ANSWERS: Homework #5
Due: 7-16-07
APEC 3001
Applied Microeconomics:
Consumers, Producers, and Markets
(Summer 2007)
Instructor: Hurley

Please show all the work you do to solve a problem.

1. How do the Cournot and Bertrand duopoly models differ? Which model results in a higher price? Which results in a higher quantity?

   Answer: In the Cournot duopoly model, firms simultaneously choose their level of output, while in the Bertrand duopoly model, firms simultaneously choose price. The Bertrand duopoly model results in a lower price and higher output than the Cournot duopoly model.

2. What are two important differences between a monopoly and monopolistic competition?

   Answer: 1) For a monopoly's product there are no close substitutes, while for a monopolistic competitor's product there are lots of close substitutes. 2) Monopoly's can earn positive economic profits in the long run because there is not free entry and exit, while a monopolistic competitor cannot earn positive economic profits in the long run because there is free entry and exit.

3. What is a Pareto optimal allocation?

   Answer: An allocation where it is impossible to make one person better off without making at least one other person worse off.

4. What is the First Welfare Theorem?

   Answer: Equilibrium in competitive markets is Pareto Optimal.

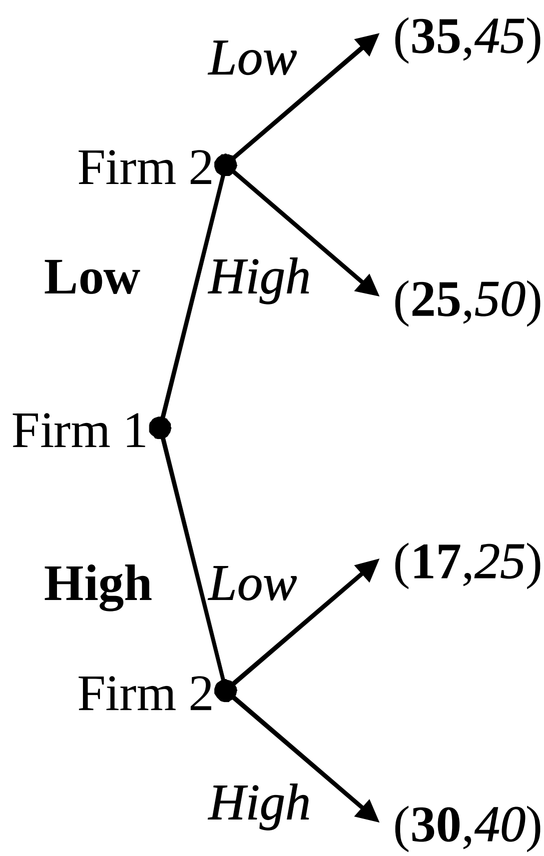
5. In the Table below, Firm 1 gets to choose the row by choosing either **Up** or **Down**, while Firm 2 gets to choose the column by choosing either *Right* or *Left*. Firm 1's profits are denoted in **bold**, while Firm 2's profits are denoted in *italics*. What is the Nash equilibrium strategy for each firm?
   a. Firm 1 choosing **Up** and Firm 2 choosing *Right*.
   b. Firm 1 choosing **Up** and Firm 2 choosing *Left*.
   c. Firm 1 choosing **Down** and Firm 2 choosing *Right*.
   d. Firm 1 choosing **Down** and Firm 2 choosing *Left*.

		Firm 2	
		Left	*Right*
Firm 1	**Up**	**120**, *75*	**150**, *50*
	Down	**90**, *100*	**125**, *150*

Answer: b. If Firm 1 chooses **Up**, Firm 2 maximizes profit by choosing *Left* because *75* > *50*. If Firm 1 chooses **Down**, Firm 2 maximizes profit by choosing *Right* because *150* > *100*. If Firm 2 chooses *Left*, Firm 1 maximizes profit by choosing **Up** because **120** > **90**. If Firm 2 chooses *Right*, Firm 1 maximizes profit by choosing **Up** because **150** > **125**. So, Firm 1's profit maximizing strategy is to choose **Up** regardless of what Firm 2 does. If Firm 1 chooses **Up**, Firm 2's best response is *Left*.

6. Consider the game in the figure below. Firm 1 chooses a **High** or **Low** output. Firm 2 then gets to choose a *High* or *Low* output after seeing Firm 1's choice. Firm 1's profit is the first number in parentheses in **Bold**, while Firm 2's profit is the second number in parentheses in *italics*. What are Firm 1's and Firm 2's equilibrium strategies?
   a. Firm 1 chooses **Low** and Firm 2 choose *Low* if Firm 1 chooses **Low** and *High* if Firm 1 chooses **High**.
   b. Firm 1 chooses **Low** and Firm 2 choose *High* if Firm 1 chooses **Low** and *High* if Firm 1 chooses **High**.
   c. Firm 1 chooses **High** and Firm 2 choose *Low* if Firm 1 chooses **Low** and *High* if Firm 1 chooses **High**.
   d. Firm 1 chooses **High** and Firm 2 choose *High* if Firm 1 chooses **Low** and *High* if Firm 1 chooses **High**.

   Answer: d. If Firm 1 chooses **Low**, Firm 2 will choose *High* because *50* > *45*. If Firm 1 chooses **High**, Firm 2 will choose *High* because *40* > *25*. Therefore, Firm 1 should choose **High** because **30** > **25**.



7. Suppose demand is $P = 300 – 5Q$ and that there are only two firms that produce for this market such that $Q = Q_1 + Q_2$ where Q_1 is Firm 1's output and Q_2 is Firm 2's output. If these firms have identical marginal costs $MC_1 = MC_2 = 60$, what is the Stackelberg equilibrium price (P*) and industry output (Q*) if firm 1 chooses its output first?
   a. P* = 60 and Q* = 48.
   b. P* = 120 and Q* = 36.
   c. P* = 140 and Q* = 32.
   d. P* = 180 and Q* = 24.

   Answer: b. Total revenue for Firm 2 is $TR_2 = 300Q_2 – 5(Q_2^2 + Q_2Q_1)$ so marginal revenue is $MR_2 = TR_2' = 300 – 10 Q_2 -5Q_1$. Setting this equal to marginal cost and solving: $300 – 10 Q_2 -5Q_1 = 60 \Rightarrow Q_2 = 24 - 0.5Q_1$, which is firm 2's reaction or best response function. Now when firm 1 goes to make its choice of output, it knows Firm 2 will respond based on $Q_2 = 24 - 0.5Q_1$, so Firm 1's total and marginal revenue are $TR_1 = 300Q_1 – 5(Q_1^2 + Q_1(24 - 0.5Q_1))= 180Q_1 – 2.5Q_1^2$ and $MR_1 = 180 – 5Q_1$. Setting marginal revenue equal to marginal cost and solving yields $180 – 5Q_1^* = 60$ or $Q_1^* = 24$. Substituting into Firm 2's reaction function then yields $Q_2^* = 24 - 0.5(24) = 12$ such that $Q^* = 24 + 12 = 36$ and $P^* = 300 – 5(36) = 120$.

8. Which of the following conditions below, does **not** have to hold for a Pareto optimal allocation of resources in a general equilibrium with production?
   a. Firms must equate their marginal rates of technical substitution.
   b. Consumers must equate the marginal rates of substitution.
   c. Marginal rates of technical substitution must equal the marginal rate of transformation.
   d. Marginal rates of substitution must equal the marginal rate of transformation.

   Answer: d. For firms to use the Pareto optimal level of inputs, they must equate their marginal rates of technical substitution, so a. must be true for Pareto optimality. For consumer to consume a Pareto optimal allocations of goods, their marginal rates of substitutions must be equal, so b. must true for Pareto optimality. For firms to produce the right amount of good for consumers, the marginal rate of substitution must equal the marginal rate of transformation, so d. must be true for Pareto optimality. By process of elimination, c. need not be true for Pareto optimality

9. Suppose we have an exchange economy with only two people, Mason and Spencer, and two goods, candy and gum. Mason's utility function is $U_M = C_M^{0.5}G_M$ where C_M and G_M are the quantity of candy and gum Mason consumes. Spencer's utility function is $U_S = C_SG_S^{0.5}$ where C_S and G_S are the quantity of candy and gum Spencer consumes.
   a) Find Mason's marginal rate of substitution: $MRS^M = MU_G^M/MU_C^M$ where MU_G^M and MU_C^M are Mason's marginal utility of gum and candy. Find Spencer's marginal rate of substitution: $MRS^S = MU_G^S/MU_C^S$ where MU_G^S and MU_C^S are Spencer's marginal utility of gum and candy.
   b) Suppose Dana gives Mason and Spencer each 75 pieces of candy and 75 pieces of gum. Show why this is not a Pareto efficient allocation of candy and gum.
   c) Suppose Dana allowed Mason and Spencer trade candy and gum provided the price of gum is \$0.01 and the price of candy is \$0.01. Show whether or not allowing this trade at these prices will result in a Pareto efficient allocation of candy and gum.

Answer:

a)

$MU_G^M = 1.0C_M^{0.5}G_M^{1.0-1} = C_M^{0.5}$ and $MU_C^M = 0.5C_M^{0.5-1}G_M^{1.0} = 0.5C_M^{-0.5}G_M$, such that $MRS^M = C_M^{0.5} / (0.5C_M^{-0.5}G_M^{1.0}) = 2C_M/G_M$.

$MU_G^S = 0.5C_SG_S^{0.5-1} = 0.5C_SG_S^{-0.5}$ and $MU_C^S = 1.0C_S^{1.0-1}G_S^{0.5} = G_S^{0.5}$, such that $MRS^S = 0.5C_S^{1.0}G_S^{-0.5} / (G_S^{0.5}) = 0.5C_S/G_S$.

b) To see if this is a Pareto efficient allocation, we must check to see if Mason's and Spencer's marginal rate of substitution is the same given their allocation of candy and gum. For Mason, $MRS^M = 2C_M/G_M = 2\times75/75 = 2$. For Spencer, $MRS^S = 2C_S/G_S = 0.5\times75/75 = 0.5$. Since $MRS^M \neq MRS^S$, this allocation is not Pareto efficient.

c) If candy and gum are each worth \$0.01 a piece, both Mason's and Spencer's initial allocation of candy and gum is worth \$1.50, so it is like they have \$1.50 in income. To find Mason's optimal mix of candy and gum, we can set his marginal rate of substitution equal to the ratio of prices: $MRS^M = P_G/P_C$ or $2C_M/G_M = \$0.01/\$0.01 = 1$ or $G_M = 2C_M$. To find out how much candy and gum he wants to consume given he has \$1.50 to start, we can substitute $G_M = 2C_M$ into his budget constraint and then solve for C_M: $\$1.50 = \$0.01G_M + \$0.01C_M = \$0.01\times2C_M + \$0.01C_M = \$0.03C_M$ or $C_M = 50$ such that $G_M = 2\times50 = 100$. We can also do the same for Spencer: $MRS^S = P_G/P_C$ or $0.5C_S/G_S = \$0.01/\$0.01 = 1$ or $C_S = 2G_S$, and $\$1.50 = \$0.01G_S + \$0.01C_S = \$0.01G_S + \$0.01\times2G_S = \$0.03G_S$ or $G_S = 50$ such that $C_S = 2\times50 = 100$. Now notice that $C_M + C_S = 150$ and $G_M + G_S = 150$, which is exactly equal to the total amount of candy and gum given to Mason and Spencer by Dana. There is no shortage or surplus. Therefore, Mason will trade Spencer 25 pieces and candy for 25 pieces of gum resulting in a Pareto efficient allocation.

10. Suppose we have two firms that produce similar but not identical products, so they behave as monopolistic competitors. The Demand for Firm 1's product is $P_1 = 150 – 5Q_1 – 10Q_2$. The Demand for Firm 2's product is $P_2 = 200 – 10Q_2 – 10Q_1$. For simplicity, assume constant marginal costs equal to 0.
   a) Find each firm's total and marginal revenue.
   b) Derive each firm's reaction/best response function.
   c) Find the Nash equilibrium quantity and price for each firm.

   Answer:
   a) Firm 1's total revenue is $TR_1 = P_1Q_1 = (150 – 5Q_1 – 10Q_2)Q_1 = 150Q_1 – 5Q_1^2 – 10Q_2Q_1$. Firm 2's total revenue is $TR_2 = P_2Q_2 = (200 – 10Q_2 – 10Q_1)Q_2 = 200Q_2 – 10Q_2^2 – 10Q_1Q_2$.

      Firm 1's marginal revenue is $MR_1 = \Delta TR_1/\Delta Q_1 = 150 – 10Q_1 – 10Q_2$.
      Firm 2's marginal revenue is $MR_2 = \Delta TR_2/\Delta Q_2 = 200 – 20Q_2 – 10Q_1$.

   b) To find firm 1's reaction/ best response function, we need to set its marginal revenue equal to its marginal cost and then we need to solve for its output: $150 – 10Q_1^* – 10Q_2 = 0 \Rightarrow 10Q_1^* = 150 – 10Q_2 \Rightarrow Q_1^* = 15 – Q_2$.

      Similarly, for firm 2: $200 – 20Q_2^* – 10Q_1 = 0 \Rightarrow 20Q_2^* = 200 – 10Q_1 \Rightarrow Q_2^* = 10 – 0.5Q_1$.

   c) To find the Nash equilibrium outputs, we need to find the Q_1^* and Q_2^* that simultaneously solve the firms reaction/best response functions:

      $Q_1^* = 15 – Q_2^*$ and $Q_2^* = 10 – 0.5Q_1^* \Rightarrow Q_1^* = 15 – (10 – 0.5Q_1^*) \Rightarrow Q_1^* = 15 – 10 + 0.5Q_1^* \Rightarrow 0.5Q_1^* = 5 \Rightarrow Q_1^* = 10$ and $Q_2^* = 10 – 0.5\times10 = 5$.

      To find the Nash equilibrium prices, we now need to substitute these equilibrium quantities back into the demand functions:
      $P_1^* = 150 – 5Q_1^* – 10Q_2^* = 150 – 5\times10 – 10\times5 = 50$ and
      $P_2^* = 200 – 10Q_2^* – 10Q_1^* = 200 – 10\times5 – 10\times10 = 50$.