

Practice with Derivatives And Partial Derivatives:

The derivative of a function $f(x)$ is denoted as $f'(x)$. It tells you the slope of the line tangent to the function at x . Explicitly, it is defined as $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

For the derivatives we will use in this class, you will not need to start from this definition because the derivative of many useful functions has already been solved.

All of the functions we will use in this class can be written as combinations of the function $f(x) = a x^b$ where a and b are some constant values. $f(x) = a x^b$ means we want to raise x to the b th power and then multiply the result by a . For example, if $a = 10$ and $b = 2$, $f(x) = 10 x^2$.

For functions of the form $f(x) = a x^b$, $f'(x) = ab x^{b-1}$.
For functions of the form, $f(x) = a$, $f'(x) = 0$.

Examples:

For $f(x) = 10 x^2$, $f'(x) = 10(2) x^{2-1} = 20 x^1 = 20 x$.

For $f(x) = 5 x^{-2} = 5 / x^2$, $f'(x) = 5(-2) x^{-2-1} = -10 x^{-3} = -10 / x^3$.

Problems:

Find the derivatives of

- a) $f(x) = 2 x^4$
- b) $f(x) = 4 x^{0.5}$
- c) $f(x) = 6 x^{-1}$
- d) $f(x) = 4 x^{-0.25}$
- e) $f(x) = 35$

Properties of Derivatives:

Let $f(x)$, $g(x)$, and $h(x)$ be three functions for which we can calculate derivatives:

Additive Property:

- (i) If $h(x) = f(x) + g(x)$, $h'(x) = f'(x) + g'(x)$.

Suppose we have a function like $h(x) = a x^b + c x^d$ where a , b , c , and d are constants.

We can set $f(x) = a x^b$ and $g(x) = c x^d$ such that $h(x) = f(x) + g(x)$. From above we know $f'(x) = ab x^{b-1}$ and $g'(x) = cd x^{d-1}$, so that

$$h'(x) = f'(x) + g'(x) = ab x^{b-1} + cd x^{d-1}.$$

Examples:

For $h(x) = 10x^2 + 5x^{-2}$, $f(x) = 10x^2$ and $g(x) = 5x^{-2}$,
so $h'(x) = 20x - 10/x^3$.

For $h(x) = 6x^{0.5} + 8x^{0.25}$, $f(x) = 6x^{0.5}$ and $g(x) = 8x^{0.25}$,
so $h'(x) = 6(0.5)x^{0.5-1} + 8(0.25)x^{0.25-1} = 3x^{-0.5} + 2x^{-0.75}$.

Problems:

Find the derivatives of

- f) $h(x) = 2x^2 + 9x^{-3}$
- g) $h(x) = 4x^{-0.5} + 7x^4$
- h) $h(x) = 6 + 5x^2$
- i) $h(x) = 3 + 4x + 5x^2$
- j) $h(x) = 1 + 3x + 8x^3 + 10x^4$

Multiplicative Property:

(ii) If $h(x) = f(x)g(x)$, $h'(x) = f'(x)g(x) + f(x)g'(x)$.

Suppose we have a function like $h(x) = (ax^b + c)(dx^k + m)$ where a, b, c, d, k and m are constants. We can set $f(x) = (ax^b + c)$ and $g(x) = (dx^k + m)$ such that $h(x) = f(x)g(x)$. From above we know $f'(x) = abx^{b-1}$ and $g'(x) = dkx^{k-1}$, so that $h'(x) = f'(x)g(x) + f(x)g'(x) = abx^{b-1}(dx^k + m) + (ax^b + c)dkx^{k-1}$.

Examples:

For $h(x) = (10 + 5x^2)(2 - x)$, $f(x) = (10 + 5x^2)$ and $g(x) = (2 - x)$,
so $h'(x) = 10x(2 - x) + (10 + 5x^2)(-1)$.

For $h(x) = (6 + 2x^3)(6 - x^{-1})$, $f(x) = (6 + 2x^3)$ and $g(x) = (6 - x^{-1})$,
so $h'(x) = 6x^2(6 - x^{-1}) + (6 + 2x^3)x^{-2}$.

Problems:

Find the derivatives of

- k) $h(x) = (3 + x^4)(2 + x^3)$
- l) $h(x) = (20 + x^{-2})(4 - x^6)$
- m) $h(x) = (10 + x^5)(8 - x^{-4})$
- n) $h(x) = (3x + x^2)(12 + x^8)$
- o) $h(x) = (1 + x^2)(4 + x^3)(2 + x)$

Chain Rule:

(iii) If $h(x) = f(g(x))$, $h'(x) = f'(g(x)) g'(x)$.

Suppose we have a function like $h(x) = a (c + d x^k)^b$ where a, b, c, d , and k are constants. We can set $f(x) = a x^b$ and $g(x) = c + d x^k$ such that $h(x) = f(g(x))$. From above we know $f'(x) = ab x^{b-1}$ and $g'(x) = dk x^{k-1}$, so that $h'(x) = f'(g(x))g'(x) = ab (c + d x^k)^{b-1} dk x^{k-1}$.

Examples:

For $h(x) = (1 + 4 x^2)^3$, $f(x) = x^3$ and $g(x) = 1 + 4 x^2$,
so $h'(x) = 3(1 + 4 x^2)^2 8 x$.

For $h(x) = (8 + 6 x^{0.5})^{-1}$, $f(x) = x^{-1}$ and $g(x) = 8 + 6 x^{0.5}$,
so $h'(x) = -1(8 + 6 x^{0.5})^{-2} 3 x^{-0.5}$.

Problems:

Find the derivatives of

- p) $h(x) = (7 + x^8)^{-4}$
- q) $h(x) = (15 + x^{-2})^6$
- r) $h(x) = (12 + 4x^{0.25})^{0.5}$
- s) $h(x) = (4x + 0.5 x^2)^{-3}$
- t) $h(x) = (14 + 4 x^2 - 0.25 x^8)^{-0.5}$

Quotient Property:

(iv) If $h(x) = f(x)/g(x)$, $h'(x) = (f'(x) g(x) - f(x)g'(x))/g(x)^2$.

Suppose we have a function like $(a x^b + c)/(d x^k + m)$ where a, b, c, d, k and m are constants. We can set $f(x) = a x^b + c$ and $g(x) = d x^k + m$ such that $h(x) = f(x)/g(x)$. From above we know $f'(x) = ab x^{b-1}$ and $g'(x) = dk x^{k-1}$, so that $h'(x) = (ab x^{b-1}(d x^k + m) - (a x^b + c) dk x^{k-1}) / (d x^k + m)^2$.

Examples:

For $h(x) = (10 + 5 x^2)/(2 - x)$, $f(x) = 10 + 5 x^2$ and $g(x) = 2 - x$,
so $h'(x) = (10 x (2 - x) - (10 + 5 x^2) (-x)) / (2 - x)^2$.

For $h(x) = (6 + 2 x^3) / (6 - x^{-1})$, $f(x) = (6 + 2 x^3)$ and $g(x) = (6 - x^{-1})$,
so $h'(x) = (6 x^2(6 - x^{-1}) - (6 + 2 x^3) x^{-2}) / (6 - x^{-1})^2$.

Problems:

Find the derivatives of

- u) $h(x) = (25 + 4x^{0.25}) / (8 + 7x^2)$
- v) $h(x) = (4 + 3x^{-1}) / (5 - 6x^3)$
- w) $h(x) = (12 + 0.5x^2) / (16 + 5x^{-3})$
- x) $h(x) = (14x - 18x^5) / (17 + x^8)$
- y) $h(x) = (13 + 5x^2 + 3x^4) / (1 + 13x + 5x^2)$

Partial Derivatives:

Partial derivatives are just derivatives when a function depends on more than one variable: e.g. $y = f(x, z)$. To take a partial derivative, you need to know with respect to what variable. That is, which variable is of interest to you. When taking the partial derivative of $f(x, z)$ with respect to x , we can think of it as taking the derivative of the function $h(x)$ where $h(x) = f(x, z)$. When taking the partial derivative of $f(x, z)$ with respect to z , we can think of it as taking the derivative of the function $g(z)$ where $g(z) = f(x, z)$. You treat everything but the variable of interest as some constant value. As mentioned before, in this class, $h(x)$ can always be written as ax^b where a and b are some constants and $g(z)$ can always be written as cz^d where c and d are some constants. For example, if $f(x, z) = 10x^2z^4$, $h(x) = ax^b$ where $a = 10z^4$ and $b = 2$ and $g(z) = cz^d$ where $c = 10x^2$ and $d = 4$. After writing the function in this form, everything we have done up to this point still applies. We can take the derivative of $f(x, z) = 10x^2z^4$ with respect to x , $f'_x(x, z)$, by taking the derivative of $h(x) = ax^b$, which equals $h'(x) = abx^{b-1} = 10z^4(2)x^{2-1} = 20z^4x$. We can take the derivative of $f(x, z) = 10x^2z^4$ with respect to z , $f'_z(x, z)$, by taking the derivative of $g(z) = cz^d$, which equals $g'(z) = cdz^{d-1} = 10x^2(4)z^{4-1} = 40x^2z^3$.

Problems:

Find the derivatives with respect to x of

- z) $f(x, z) = 5x^{0.5}z^{0.5}$
- aa) $f(x, z) = 5x^3 + 10z^2$
- bb) $f(x, z) = 5x^4/z$
- cc) $f(x, z) = x^2 - 8xz + 4z^2$

Now find the derivatives of z) – cc) with respect to z