Adjustment under trade liberalization, labor market segmentation, and informal employment: A dynamic general equilibrium analysis of a three-sector-open economy *(Preliminary; please do not cite)*

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Abstract

This paper analyses the effects of trade reforms in an economy with an informal sector and significant informal employment, defined as employment which does not abide with labor market regulations, including minimum wage and social security laws. Foreign trade reforms subject domestic firms to increased foreign competition, leading them to seek ways to cut back production costs, most notably labor costs. Cutting labor costs can be accomplished in one of three ways, including laying off workers (who subsequently look for employment in the informal sector) and possibly replacing them with part time workers; cutting down or eliminating worker benefits, putting the workers in informally employed status; or establishing subcontracting relationships with smaller scale firms which already employ workers informally. In this paper, we concentrate on the first effect. The effects of increased exposure to foreign competition are examined in the context of a dynamic general equilibrium model of a small open economy with three sectors including an informal sector, a formal sector, an agricultural sector, and a segmented labor market.

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1 Introduction

With the implementation of structural adjustment programs in developing countries during the 1980's and and early 1990's, a vast body of research on economic transformation in these countries has emerged. This literature has particularly focused on the transformation and changes in the labor markets, including flexibility issues and the employment effects of such programs. One principal argument set forth by this literature is that export-oriented growth strategy as part of structural adjustment programs has created a potential for increasing employment. At the heart of this argument is the comparative advantage theory. With increased degree of trade liberalization and export volume, it is expected that labor demand would increase due to labor intensity of production in developing countries (Krueger, 1983). However following the 1980's in Turkey, despite the increases in export volume and significant falls in real wages, the rate of increase in employment has remained below that occured during the period of import-substitution industrialization strategy (Ansal et al., 2000). One possible explanation as to why expected increases in employment have not materialized in response to increases in degree of trade liberalization is provided by rigidities in labor markets. However, one can say that developing country experiences and empirical studies do not support this view (Amsden and Hoeven, 1996; Boratav et al., 1996). Onaran (2003) has studied the effects of foreign trade on employment in Turkey in an empirical study, however, what is implied by employment is formal employment, only. Results from this study confirm that significant increases in the export volume following trade liberalization measures in Turkey after the 1980's have not led to equally significant increases in labor demand and employment.

In this study, we examine the relationship between the changes in degree of trade liberalization and employment, considering both formal and informal employment types. Goldberg and Pavcnik (2003) argue that foreign trade reforms expose establishments in the formal sector to increased foreign competition, and thus leading them to seek ways to cut back production costs, most notably labor costs. Cutting labor costs can be accomplished in one of three ways: the establishment can lay off workers, and those without a job can look for employment in the informal sector with lower pay. Secondly, the establishment may cut down or eliminate worker benefits, putting the workers in informally employed status. Lastly, in order to cut labor costs, the firm may establish subcontracting relationships with smaller scale firms which already employ workers informally. In the present study, we focus on the first effect described above.

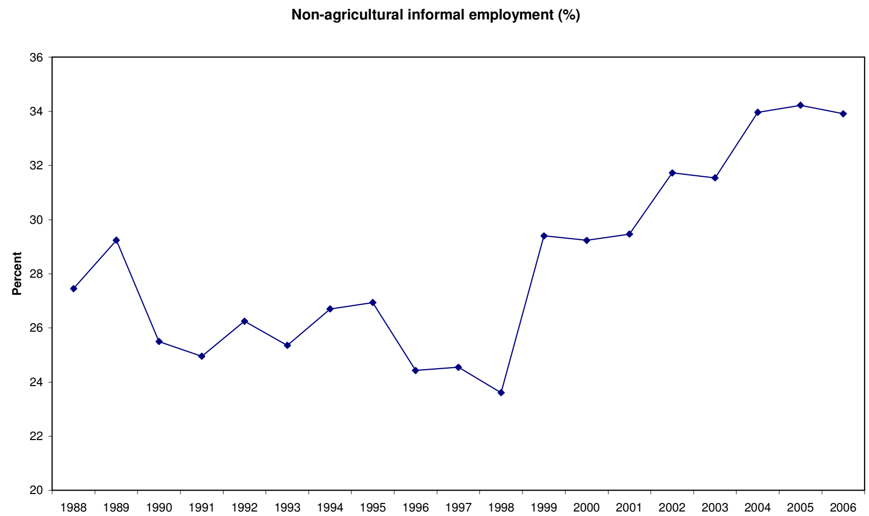


Figure 1: Informal employment in total non-agricultural employment (%), Turkey

Turkish Statistical Institute (TURKSTAT) defines informal employment in Turkey as employment not covered by any social security institution. Accordingly, in 2006, 48.5 percent of all employment in Turkey was informal, 49 percent of which was in agriculture, and the remaining 51 percent in non-agricultural sectors. Furthermore, 34 percent of all employed in non-agricultural sectors were informal. Figure 1 shows the progression of informal employment in non-agricultural sectors over the last 20 years. Figure 2, on the other hand, depicts that the share of non-agricultural informal employment in total informal employment has risen considerably over the last two decades when trade liberalization policies have been in effect: it has risen from 25 percent in 1988 to 51 percent in 2006. One important implication from these figures is that with the fall in employment in agriculture, informal non-agricultural employment has started rising over the years. That is, with the shift of labor from agricultural sector to non-agricultural sectors, the shift has mainly concentrated towards informal employment, rather than formal employment as more employment opportunuties are created in non-agricultural sectors. What's more, we also observe a shift within the non-agricultural sectors from formal towards informal employment in the last two decades.

The purpose of this study is to analyze the effects of various policies, including trade liberalization policies, on output, wages and employment in a dynamic general equilibrium

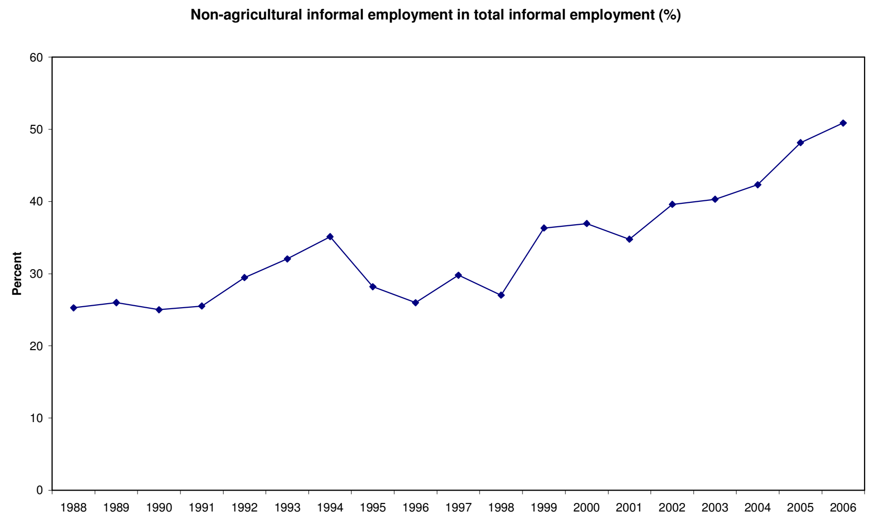


Figure 2: Non-agricultural informal employment in total informal employment (%), Turkey

model of a small open economy with three sectors including an informal sector, a formal sector, an agricultural sector, and a segmented labor market. Section 2 develops the theoretical model framework, introducing the labor market structure, the production sectors, and the behavior of households. Competitive equilibrium is defined and characterized in this section, steady state and transition path equilibria are also solved for. Section 3 presents the comparative statics analysis of various policy changes, including increasing subsidies to the export sector, reducing tariffs in agricultural sector, changes in minimum wage, and changes in the exchange rate, and a numerical solution to the model with model calibration with policy simulation results and finally Section 4 summarizes the main findings of the paper, introduces further study, and concludes.

2 The framework of the theoretical model

In the theoretical model, we examine a small open economy with three production sectors. The production sectors included in the model economy are the agricultural sector, the informal sector and the formal sector. The primary objective in constructing the theoretical model is to analyze the linkages between the formal and informal sectors as capital accumulates and as the economy grows through time. The linkages between these two sectors materialize through the workings of the labor market. The secondary objective is to observe the changes in the production sectors as the economy exposes its markets to increased foreign competition.

In the model economy, in addition to three production sectors, there are three economic agents: the producer, the household and the government. The production takes place using four production factors: capital, skilled labor, unskilled labor, and land. The household owns all production factors, and generates income from renting them. The formal sector utilizes capital, unskilled labor and skilled labor in production, and produces a traded good which is both an investment and a consumption good. The informal sector uses capital and unskilled labor in production, and produces a non-traded consumption good. The agricultural sector rents land and hires unskilled labor in production, and produces a traded pure consumption good. Although foreign trade of goods are allowed in the model, there is no international mobility in labor and capital. Within the economy, capital is perfectly mobile across all sectors, while the labor market is segmented. Land can be rented in and out only within the agricultural sector. Finally, the government only serves to collect taxes and tariffs, and distribute subsidies and transfers, and has no consumption and investment behavior.

2.1 Labor market structure

One important feature of the theoretical model is that the labor market is segmented. The literature on segmented labor markets has gained momentum especially with Mazumdar (1983), and subsequently has focused on the formal versus informal labor markets analysis. In the present study, in modelling the labor markets, we follow the structure introduced in Agenor and Aizenman (1999). In the model, two types of labor are defined: skilled and unskilled. Skilled labor is employed only in the formal sector, while unskilled labor is employed in all production sectors. In segmented labor markets, distinct wages arise. The wage of the unskilled labor employed in informal and agricultural sectors is determined in a fully competitive labor market (i.e. an informal labor market), and is fully flexible. On the other hand, unskilled labor employed in the formal sector is paid a legally determined minimum wage. Lastly, skilled labor employed in the formal sector earns an efficiency wage above the market equilibrium wage. Once the formal sector decides on how much unskilled and skilled labor to hire, any labor that is not hired by the formal sector is absorbed by the informal labor market (to be employed in the informal and agricultural sectors). As a consequence, there is no unemployment in the model. Since any skilled labor that is not hired in the formal sector can also be seeking employment in the informal labor market, there may well emerge an ineffcent allocation of labor.

2.2 Production sectors

As mentioned before, production takes place in three sectors. Producers in all three sectors have a similar motive: minimize costs and maximize profits. They all face a constant returns to scale, Cobb-Douglas-type production technology.

2.2.1 Formal sector

Production in the formal sector follows a Cobb-Douglas production technology:

$$Y_F = B_F(\varepsilon L_s)^{\delta_1} L_{u,F}^{\delta_2} K_F^{\delta_3}$$

where Y_F is the formal sector production volume, L_s is the formal sector skilled labor use, $L_{u,F}$ is the formal sector unskilled labor use, K_F is the formal sector capital use, ε is the skilled worker effort coefficent, and $B_F > 0$ is a constant. Here, $\delta_1, \delta_2, \delta_3 \in (0, 1)$ and $\delta_1+\delta_2+\delta_3 = 1$.

Skilled worker effort The skilled worker effort analysis in this study coincides with that in Agenor and Aizenman (1999). Skilled labor has a preference between showing an effort of ε and working (earning a wage of ω), and not working (i.e. showing an effort of only $1 - \varepsilon$), summarized by the utility function $u(\omega, \varepsilon)$:

$$\begin{aligned} u(\omega, \varepsilon) &= \ln[\omega^{\gamma}(1-\varepsilon)^{1-\gamma}] \\ 0 &< \gamma < 1 \end{aligned}$$

Assume that with probability $0<\phi < 1$, a skilled worker employed in the formal sector is caught shirking on the job. If the worker is caught shirking on the job with probability ϕ, then the worker will be fired from the formal sector job paying ω_s, and will be compelled to look for a job in the informal labor market with wage ω_I. Accordingly, the total expected utility that the worker gains by showing effort ε and earning a wage of ω_s must be at least as much as the total expected utility gained by not showing any effort and shirking on the job ($\varepsilon = 0$):

$$\gamma \ln \omega_s + (1-\gamma)\ln(1-\varepsilon) \geq \phi\gamma \ln \omega_I + (1-\phi)\gamma \ln \omega_s$$

In equilibrium, the worker is indifferent between showing or not showing any effort:

$$\gamma \ln \omega_s + (1-\gamma)\ln(1-\varepsilon) = \phi\gamma \ln \omega_I + (1-\phi)\gamma \ln \omega_s$$

which implies that

$$(1-\varepsilon)^{1-\gamma} = \left(\frac{\omega_I}{\omega_s}\right)^{\phi\gamma}$$

or,

$$\varepsilon = 1 - \left(\frac{\omega_I}{\omega_s}\right)^{\beta}, \ \beta = \frac{\phi\gamma}{1-\gamma} > 0 \tag{1}$$

This equation indicates that the effort that skilled worker shows in equilibrium increases with formal sector skilled worker wage, and decreases with informal labor market wage.

Formal sector analysis Representative producer in the formal sector chooses the allocation of capital and skilled and unskilled labor amounts, along with the wages to be paid to the skilled worker that minimize total costs. As previously shown, skilled labor wage depends on the skilled worker effort, while the wage of unskilled labor, minimum wage $\bar{\omega}_u$, and the unit cost of capital or the interest rate r are taken as given by the producer. Accordingly, the cost minimization problem of the formal sector producer is given by

$$\min_{\omega_s, L_s, L_{u,F}, K_F} \ \omega_s L_s + \bar{\omega}_u L_{u,F} + r K_F$$

$$\begin{aligned} \text{s.t. } B_F(\varepsilon L_s)^{\delta_1} L_{u,F}^{\delta_2} K_F^{\delta_3} &\geq Y_F \\ L_s, L_{u,F}, K_F &\geq 0 \end{aligned}$$

where

$$\varepsilon = 1 - \left(\frac{\omega_I}{\omega_s}\right)^{\beta}$$

From the minimization problem above, we obtain

$$L_s = (\frac{\delta_1}{\delta_2})(\frac{\bar{\omega}_u}{\omega_s})L_{u,F} \tag{2}$$

$$K_F = (\frac{\delta_3}{\delta_2})(\frac{\bar{\omega}_u}{r})L_{u,F} \tag{3}$$

$$\frac{\omega_I}{\omega_s} = \frac{1}{\sigma}, \ \sigma = (1+\beta)^{1/\beta} \tag{4}$$

$$\varepsilon = \frac{\beta}{1+\beta} \tag{5}$$

That is, in equilibrium, effort ε is a constant, and is a function of the probability of getting caught when shirking, and γ (share of utility gained by working and earning a wage). Using (2), (3) and (5), we have the following factor demand functions:

$$L^*_{u,F} = Y_F B_F^{-1} (\beta/1+\beta)^{-\delta_1} (\frac{\delta_1}{\delta_2})^{-\delta_1} (\frac{\delta_3}{\delta_2})^{-\delta_3} (\frac{\bar{\omega}_u}{\omega_s})^{-\delta_1} (\frac{\bar{\omega}_u}{r})^{-\delta_3} \quad (6)$$

$$L^*_{s} = Y_F B_F^{-1} (\beta/1+\beta)^{-\delta_1} (\frac{\delta_1}{\delta_2})^{1-\delta_1} (\frac{\delta_3}{\delta_2})^{-\delta_3} (\frac{\bar{\omega}_u}{\omega_s})^{1-\delta_1} (\frac{\bar{\omega}_u}{r})^{-\delta_3} \quad (7)$$

$$K^*_{F} = Y_F B_F^{-1} (\beta/1+\beta)^{-\delta_1} (\frac{\delta_1}{\delta_2})^{-\delta_1} (\frac{\delta_3}{\delta_2})^{1-\delta_3} (\frac{\bar{\omega}_u}{\omega_s})^{-\delta_1} (\frac{\bar{\omega}_u}{r})^{1-\delta_3} \quad (8)$$

The resulting minimum total cost of the formal sector firm is found as

$$\begin{aligned} TC_F &= \omega_s L^*_s + \bar{\omega}_u L^*_{u,F} + r K^*_F \\ &= Y_F \left[\frac{\omega_s(1+\beta)}{\beta} \right]^{\delta_1} \bar{\omega}_u^{\delta_2} r^{\delta_3} \end{aligned}$$

Here, $B_F \equiv \delta_1^{-\delta_1} \delta_2^{-\delta_2} \delta_3^{-\delta_3}$. Under perfect competition in goods markets,

$$\begin{aligned} p_F &= MC_F \\ MC_F &= \frac{\partial TC_F}{\partial Y_F} \end{aligned}$$

where MC_F is the marginal cost in the formal sector. Then, it must be that

$$p_F = \left[\frac{\omega_s(1+\beta)}{\beta} \right]^{\delta_1} \bar{\omega}_u^{\delta_2} r^{\delta_3}$$

in equilibrium. Unit price p_F in formal sector is defined as

$$p_F \equiv p_F^W E(1+\tau_F)$$

where p_F^W is the world price of the product, E is the exchange rate, and τ_F represents the subsidies to the formal (export) sector.

2.2.2 Informal sector

Using a constant returns to scale-Cobb-Douglas technology, the informal sector firm produces output Y_I,

$$Y_I = B_I L_{u,I}^{\eta} K_I^{1-\eta}$$

where $L_{u,I}$ is the informal sector unskilled labor use, K_I is the informal sector capital use, $0<\eta < 1$, and $B_I > 0$ is a constant, $B_I \equiv \eta^{-\eta}(1-\eta)^{-(1-\eta)}$. Perfectly competitive, cost-minimizing informal sector firm has the indirect cost of

$$\begin{aligned} TC_I &= \omega_I L^*_{u,I} + rK^*_I \\ &= Y_I \omega_I^{\eta} r^{1-\eta} \end{aligned}$$

Under perfect competition in product markets, profit maximization (equilibrium) condition is

$$\begin{aligned} p_I &= MC_I \\ MC_I &= \frac{\partial TC_I}{\partial Y_I} \end{aligned}$$

p_I is the unit price of the informal sector product. Then, one can rewrite the equilibrium condition as

$$p_I = \omega_I^{\eta} r^{1-\eta}$$

2.2.3 Agricultural sector

Agricultural sector uses technology

$$Y_A = B_A(L_{u,A})^{\alpha_1} K_A^{\alpha_2} T^{\alpha_3}$$

where Y_A is the agricultural output, $L_{u,A}$ is the unskilled labor use in agriculture, K_A is the capital use in agriculture, T is the fixed land factor, $B_A > 0$ is a constant, and $\alpha_1+\alpha_2+\alpha_3 = 1$ with $\alpha_1, \alpha_2, \alpha_3 \in (0, 1)$. Since land is a fixed factor, returns to scale in labor and capital in agriculture are diminishing. As in the informal sector, agricultural sector employes labor at flexible wage ω_I.Optimal agricultural output under cost minimization is found to be

$$Y_A^* = B_A^{1/\alpha_3} p_A^{\frac{\alpha_1+\alpha_2}{\alpha_3}} (\frac{\alpha_1}{\omega_I})^{\frac{\alpha_1}{\alpha_3}} (\frac{\alpha_2}{r})^{\frac{\alpha_2}{\alpha_3}} T$$

Given that the unit world price of the agricultral product is p_A^W, the domestic price is $p_A = p_A^W E(1+\tau_A)$, where τ_A is the import tariff rate. Indirect agricultural profits (or, land rents) are

$$
\begin{aligned}
\Pi_A^* &= p_A Y_A^* - [\omega_I L_{U,A}^* + r K_A^*] \\
&= p_A Y_A^* - p_A \alpha_1 Y_A^* - p_A \alpha_2 Y_A^* \\
&= p_A Y_A^* (1 - \alpha_1 - \alpha_2) \\
&= p_A B_A^{1/\alpha_3} p_A^{\frac{\alpha_1+\alpha_2}{\alpha_3}} (\frac{\alpha_1}{\omega_I})^{\frac{\alpha_1}{\alpha_3}} (\frac{\alpha_2}{r})^{\frac{\alpha_2}{\alpha_3}} T \alpha_3 \\
&= p_A^{1/\alpha_3} \omega_I^{-\frac{\alpha_1}{\alpha_3}} r^{-\frac{\alpha_2}{\alpha_3}} T
\end{aligned}
$$

Here, $B_A \equiv \alpha_1^{-\alpha_1} \alpha_2^{-\alpha_2} \alpha_3^{-\alpha_3}$ is assumed. From now on, for simplicity, agricultural land factor is normalized to 1, $T = 1$.

2.3 Household behavior

There is a representative household who consumes and realized expenditures on all three types of goods: an agricultural good, a formally produced good, and an informally produced good. The representative household has a two-stage consumption choice problem: intertemporally, the representative household decides how much to save and how much to spend on total consumption, and within each period she chooses how to allocate total spending among three different consumption items. The instantaneous composite consumption function of the representative household is given as

$$c' = B_c c_F^{\lambda_1} c_A^{\lambda_2} c_I^{\lambda_3}$$

where c_F is the consumption of formally produced good, c_A is the consumption of agricultural good, and c_I is the consumption of informal good. Here, $\lambda_1 + \lambda_2 + \lambda_3 = 1$, $\lambda_1, \lambda_2, \lambda_3 \in (0, 1)$, and $B_c > 0$ is a constant. Then, in every period, the representative household minimizes total expenditures to choose (c_F, c_I, c_A) such that she solves the problem

$$\min \; p_F c_F + p_A c_A + p_I c_I$$

$$
\begin{aligned}
\text{s.t. } B_c c_F^{\lambda_1} c_A^{\lambda_2} c_I^{\lambda_3} &\geq c' \\
c_F, c_A, c_I &> 0
\end{aligned}
$$

Udner the assumption that $B_c \equiv \lambda_1^{-\lambda_1} \lambda_2^{-\lambda_2} \lambda_3^{-\lambda_3}$, the minimum total expenditures in every period are

$$\mathsf{E} = p_F^{\lambda_1} p_A^{\lambda_2} p_I^{\lambda_3} c'$$

Representative household demand for each type of good can be found to be

$$c_F = \frac{\partial \mathsf{E}}{\partial p_F} = \lambda_1 p_F^{\lambda_1 - 1} p_A^{\lambda_2} p_I^{\lambda_3} c' = \lambda_1 \frac{\mathsf{E}}{p_F} \tag{9}$$

$$c_A = \frac{\partial \mathsf{E}}{\partial p_A} = \lambda_2 p_F^{\lambda_1} p_A^{\lambda_2 - 1} p_I^{\lambda_3} c' = \lambda_2 \frac{\mathsf{E}}{p_A} \tag{10}$$

$$c_I = \frac{\partial \mathsf{E}}{\partial p_I} = \lambda_3 p_F^{\lambda_1} p_A^{\lambda_2} p_I^{\lambda_3 - 1} c' = \lambda_3 \frac{\mathsf{E}}{p_I} \tag{11}$$

Intertemporally, the representative household wishes to maximize the present value of discounted intertemporal utility, U, as given by the function

$$\max_{(c',a)} \int_0^\infty \frac{c'(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to the intertemporal budget constraint, the transversality constraint, non-negativity and initial asset value constraints:

$$\begin{aligned} \text{s.to } \dot{a}(t) &= \Omega(t) + r(t)a(t) + \Upsilon(t) - \mathsf{E}(\mathsf{t}) \\ & \lim_{t\to\infty} \int_{t=0}^{\infty} a(t)v(t) = 0 \\ c'(t) &> 0 \\ a(0) &\leq a_0 \end{aligned}$$

In the intertemporal budget constraint, a represents per capita assets, $\dot{a}$ the accumulated assets; Ω represents income from all types of labor, ra is the return on assets owned, Υ is transfers from government, E is total expenditures on consumption, $\rho > 0$ is a constant denoring the rate of time preference, and finally $\frac{1}{\theta}$ is the elasticity of intertemporal substitution. Solution to the intertemporal problem of the representative household implies the Ramsey rule for optimal saving:

$$\frac{\dot{c}'(t)}{c'(t)} = \frac{1}{\theta}\left[r(t) - \rho - \lambda_1 \frac{\dot{p}_F(t)}{p_F(t)} - \lambda_2 \frac{\dot{p}_A(t)}{p_A(t)} - \lambda_3 \frac{\dot{p}_I(t)}{p_I(t)}\right] \tag{12}$$

Since the prices of goods subject to international trade are taken as given (are constants unless otherwise stated), $\frac{\dot{p}_F(t)}{p_F(t)} = \frac{\dot{p}_A(t)}{p_A(t)} = 0$, then

$$\frac{\dot{c}'(t)}{c'(t)} = \frac{1}{\theta}\left[r(t) - \rho - \lambda_3 \frac{\dot{p}_I(t)}{p_I(t)}\right] \tag{13}$$

Accordingly, the evolution (or, the growth) of the representative household's composite consumption mainly depends on the interest rate, the rate of time preference, and the rate of change in price of home-good (the informal sector good).

2.4 Competitive Equilibrium

Definition. A competitive equilibrium for this economy is a list of sequences of output prices, consumption levels, wage rates, capital and land rental rates, and production plans for each of the sectors, such that

(i) given output and factor prices, the representative household maximizes the present value of her discounted intertemporal utility;

(ii) given output and factor prices, representative firms in each sector maximize profits;

(iii) market clears in the non-tradeable (informal) goods market;

(iv) capital market clears;

(v) informal labor market clears;

(vi) skilled labor is indifferent between shirking (not showing any effort) and not shirking on the job, as such, skilled labor wage depends on equilibrium effort;

(vii) Walras' Law holds;

(viii) no-arbitrage condition holds between capital and land assets;

(ix) total taxes collected by the government equal total transfers plus total subsidies paid by the government, i.e. government budget balances every period.

2.4.1 Characterization of competitive equilibrium

In equilibrium, we have stated that profit maximization occurs in formal and informal sectors implies

$$\begin{aligned} MC_F(\omega_s, \bar{\omega}_u, r) &= p_F \\ MC_I(\omega_I, r) &= p_I \end{aligned}$$

That is, at any point of eqilibrium, it must be true that

$$\begin{aligned} \left[\frac{\omega_s(1+\beta)}{\beta}\right]^{\delta_1} \bar{\omega}_u^{\delta_2} r^{\delta_3} &= p_F \\ \omega_I^{\eta} r^{1-\eta} &= p_I \end{aligned}$$

Above, p_F is exogenously given, while p_I is an endogenous variable. In addition, we have found that in equilibrium, formal sector skilled labor wages are a multiple of the flexible informal labor wages:

$$\omega_s = \sigma\omega_I$$

Using these three equilibrium conditions, we can express

$$\begin{aligned} r &= r(p_I), && (14) \\ \omega_I &= w(p_I). && (15) \end{aligned}$$

As mentioned before, there are two types of labor in the economy: skilled and unskilled. Let's say that skilled labor supply is L_s^s, and unskilled labor supply is L_u^s. If total economywide labor supply is L, it must be that

$$L_s^s + L_u^s = L$$

In the formal sector, skilled labor demand is

$$L_s^d = \frac{\partial MC_F}{\partial \omega_s} Y_F$$

and unskilled labor demand is

$$L_{u,F}^d = \frac{\partial MC_F}{\partial \bar{\omega}_u} Y_F$$

By construction of our economy, we know that whoever is not hired in the formal sector, either as skilled or unskilled labor, will be absorbed as unskilled labor in the informal labor market, under wage ω_I. Then,

$$L_s^s - L_s^d + L_u^s - L_{u,F}^d = L_u^d$$

Here,

$$\begin{aligned} L_u^d &= L_{u,A}^d + L_{u,I}^d \\ &= -\frac{\partial \Pi_A^*}{\partial \omega_I} + \frac{\partial MC_I}{\partial \omega_I} Y_I \end{aligned}$$

That is,

$$L_s^s - \frac{\partial MC_F}{\partial \omega_s} Y_F + L_u^s - \frac{\partial MC_F}{\partial \bar{\omega}_u} Y_F = -\frac{\partial \Pi_A^*}{\partial \omega_I} + \frac{\partial MC_I}{\partial \omega_I} Y_I$$

or,

$$\underbrace{-\frac{\partial \Pi_A^*}{\partial \omega_I} + \frac{\partial MC_F}{\partial \bar{\omega}_u} Y_F + \frac{\partial MC_F}{\partial \omega_s} Y_F + \frac{\partial MC_I}{\partial \omega_I} Y_I}_{\text{Total labor demand}} = \underbrace{L}_{\text{Total labor supply}} \qquad (16)$$

Similarly, capital market clearing condition is given as

$$\underbrace{-\frac{\partial \Pi_A^*}{\partial r} + \frac{\partial MC_F}{\partial r} Y_F + \frac{\partial MC_I}{\partial r} Y_I}_{\text{Total capital demand}} = \underbrace{K}_{\text{Capital stock}} \tag{17}$$

Expressing both factor market clearing conditions in per capita terms, we obtain[1]

$$-\frac{\partial \Pi_A^*}{\partial \omega_I} + \frac{\partial MC_F}{\partial \bar{\omega}_u} y_F + \frac{\partial MC_F}{\partial \omega_s} y_F + \frac{\partial MC_I}{\partial \omega_I} y_I = 1 \tag{18}$$

$$-\frac{\partial \Pi_A^*}{\partial r} + \frac{\partial MC_F}{\partial r} y_F + \frac{\partial MC_I}{\partial r} y_I = k \tag{19}$$

We note that labor market clearing and capital market clearing conditions are linear in both y_F and in y_I. Substituting for ω_I and for r in (18) and (19), one can solve for functions of per capita output y_F and y_I in terms of p_I and k (and the relevant exogenously given variables):

$$\begin{aligned} y_F &= y_F(p_I, k) \\ y_I &= y_I(p_I, k) \end{aligned}$$

On the other hand, imposing market clearing in the informal goods market, i.e.,

$$c_I = y_I(p_I, k)$$

we have

$$\begin{aligned} \lambda_3 c' p_F^{\lambda_1} p_A^{\lambda_2} p_I^{\lambda_3 - 1} &= y_I(p_I, k) \\ c' p_F^{\lambda_1} p_A^{\lambda_2} p^{\lambda_3} &= \frac{p_I y_I(p_I, k)}{\lambda_3} \end{aligned} \tag{20}$$

[1] In explicit form, one can write labor market clearing condition and capital market clearing condition, respectively, as follows:

$$\begin{aligned} & \frac{\alpha_1}{\alpha_3} p_A^{1/\alpha_3} \omega_I^{-\frac{\alpha_1}{\alpha_3}-1} r^{-\frac{\alpha_2}{\alpha_3}} + \delta_2 [\frac{\omega_s(1+\beta)}{\beta}]^{\delta_1} \overline{\omega}_u^{\delta_2 - 1} r^{\delta_3} y_F \\ & + \delta_1 (\frac{1+\beta}{\beta})^{\delta_1} \omega_s^{\delta_1 - 1} \overline{\omega}_u^{\delta_2} r^{\delta_3} y_F + \eta \omega_I^{\eta - 1} r^{1-\eta} y_I \\ = \; & 1 \\ & \frac{\alpha_2}{\alpha_3} p_A^{1/\alpha_3} \omega_I^{-\frac{\alpha_1}{\alpha_3}} r^{-\frac{\alpha_2}{\alpha_3}-1} + \delta_3 [\frac{\omega_s(1+\beta)}{\beta}]^{\delta_1} \overline{\omega}_u^{\delta_2} r^{\delta_3 - 1} y_F \\ & + (1-\eta) \omega_I^{\eta} r^{-\eta} y_I \\ = \; & k \end{aligned}$$

At the same time, we know from the representative household's intertemporal utility maximization that

$$\begin{aligned}\frac{\dot{c}'}{c'} + \lambda_3 \frac{\dot{p}_I}{p_I} &= \frac{1}{\theta}\left[r - \rho - \lambda_3 \frac{\dot{p}_I}{p_I}\right] + \lambda_3 \frac{\dot{p}_I}{p_I} \\ &= \frac{1}{\theta}(r-\rho) + \left(\frac{\theta - 1}{\theta}\right)\lambda_3 \frac{\dot{p}_I}{p_I} \end{aligned} \tag{21}$$

Total time-differentiating both sides of (20), we have

$$\begin{aligned} &\lambda_3 p_F^{\lambda_1} p_A^{\lambda_2} p_I^{\lambda_3} \dot{c}' + \lambda_3 p_F^{\lambda_1} p_A^{\lambda_2} p_I^{\lambda_3 - 1} \dot{p}_I c' \lambda_3 \\ = \; & \dot{p}_I y_I(p_I, k) + p_I \left[\frac{\partial y_I(p_I,k)}{\partial p_I}\dot{p}_I + \frac{\partial y_I(p_I,k)}{\partial k}\dot{k}\right] \end{aligned} \tag{22}$$

Rearranging (22),

$$\begin{aligned} &\lambda_3 p_F^{\lambda_1} p_A^{\lambda_2} p_I^{\lambda_3} c' \left[\frac{\dot{c}'}{c'} + \lambda_3 \frac{\dot{p}_I}{p_I}\right] \\ = \; & \dot{p}_I y_I(p_I, k) + p_I \left[\frac{\partial y_I(p_I,k)}{\partial p_I}\dot{p}_I + \frac{\partial y_I(p_I,k)}{\partial k}\dot{k}\right] \end{aligned}$$

or,

$$\begin{aligned} &p_I y_I(p_I, k) \left[\frac{\dot{c}'}{c'} + \lambda_3 \frac{\dot{p}_I}{p_I}\right] \\ = \; & \dot{p}_I y_I(p_I, k) + p_I \left[\frac{\partial y_I(p_I,k)}{\partial p_I}\dot{p}_I + \frac{\partial y_I(p_I,k)}{\partial k}\dot{k}\right] \end{aligned}$$

Using (21) and rearranging, this becomes

$$\begin{aligned} &p_I y_I(p_I, k) \left[\frac{1}{\theta}(r-\rho) + \left(\frac{\theta - 1}{\theta}\right)\lambda_3 \frac{\dot{p}_I}{p_I}\right] \\ = \; & \dot{p}_I y_I(p_I, k) + p_I \left[\frac{\partial y_I(p_I,k)}{\partial p_I}\dot{p}_I + \frac{\partial y_I(p_I,k)}{\partial k}\dot{k}\right] \end{aligned}$$

which yields us an expression for the time derivative of p_I :

$$\dot{p}_I = \frac{p_I \frac{\partial y_I(p_I,k)}{\partial k}\dot{k} - \frac{p_I}{\theta} y_I(p_I,k)[r(p_I) - \rho]}{y_I(p_I,k)\left(\frac{\theta-1}{\theta}\right)\lambda_3 - \left[y_I(p_I,k) + p_I \frac{\partial y_I(p_I,k)}{\partial p_I}\right]} \tag{23}$$

The last step in characterization involves deriving the $\dot{k}$ equation in terms of p_I, k and the other relevant exogenous variables and parameters of the model. The intertemporal budget constraint of the representative household can be expressed as[2]

$$\begin{aligned}\dot{k} &= \frac{1}{p_F}\Omega(p_I,k) + r(p_I)k + \frac{1}{p_F}\pi(p_I) + \frac{1}{p_F}\Upsilon(p_I,k) - \frac{1}{p_F}\mathsf{E}(p_I,k) \\ &= f_1(p_I,k) \end{aligned} \tag{24}$$

[2] This function for capital per capita accumulation is derived from the representative household's intertemporal budget constraint. Assuming that capital markets are closed to international flows, we can say that total per capita assets are composed of capital holdings and land holdings as follows:

$$a = p_k k + p_T T$$

Then,

$$\dot{a} = p_k \dot{k} + \dot{p}_T T$$

Plugging this in the representative household's intertemporal budget constraint,

$$p_k \dot{k} + \dot{p}_T T = \Omega + r(p_k k + p_T T) + \Upsilon - \mathsf{E}$$

or,

$$\dot{k} = \frac{1}{p_k}[\Omega + r(p_k k + p_T) + \Upsilon - \dot{p}_T - \mathsf{E}]$$

where $T = 1$. Under the assumption that there are constant returns to scale in all production processes, it has to be the case that,

$$r = \frac{\pi}{p_T} + \frac{\dot{p}_T}{p_T}$$

which is also the equilibrium indifference condition (or, the arbitrage condition) for the household. This condition assures that the household is indifferent in terms of the returns to land and the returns to capital in equilibrium. Accordingly, the household's intertemporal budget constarint can be re-expressed as the economy's resource constraint:

$$\begin{aligned}\dot{k} &= \frac{1}{p_k}[\Omega + r p_k k + \pi + \Upsilon - \mathsf{E}] \\ \dot{k} &= \frac{1}{p_k}\Omega + rk + \frac{1}{p_k}\pi + \frac{1}{p_k}\Upsilon - \frac{1}{p_k}\mathsf{E}\end{aligned}$$

Here, the price of capital p_k is in fact the price of the formal good, then

$$p_k = p_F$$

can be replaced in the equation above.

where

$$\begin{aligned}
\Omega(p_I,k) &= [\sigma\omega_I(p_I)-\omega_I(p_I)]\times\frac{\partial MC_F}{\partial\omega_s}y_F(p_I,k) \\
&\quad +[\bar{\omega}_u-\omega_I(p_I)]\times\frac{\partial MC_F}{\partial\bar{\omega}_u}y_F(p_I,k)+\omega_I(p_I) \\
\pi(p_I) &= p_A^{1/\alpha_3}\omega_I(p_I)^{-\frac{\alpha_1}{\alpha_3}}r(p_I)^{-\frac{\alpha s_2}{\alpha_3}} \\
\Upsilon(p_I,k) &= \left[\frac{\lambda_2}{\lambda_3}\frac{p_I y_I(p_I,k)}{p_A}-y_A(p_I)\right]\times[\tau_A-\tau_F\frac{p_A}{p_F}] \\
\mathsf{E}(p_I,k) &= \frac{p_I y_I(p_I,k)}{\lambda_3}
\end{aligned}$$

Finally, replacing for (24) in (23), the resulting differential equation for $\dot{p}_I$ solely in terms of (p_I,k) can be obtained:

$$\dot{p}_I = f_2(p_I,k) \tag{25}$$

The reduced system of two differential equations (24) and (25) together with an intial condition for capital per capita, k_0, and the transversality condition characterize the dynamic competitive equilibria.

2.4.2 Steady state analysis

In the long run (steady state) equilibrium of the model economy, it must be true that

$$\begin{aligned}
\dot{k} &= 0 \\
\dot{c}' &= 0 \\
\dot{p}_I &= 0
\end{aligned}$$

that is, all endogenous variables are constant. Accordingly, at the steady state equilibrium, eqution (13) implies

$$r_{ss}=\rho$$

where r_{ss} is the steady state value of the capital rental rate. Under the steady state condition, the informal labor market wage and the price of informal sector good at the steady state become

$$(\omega_I^*)_{ss}=\frac{p_F^{1/\delta_1}}{\beta(1+\beta)^{\frac{1}{\beta}-1}\bar{\omega}_u^{\delta_2/\delta_1}\rho^{\delta_3/\delta_1}} \tag{26}$$

$$(p_I)_{ss} = \frac{\rho^{\frac{\delta_1-\eta(\delta_1+\delta_3)}{\delta_1}} p_F^{\frac{\eta}{\delta_1}}}{[\beta(1+\beta)^{\frac{1}{\beta}-1}]^{\eta}\bar{\omega}_u^{\frac{\delta_2}{\delta_1}\eta}} \tag{27}$$

This allows us to rewrite the labor market and capital market clearing conditions at the steady state as follows:

$$\begin{aligned} & \frac{\alpha_1}{\alpha_3}(p_A)^{1/\alpha_3}(\omega_I^*)_{ss}^{-\frac{\alpha_1+\alpha_3}{\alpha_3}} r_{ss}^{-\frac{\alpha_2}{\alpha_3}} + \delta_2[\beta(1+\beta)^{\frac{1}{\beta}-1}(\omega_I^*)_{ss}]^{\delta_1}\bar{\omega}_u^{\delta_2-1} r_{ss}^{\delta_3}(y_F^*)_{ss} \\ & +(\frac{1+\beta}{\beta})^{\delta_1}\delta_1[\frac{\beta}{1+\beta}(\omega_I^*)_{ss}]^{\delta_1-1}\bar{\omega}_u^{\delta_2} r_{ss}^{\delta_3}(y_F^*)_{ss} + \eta(\omega_I^*)_{ss}^{\eta-1} r_{ss}^{1-\eta}(y_I^*)_{ss} \\ = \; & 1 \qquad \text{(Labor market clearing)} \end{aligned}$$

$$\begin{aligned} & \frac{\alpha_2}{\alpha_3}(p_A)^{1/\alpha_3}(\omega_I^*)_{ss}^{-\frac{\alpha_1}{\alpha_3}} r_{ss}^{-\frac{\alpha_2+\alpha_3}{\alpha_3}} + \delta_3[\beta(1+\beta)^{\frac{1}{\beta}-1}(\omega_I^*)_{ss}]^{\delta_1}\bar{\omega}_u^{\delta_2} r_{ss}^{\delta_3-1}(y_F^*)_{ss} \\ & +(1-\eta)(\omega_I^*)_{ss}^{\eta} r_{ss}^{-\eta}(y_I^*)_{ss} \\ = \; & k_{ss} \qquad \text{(Capital market clearing)} \end{aligned}$$

These two factor market clearing conditions yield steady state formal and informal sector output values $(y_f^*)_{ss}$ and $(y_I^*)_{ss}$ in terms of k_{ss} and given parameters of the model:

$$\begin{aligned} (y_F^*)_{ss} &= y_F(k_{ss}, p_A, p_F, \beta, \bar{\omega}_u, \rho, \alpha_1, \alpha_2, \alpha_3, \delta_1, \delta_2, \delta_3, \eta) \\ (y_I^*)_{ss} &= y_I(k_{ss}, p_A, p_F, \beta, \bar{\omega}_u, \rho, \alpha_1, \alpha_2, \alpha_3, \delta_1, \delta_2, \delta_3, \eta) \end{aligned}$$

Independently, we may also find the steady state agricultural output value as

$$\begin{aligned} (y_A^*)_{ss} &= \frac{\partial(\pi_A^*)_{ss}}{\partial p_A} \\ &= \frac{1}{\alpha_3} p_A^{\frac{1}{\alpha_3}-1} \omega_{I,ss}^{-\frac{\alpha_1}{\alpha_3}} \rho^{-\frac{\alpha_2}{\alpha_3}} \end{aligned}$$

Recall that we have obtained the economy's resource constraint using the household's budget constraint as follows:

$$\dot{k} = \frac{1}{p_F}\Omega(p_I,k) + r(p_I)k + \frac{1}{p_F}\pi(p_I) + \frac{1}{p_F}\Upsilon(p_I,k) - \frac{1}{p_F}\mathsf{E}(p_I,k)$$

At the steady state with $\dot{k}=0$,

$$\frac{1}{p_F}\Omega_{ss} + r_{ss}k_{ss} + \frac{1}{p_F}\pi_{ss} + \frac{1}{p_F}\Upsilon_{ss} = \frac{1}{p_F}\mathsf{E}_{ss} \tag{28}$$

where

$$\begin{aligned}\Omega_{ss} L &= (\omega_s^*)_{ss} L_{s,ss} + \bar{\omega}_u L_{u,F,ss} + (\omega_I^*)_{ss} L_{u,I,ss} + (\omega_I^*)_{ss} L_{u,A,ss} \\ &= (\sigma\omega_I^*)_{ss} L_{s,ss} + \bar{\omega}_u L_{u,F,ss} + (\omega_I^*)_{ss} L_{u,I,ss} + (\omega_I^*)_{ss}(L - L_{s,ss} - L_{u,F,ss} - L_{u,I,ss}) \\ &= [(\sigma\omega_I^*)_{ss} - (\omega_I^*)_{ss}] L_{s,ss} + [\bar{\omega}_u - (\omega_I^*)_{ss}] L_{u,F,ss} + (\omega_I^*)_{ss} L\end{aligned}$$

is the total income due to labor. Here, $L_{s,ss}$, $L_{u,F,ss}$, $L_{u,I,ss}$ and $L_{u,A,ss}$ denote the number of workers hired at the steady state in respective sectors. In per capita terms we can write

$$\begin{aligned}\Omega_{ss} &= [(\sigma\omega_I^*)_{ss} - (\omega_I^*)_{ss}]\ell_{s,ss} + [\bar{\omega}_u - (\omega_I^*)_{ss}]\ell_{u,F,ss} + (\omega_I^*)_{ss} \\ &= [(\sigma\omega_I^*)_{ss} - (\omega_I^*)_{ss}] \times \frac{\partial MC_F}{\partial \omega_s}(y_F^*)_{ss} + [\bar{\omega}_u - (\omega_I^*)_{ss}] \times \frac{\partial MC_F}{\partial \bar{\omega}_u}(y_F^*)_{ss} + (\omega_I^*)_{ss}\end{aligned}$$

We know that $(y_F^*)_{ss}$ is a function of k_{ss}, then, we can represent Ω_{ss} as a function of k_{ss} and given paramaters of the model as such:

$$\Omega_{ss} = \Omega(k_{ss}, p_A, p_F, \beta, \bar{\omega}_u, \rho, \alpha_1, \alpha_2, \alpha_3, \delta_1, \delta_2, \delta_3, \eta)$$

On the other hand, transfers to the household, Υ_{ss}, is such that the government budget is balanced, i.e. the receipts form imports taxes are equal to subsidies and transfers paid,

$$\tau_A E p_A^W M_{A,ss} = \tau_F E p_F^W X_{F,ss} + \Upsilon_{ss}$$

where $M_{A,ss}$ and $X_{F,ss}$ represents the import volume and the export volume, respectively,

$$\begin{aligned}M_{A,ss} &= c_{A,ss} - y_{A,ss}^* \\ X_{F,ss} &= y_{F,ss}^* - c_{F,ss}\end{aligned}$$

Then,

$$\Upsilon_{ss} = \tau_A E p_A^W (c_{A,ss} - y_{A,ss}^*) - \tau_F E p_F^W (y_{F,ss}^* - c_{F,ss})$$

Under Walras' Law, it must be true that

$$p_A(y_{A,ss}^* - c_{A,ss}) + p_F(y_{F,ss}^* - c_{F,ss}) = 0$$

or,

$$y_{F,ss}^* - c_{F,ss} = \frac{p_A}{p_F}(c_{A,ss} - y_{A,ss}^*)$$

holds. Hence, transfers to household becomes

$$\Upsilon_{ss} = (c_{A,ss} - y^*_{A,ss})[\tau_A - \tau_F \frac{p_A}{p_F}] \tag{29}$$

Additionally, rents in agricultural sector π_{ss} that appear in equation (28) take the form of

$$\pi_{ss} = p_A^{1/\alpha_3} (\omega_I^*)_{ss}^{-\frac{\alpha_1}{\alpha_3}} r_{ss}^{-\frac{\alpha_2}{\alpha_3}}$$

At the steady state, total expenditures of the household are

$$\mathsf{E}_{ss} = p_F^{\lambda_1} p_A^{\lambda_2} (p_{I,ss})^{\lambda_3} c'_{ss}$$

We know that goods market clearing condition in the informal sector is given by

$$\begin{aligned}
(c_I^*)_{ss} &= (y_I^*)_{ss} \\
\lambda_3 c'_{ss} p_F^{\lambda_1} p_A^{\lambda_2} (p_{I,ss})^{\lambda_3 - 1} &= (y_I^*)_{ss} \\
&\implies c'_{ss} p_F^{\lambda_1} p_A^{\lambda_2} (p_{I,ss})^{\lambda_3} = \frac{(p_{I,ss})(y_I^*)_{ss}}{\lambda_3} \\
&\implies \mathsf{E}_{ss} = \frac{(p_{I,ss})(y_I^*)_{ss}}{\lambda_3} \\
&\implies \mathsf{E}_{ss} = \mathsf{E}(k_{ss}) \qquad (30)
\end{aligned}$$

In equation (29) above, $c_{A,ss}$ appears as an unknown in the equation. In fact, we can write $c_{A,ss}$ as

$$c_{A,ss} = \lambda_2 \frac{\mathsf{E}(k_{ss})}{p_A}$$

or, transfers at the steady state are now equal to

$$\begin{aligned}
\Upsilon_{ss}(k_{ss}) &= (\lambda_2 \frac{\mathsf{E}(k_{ss})}{p_A} - y^*_{A,ss})[\tau_A - \tau_F \frac{p_A}{p_F}] \\
&= (\frac{\lambda_2}{\lambda_3} \frac{p_{I,ss}(y_I^*)_{ss}}{p_A} - y^*_{A,ss})[\tau_A - \tau_F \frac{p_A}{p_F}]
\end{aligned}$$

Finally, using equations (28) and (30),

$$\begin{aligned}
\Omega_{ss}(k_{ss}) + \rho p_F k_{ss} + \pi_{ss} + \Upsilon_{ss}(k_{ss}) &= \frac{(p_{I,ss})(y_I^*)_{ss}}{\lambda_3} \\
&= \frac{(p_{I,ss}) \times y_I(k_{ss})}{\lambda_3} \qquad (31)
\end{aligned}$$

(31) can be solved for a unique k_{ss}. Once k_{ss} is obtained, one can solve for the values of remaining endogenous variables of the model, such as the output, or the consumption levels at the steady state.

2.4.3 Transition path equilibria

Given the steady state values, differential equations (24) with (25), and an initial condition for k_0, we use the Time Elimination Method to solve for the transition path equilibria. The two differential equations

$$\begin{aligned}\dot{k}(t) &= f_1(p_I(t), k(t)) \\ \dot{p}_I(t) &= f_2(p_I(t), k(t))\end{aligned}$$

help us characterize the equilibria at any given time period t. Given the two differential equations above, let's assume that a differentiable policy function such as

$$p_I = P(k)$$

exists. If such a policy function exists, then, the slope of the function P at any given point can be found as

$$\begin{aligned}\dot{p}_I &= \frac{\partial P(k)}{\partial k}\dot{k} \\ \frac{\dot{p}_I}{\dot{k}} &= \frac{\partial P(k)}{\partial k} \qquad (32)\end{aligned}$$

However, this slope is not defined at the steady state because of the fact that steady state requires

$$\dot{p}_I = \dot{k} = 0$$

This indeterminacy problem makes it impossible to integrate backwards from the steady state. In order to avoid this problem, we adopt the Eigenvalues-Eigenvectors Approach to Time-Elimination Method (Mulligan and Sala-i-Martin, 1991). According to this approach, the slope of the function P in the neighborhood of the steady state is the ratio of the coordinates of the eigenvector corresponding to the negative eigenvalue of the Jacobian of the linearized system of differential equations at the steady state:

$$\begin{bmatrix}\dot{k}(t) \\ \dot{p}_I(t)\end{bmatrix} = \underbrace{\begin{bmatrix}\frac{\partial f_1(p_I,k)}{\partial k}|_{p_{I,ss},k_{ss}} & \frac{\partial f_1(p_I,k)}{\partial p_I}|_{p_{I,ss},k_{ss}} \\ \frac{\partial f_2(p_I,k)}{\partial k}|_{p_{I,ss},k_{ss}} & \frac{\partial f_2(p_I,k)}{\partial p_I}|_{p_{I,ss},k_{ss}}\end{bmatrix}}_{J} \times \begin{bmatrix}k(t) - k_{ss} \\ p_I(t) - p_{I,ss}\end{bmatrix}$$

This procedure allows us to find a value for

$$p_I = P(k)\,|_{k \cong k_{ss}}$$

Then, using numerical methods, we solve for the remaining values of $P(k(t))$ over the range of $k(t) \in [k(0), k_{ss}), \forall t$. Having solved for these values, integrating the diffrential equation $\dot{k}(t) = f_1(P(k(t)), k(t))$ forward with respect to time, a time path for $k(t)$ can be obtained. The final step in the procedure is to return to the policy function to derive the time path for $p_I(t)$. Having found the time paths of p_I and k, now one can derive the time paths of the remaining endogenous variables (such as ω_I, y_I, y_F) of the model.

3 Numerical Solution

3.1 Data

The model is calibrated to an economy summarized in a simple 3-sector economy Social Accounting Matrix (SAM). The SAM is constructed for the 2006 Turkish economy. In constructing the SAM, data on national accounts, sectoral employment and household consumption form TURKSTAT were used. The magnitude of the informal sector production was determined using the method provided in Kelley (1994), however unlike Kelley, enformal employment in our model is the 'employment without social security coverage' (as defined in TURKSTAT), rather than 'self-employment'.

In the SAM, the agricultural production is about 11 percent of the GDP, while the formal manufacturing sector makes up fo the 74 percent of GDP. The remaining 15 percent is due to the informal production. In our model economy, government services are not included in the GDP measurement. In the model, labor force is divided into two as skilled and unskilled. For simplicity, skilled labor is defined as labor with the minimum education level of junior high school or vocational school. Accordingly, the production elasticity of skilled balor in the formal sector is found as 17 percent, while the production elasticity of unskilled labor is 6 percent. Rest of the payments (77 percent) are made to capital. As expected, the highest capital elasticity in production is in the formal sector. The table below summarizes the calibrated production elasticities of factors of production in each sector (land elasticity in agriculture is assumed):

Production	Symbol	Value
Skilled labor elasticity in formal sect.	δ_1	0.17
Unskilled labor elast. in formal sect.	δ_2	0.06
Capital elasticity in formal sect.	δ_3	0.77
Labor elasticity in informal sect.	η	0.499
Capital elasticity in informal sect.	$1\text{-}\eta$	0.5
Labor elasticity in agriculture	α_1	0.45
Land elasticity in agriculture	α_2	0.15
Capital elasticity in agriculture	α_3	0.4

Calibrated production parameters

On the other hand, according to TURKSTAT data, the share of expenditures on food (mainly agricultural products) is 29 percent in 2006. In equilibrium we assume the informal sector clears domestically hence the expenditures on informal sector goods is the same as the value of production in informal sector. Thus, we find that the share of expenditures to informal sector goods is about 19 percent of total expenditures. The remaining expenditures are made on formal sector goods, at 52 percent of total. The remaining parameter values in the model (the time preference rate and the intertemporal elasticity of substitution) are taken exogenously:

Consumption	Symbol	Value
Expenditure share of formal good	λ_1	0.52
Expenditure share of agr. good	λ_2	0.29
Expenditure share of informal good	λ_3	0.19
Elasticity of intertemporal subst.	$1/\theta$	0.9
Time preference rate	ρ	0.042

Consumption parameters

3.2 Comparative Statics

In the comparative statics analysis, we examine how and in what direction certain endogenous variable values at the steady state are affected as a response to various exogenous policy changes. One of those is provided in the summary table below:

Policy Simulation	Introduce formal sector subsidies
Flexible wages	↑
Home-good prices	↑
Accumulated capital	↑
Income	↓
Formal production	↓
Informal production	↑
Agricultural production	↓
Agricultural profits	↓
Formal skilled labor	↓
Formal unskilled labor	↑
Informal unskilled labor	↑
Agricultural labor	↓
Formal sector capital use	↑
Informal sector capital use	↑

Steady-state effects

3.2.1 A change in subsidy rate (τ_F) in formal sector

Subsidy rate in the formal sector appears in the model in the price of the formal sector good, p_F. A change in the subsidy rate produces the following effects in the equilibrium:

$$\frac{\partial(\omega_I^*)_{ss}}{\partial\tau_F} > 0$$
$$\frac{\partial(p_I)_{ss}}{\partial\tau_F} > 0$$

In steady state equilibrium, holding everything else constant, an increase in the subsidy rate in the formal sector allows for an increase in the skilled labor wages ω_s :

$$p_F = \left[\frac{\omega_{s,ss}(1+\beta)}{\beta}\right]^{\delta_1} \bar{\omega}_u^{\delta_2} \rho^{\delta_3}$$

Since $\omega_{s,ss} = \sigma\omega_{I,ss}$, for a constant σ, there must be a compensating increase in informal flexible wage $\omega_{I,ss}$. An increase in unit labor cost in the informal sector will cause the unit price to increase:

$$p_{I,ss} = \omega_{I,ss}^{\eta}\rho^{1-\eta}$$

Returning back t the labor markets, we observe that the wage of the formal skilled labor rise relative to the formal unskilled labor, since the wage of the formal unskilled labor is constant. These two types of labor in the model are perfectly substitutable (Cobb-Douglas production function), then the formal producer shifts towards hiring relatively more unskilled versus skilled labor. Formal sector production has a higher elasticity in skilled labor compared to unskilled labor, thus, despite the slight increase in cpital use, formal sector output declines.

Increasing relative price of the informal sector good makes this sector relatively more attractive and pulls factors of production towards this sector. Particularly, the skilled labor released from the formal sector will be hired in the informal sector as unskilled labor. Additionally, informal sector will also pull unskilled labor out of the agricultural sector, leading to a decrease in the output of this sector.

3.2.2 A change in the import tariff (τ_A) rate

A change in the import tariff rate will directly affect agricultural production and agricultural profits:

$$\begin{aligned} p_A &= p_A^W E(1+\tau_A) \\ \Pi_A^* &= p_A^{1/\alpha_3}\omega_I^{-\frac{\alpha_1}{\alpha_3}} r^{-\frac{\alpha_2}{\alpha_3}} T \end{aligned}$$

A fall in this rate (thus a higher degree of trade liberalization) will bring about lower agricultural prices, holding everything else constant, lower agricultural profits, which will prompt the producer to reduce demand for factors of production, namely unskilled labor and capital. Unskilled labor released from agricultural sector will be employed in the informal sector within the same labor market, increasing the informal sector production.

3.2.3 A change in the minimum wage ($\bar{\omega}_U$)

Minimum wage is the wage paid to the unskilled labor in the formal sector. An change in the minimum wage creates the following general equilibrium effects in the economy:

$$\begin{aligned} \frac{\partial(\omega_I^*)_{ss}}{\partial\bar{\omega}_U} &< 0 \\ \frac{\partial(p_I)_{ss}}{\partial\bar{\omega}_U} &< 0 \end{aligned}$$

That is, an increase in the minimum wage will reduce the demand for unskilled labor in the formal sector, holding all else constant, and the producer will try to compensate for the fall in the unskilled labor with other factors of production. Unskilled labor released from the formal sector will seek jobs in the informal labor market, reducing equilibrium wages there. With the fall in informal (unskilled) labor wages, equilibrium wages of the skilled labor, $\sigma\omega_{I,ss}$ will also decline. Some of the unskilled labor released from the formal sector will be hired in the informal sector, expanding output, and reducing unit prices, p_I, as marginal costs decline in this sector. Our conclusion regarding the change in the minimum wage concurs with the findings of Agenor and Aizenman (1999), arguing that there will be a fall in informal labor market wages if there's a rise in minimum wage.

3.2.4 A change in the exchange rate (E)

Exchange rate in our model is introduced through the prices of internationally traded goods. The general equilibrium effects of a change in the exchange rate can be best observed through the effects on the formal sector good price. In particular,

$$\begin{aligned} \frac{\partial(\omega_I^*)_{ss}}{\partial E} &> 0 \\ \frac{\partial(p_I)_{ss}}{\partial E} &> 0 \end{aligned}$$

i.e. an increase in the (nominal) exchange rate (devaluation, or depreciation in domestic currency) will lead to an increase in informal labor wages and price of the informal sector good. An increase in the exchange rate will shift factors of production towards the traded goods sectors. In fact, an increase in demand for skilled labor in the formal sector will pull ineffiœntly allocated skilled labor from the informal labor market towards formal sector employment. This reallocation of skilled labor requires an increase in skilled labor wages,

$\sigma\omega^*_{I,ss}$. Actually, with the exit of labor from the informal labor market, the informal labor wages $(\omega^*_I)_{ss}$ increase, which justifies the increase in skilled labor wages, with σ constant. An increase in unit labor costs will lead to a contraction in informal sector production, and consequently, a rise in the price of the informal sector product (to match the rise in the marginal costs). On the other hand, a fall in exchange rate, or an appreciation, would lead to an expansion of the informal sector employment and production with general equilibrium effects opposite to what has been described above.[3]

Here, we also need to mention the effect of changes in world prices. A fall in world prices (concerning both traded goods) would have similar effects on employment and output as an exchange rate appreaciation, expanding informal employment and informal sector, while contracting the traded sector outputs.

3.3 Dynamic solution of the base model

According to the dynamic solution of the base model without policy, capital accumulation over time will lead to shifts in factors of production (labor and capital) across sectors. Accordingly, the agricultural sector (initially with a share of 11 percent in GDP) reduces down to a negligable share (almost all agricultural consumption goods are now imported), formal sector share increases from an initial 74 percent to 81 percent, while the informal sector share increases from an initial 15 percent to 19 percent in the long run (Figure 3):

As income increases with capital accumulation, home good (informal sector) prices also increase. Although the informal sector uses a relatively less capital-intensive technology than the formal sector, as the relative price of the informal good increases, the informal sector is able to compete for capital with the formal sector. During transtion with the accumulation of capital, the marginal productivity of labor in informal labor market increases, hence the flexible wage rises over time. Rising wages eventually lead to decreased demand for labor in the informal sector (even though the production rises; increased production is made possible by increased use of capital). Recal that skilled labor wages are a multiple of flexible wages, hence with the increase in flexible wages, there is also inevitably a rise in skilled labor wages in the formal sector. Since unskilled labor (whose wages are fixed) is perfectly substitutable with skilled labor in the formal sector, producer will decrease demand for skilled and increase demand for unskilled labor.

[3] Similar effects of exchange rate appreciation on informal sector (or, non-tradeable sector) employment have been mentioned in Goldberg and Pavcnik (2003).

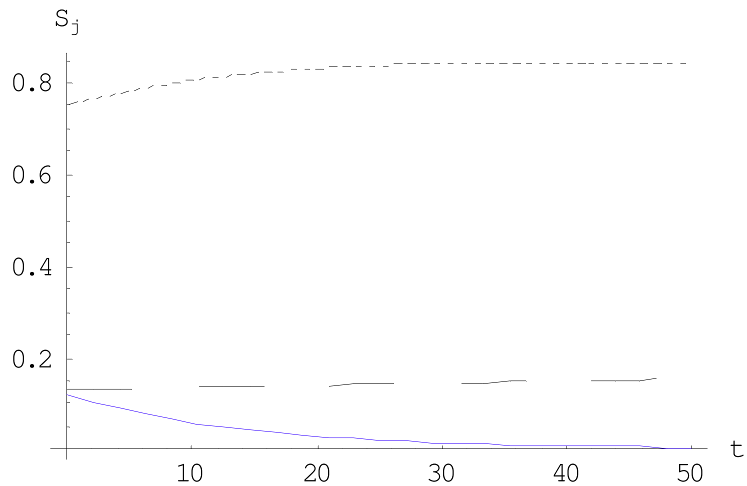


Figure 3: Sectoral Shares

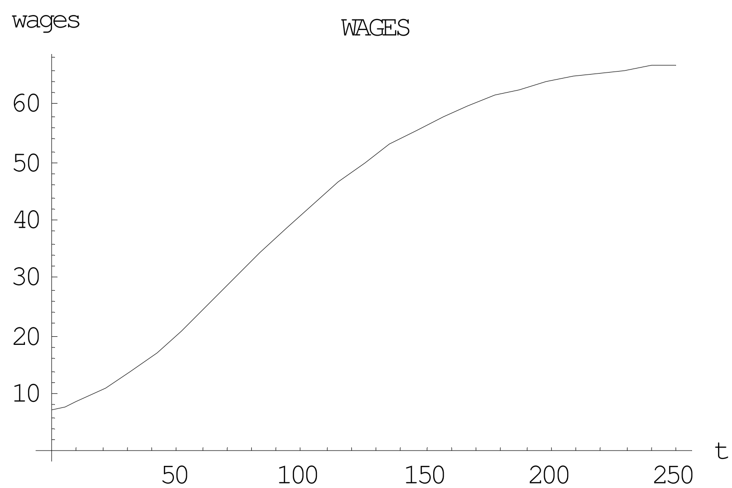


Figure 4: Flexible wages

Figure 5: Informal good prices

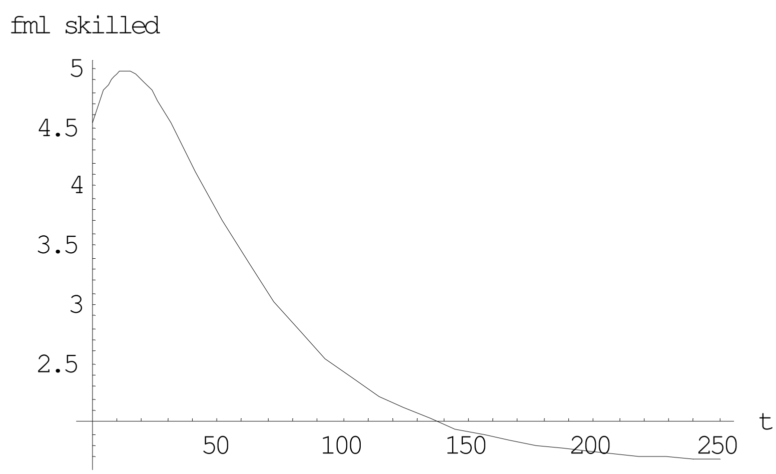
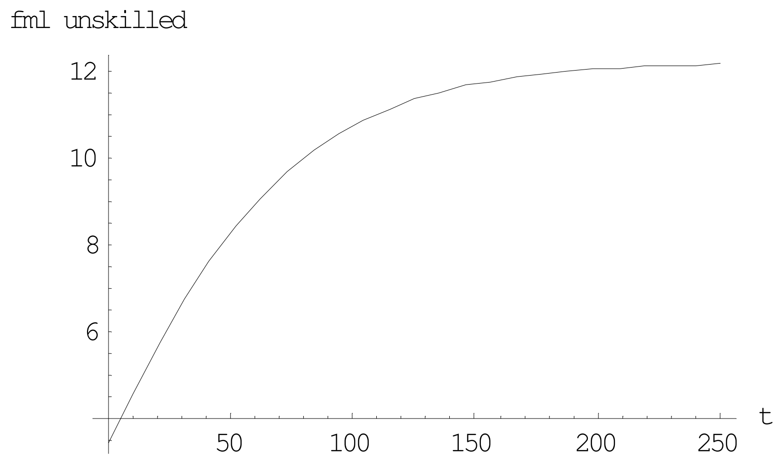


Figure 6: Formal sector skilled labor



Formal sector unskilled labor

4 Conclusion

In this study, we examined an small-open economy with 3 sectors of production and a segmented labor market. Under the dynamic general equilibrium framework, we were able to trace the evolution of the baseline economy, as capital accumulates and as income grows over time. The dynamic framework also allows us to follow the shifts in the factors market, particularly the segmented labor market. One interesting result from our model is that as the economy grows towards the long run equilibrium, formal sector unskilled labor use increases while skilled labor use declines. This occurs because during transition skilled labor wages rise relative to unskilled labor wages (the minimum wage which is fixed), and formal sector producers substitute unskilled labor for skilled.

Using the model, we also analyse the steady state effects of various policy changes, such as introduction of subsidies in the formal (export) sector. One of the most noteworthy results we obtain form this simulation exercise is that increasing subsidies in the formal sector leads to an increase in the wages of the skilled labor, and hence a decreased demand for skilled and an increased demand for unskilled labor. Since the production elasticity of skilled labor is higher in the formal sector, such changes will lead to a decreased production volume of the formal sector output (although there is an increased use of capital, it will not be enough to compansate for the declining use of skilled labor). At the same time, skilled labor not employed in the formal sector will be hired in the informal sector as unskilled labor, leading to an increase in output in this sector. This result is similar to the main result obtained in Goldberg and Pavcnik (2003).

Future improvements in this study include establishing a link between the formal and informal sectors through a subcontracting relationsip. In this sense, the informal sector will not only be producing for the home good market, and also will be supplying intermediate goods to the formal sector. Such a link will provide a more accurate view of the flows of labor across sectors.

BIBLIOGRAPHY

Agénor, P.-R. and J. Aizenman (1999), "Macroeconomic Adjustment with segmented labor markets", *Journal of Development Economics*, 58, pp. 277–296.

Amsden, A.H., and R.V. Hoeven (1996), "Manufacturing Output, Employment and Real Wages in the 1980s: Labor's Loss Until the Century's End," *The Journal of Development Studies,* 32 (4), pp. 506-530.

Ansal, H., Küçükçiftçi, S., Onaran,Ö. and B.Z. Orbay (2000), *Türkiye Emek Piyasasının Yapısı ve İşsizlik*, İstanbul: Türkiye Ekonomik ve Toplumsal Tarih Vakfı.

Barro, R.J. and X. Sala-i-Martin (2004), *Economic Growth*, 2nd Edition, Cambridge, MA: The MIT Press.

Boratav, K., Türel, O. and N. Yentürk (1996), "*Adjustment, Distribution and Accumulation*," Geneva: UNCTAD Research Paper.

Goldberg, P. K. and N. Pavcnik (2003), "The Response of the Informal Sector to Trade Liberalization", *Journal of Development Economics*, 72, pp. 463–496.

Kelley, B., (1994) "The Informal Sector and the Macroeconomy: A Computable General Equilibrium Approach for Peru", *World Development,* 22(9), pp. 1393-1411.

Krueger, A.O. (1983), *Trade and Employment in Developing Countries: Synthesis and Conclusions*, Chicago: The University of Chicago Press.

Mazumdar, D. (1983), "Segmented Labor Markets in LCDs", *The American Economic Review* (Papers and Proceedings of the Ninety-fifth Annual Meeting of the American Economic Association), 73 (2), pp. 254–259.

Mulligan, C.B., and X. Sala-i-Martin (1991), "*A Note on the Time-Elimination Method for Solving Recursive Dynamic Models*", NBER Technical Working Paper no. 116.

Onaran, Ö. (2003), "Türkiye'de ihracat yönelimli büyüme politikalarının istihdam üzerindeki etkileri," in *İktisat Üzerine Yazılar II: İktisadi Kalkınma, Kriz ve İstikrar*, A.H. Köse, F. Şenses and E. Yeldan (eds.), İstanbul: İletişim, pp. 579-601.