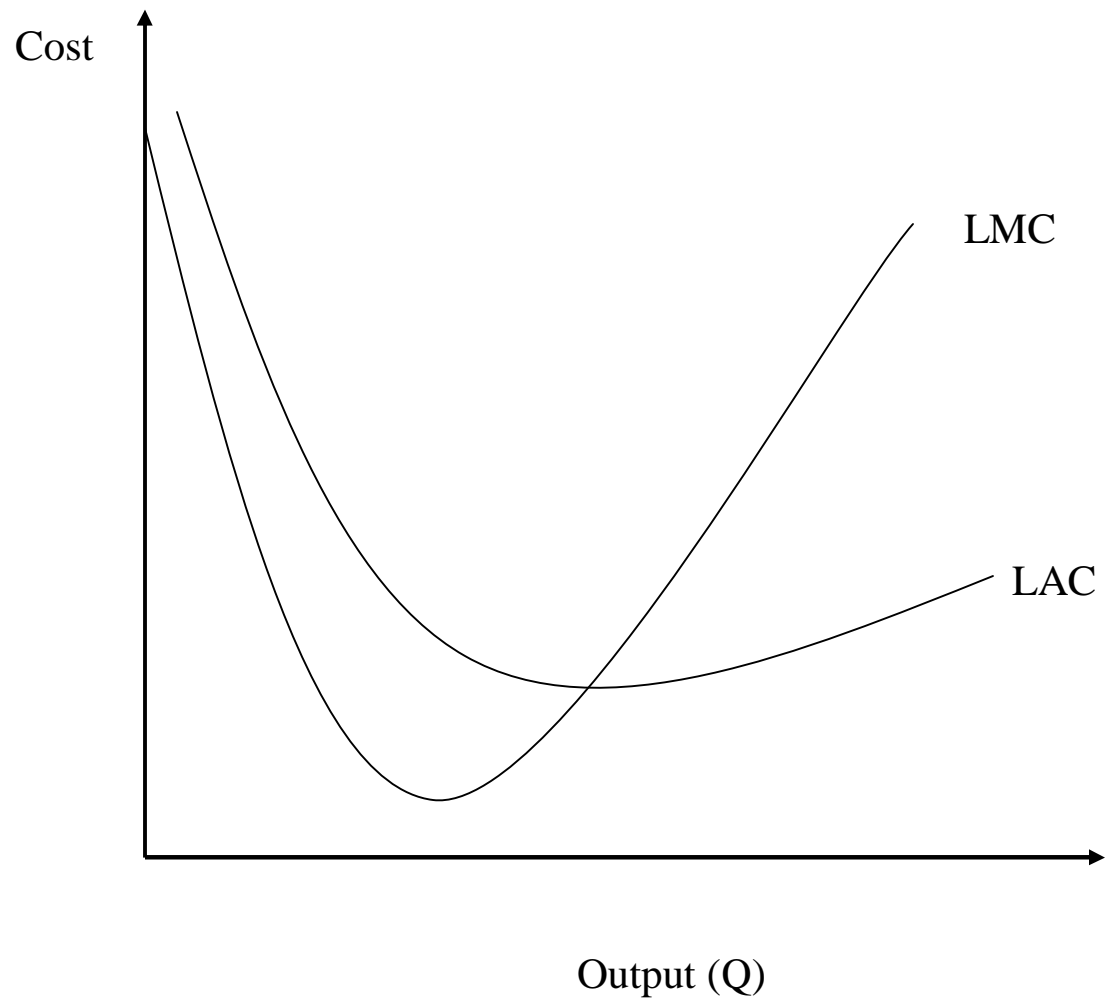


Figure 11: Types of Long Run Average Cost

