ANSWERS: Homework #3
Due: 7-2-06
APEC 3001
Applied Microeconomics:
Consumers, Producers, and Markets
(Summer 2007)
Instructor: Hurley

Please show all the work you do to solve a problem.

1. What do economists mean by long-run production?

   Answer: The shortest period of time required to alter the amount of all inputs used in a production process.

2. What is the law of diminishing returns?

   Answer: If other inputs are fixed, the increase in output from an increase in variable inputs must eventually decline.

3. What is an iso-cost curve?

   Answer: All combinations of inputs that result in the same cost of production.

4. What is the output expansion path and how can we use it?

   Answer: The locus of tangencies (minimum cost input combinations) traced out by an isocost line of a given slope as it shifts outward into the isoquant map for the production process. The output expansion path is used to derive the long-run total cost function.

5. Consider the short run production function $Q = 10L - 0.5L^2$ where labor (L) is the only variable input. What is the average product when labor equals 4?
   a. 6.
   b. 8.
   c. 12.
   d. 32.

   Answer: b. The average product of L is equal to $AP_L = Q/L = (10L - 0.5L^2)/L = 10\text{-}0.5L$. Evaluating the average product for $L = 4$ yields $AP_L = 10\text{-}0.5\times4 = 8$.

6. Consider the production function $Q = K^{0.5}L^{0.75}$ where K is capital and L is labor. Which of the following statements about this production function is true?
   a. Its exhibits decreasing returns to scale.
   b. Its exhibits constant returns to scale.
   c. Its exhibits increasing returns to scale.
   d. It exhibits decreasing, increasing, or constant returns to scale depending on how much capital and labor are used.

   Answer: c. If we multiply both labor and capital by $\alpha > 1$, output will increase by a proportion greater than α: $(\alpha K)^{0.5}(\alpha L)^{0.75} = \alpha^{0.5}\alpha^{0.75}K^{0.5}L^{0.75} = \alpha^{1.25}K^{0.5}L^{0.75} = \alpha^{1.25}Q > \alpha Q$.

7. Which of the following statements must be **true**?
   a. Short run average cost is increasing when short run marginal cost is above short run average cost.
   b. Short run average cost is increasing when short run marginal cost is below short run average cost.
   c. Short run average cost is increasing when short run marginal cost is increasing.
   d. Short run average cost is increasing when short run marginal cost is decreasing.

   Answer: a. Average cost decreases when the marginal cost is lower and increases when the marginal cost is higher. This is true for any cost that varies with output.

8. Consider the long-run total cost function $LTC = 2{,}000Q - 50Q^2 + 0.5Q^3$. Which of the following statements is true?
   a. Long-run average cost is decreasing.
   b. Long-run average cost is constant.
   c. Long-run average cost is increasing.
   d. Long-run average cost is U-Shaped.

   Answer: d. There are two ways to answer this question. First, we can find the long-run average cost and then take the derivative: $LAC = LTC/Q = (2{,}000Q - 50Q^2 + 0.5Q^3)/Q = 2{,}000 - 50Q + 0.5Q^2$, such that $LAC' = -50 + Q$. Notice that $LAC' > 0$ for $Q > 50$ and $LAC' < 0$ for $Q < 50$, which means LAC is U-Shaped.

   Alternatively, we can compare the long-run average cost to long-run marginal cost. From above, we know $LAC = 2{,}000 - 50Q + 0.5Q^2$. Long-run marginal cost is the derivative of long-run total cost: $LMC = LTC' = 2{,}000 - 100Q + 1.5Q^2$. Therefore, LAC >(<) LMC if $2{,}000 - 50Q + 0.5Q^2 >(<) 2{,}000 - 100Q + 1.5Q^2$ or $50 >(<) Q$. When $Q < 50$, LAC > LMC, so LAC must be decreasing. When $Q > 50$, LAC < LMC, so LAC must be increasing. This also implies a U-Shaped long-run average cost curve.

9. Consider the production function $Q = K^{0.5}L^{0.5}$ where K is capital and L is labor. Suppose capital is fixed at 400 in the short-run.
   a) What are the short-run total, average, and marginal product?
   b) Assuming the price of capital is r = \$100 and the price of labor is w = \$25, what are the fixed, variable, and total cost?
   c) Again assuming the price of capital is r = \$100 and the price of labor is w = \$25, what are the average fixed, average variable, and average total cost?
   d) What is the marginal cost?

Answer:

a. The short-run total product is $Q = 400^{0.5}L^{0.5} = 20L^{0.5}$. Average product is just the total product divided by the variable input labor: $AP_L = 20L^{0.5}/L = 20/L^{0.5}$. Marginal product is the derivative of the total product with respect to the variable input labor: $MP_L = 0.5\times20L^{0.5-1} = 10/L^{0.5}$.

b. Fixed cost is the amount of the fixed input multiplied by the price of the fixed input: FC = \$100×400 = \$40,000. Variable cost is the amount of the variable input defined in terms of output multiplied by the price of the variable input: VC = \$25×L = \$25L, but $Q = 20L^{0.5}$ or $L = (Q/20)^2 = Q^2/400$, so VC = $\$25\times Q^2/400 = \$Q^2/16$. Total cost is fixed cost plus variable cost: TC = $\$40{,}000 + \$Q^2/16$.

c. Average fixed cost is just fixed costs divided by output: AFC = FC/Q = \$40,000/Q. Average variable cost is just variable costs divided by output: AVC = VC/Q = $(\$Q^2/16)/Q = \$Q/16$. Average total cost is just the sum of average fixed and average variable cost: ATC = AFC + AVC = \$40,000/Q + \$Q/16.

d. Marginal cost is just the derivative of total cost with respect to output: MC = TC' = 0 + $2\times\$Q^{2-1}/16 = \$Q/8$.

10. Consider the production function $Q = K^{1.5}L^{0.5}$ where K is capital and L is labor.
    a) Find the marginal rate of technical substitution for this production function.
    b) Suppose the price of capital is r = \$60 and the price of labor is w = \$20. What is the output expansion path given these prices?
    c) What are long-run total cost, long-run average cost, and long-run marginal cost?

Answer:

a) MRTS = MP_L/MP_K. $MP_L = \frac{\partial Q}{\partial L} = 0.5\frac{K^{1.5}}{L^{0.5}}$. $MP_K = \frac{\partial Q}{\partial K} = 1.5K^{0.5}L^{0.5}$. So, MRTS = K/(3L).

b) The output expansion path is defined by the tangency of isoquants with iso-cost curves. That is, where MRTS = w/r or K/(3L) = \$20/\$60 = 1/3 $\Rightarrow$ K = L.

c) Long-run total cost is LTC = rK + wL = \$60K + \$20L, but we want this in terms of output along the output expansion path. Recall $Q = K^{1.5}L^{0.5}$. Also note that along the output expansion path K = L, so $Q = L^{1.5}L^{0.5} = L^2$ or $L = Q^{0.5}$. Again, K = L along the output expansion path, so LTC = \$60K + \$20L = \$60L + \$20L = \$80L = $\$80Q^{0.5}$. Now to find long-run average cost all we need to do is divide long-run total costs by output: $LAC = LTC/Q = \$80Q^{0.5}/Q = \$80/Q^{0.5}$. To find long-run marginal cost we need to take the derivative of long-run total cost: $LMC = LTC' = 0.5\times\$80Q^{0.5-1} = \$40/Q^{0.5}$.