Homework #3: Part 3
Tutorial on Monte Carlo Simulation Using @Risk

APEC 4501 Spring 2009

Objective: **To use historical rate of return data to fit the underlying distribution, then to use the selected distribution to simulate returns using @Risk to sample from the distribution.**

Reading: @Risk Manual (on-line) - - Ch. 4 (all)

Assignment for this in-class tutorial:

Retrieve the **S&P500 Returns data.xls** file from the class web page. The data file contains the historical rate of return of the S&P500 Index for 1980-2008. The template also contains a spreadsheet for calculating the gross return on an investment portfolio assuming the annual realized rate of return is stochastic. We will fit a distribution to the historical data and simulate a 10-year series of rates of return. We will use the simulated rates of return to calculate the gross returns and the opening/ending balances on the portfolio. We will calculate the probabilities of achieving different target rates of return on the portfolio.

Steps we will follow in class:

1. Open Excel, click on the **Start button** and select @Risk from **All Programs**.

2. Open the Excel data file **S&P500 Returns Data.xls**. First, calculate the mean and the standard deviation using this data. Select the values (B3:B31) under the title **Percentage Rate**, and then click on the icon "**Fit Distributions to Data**" in @Risk Excel Toolbar.

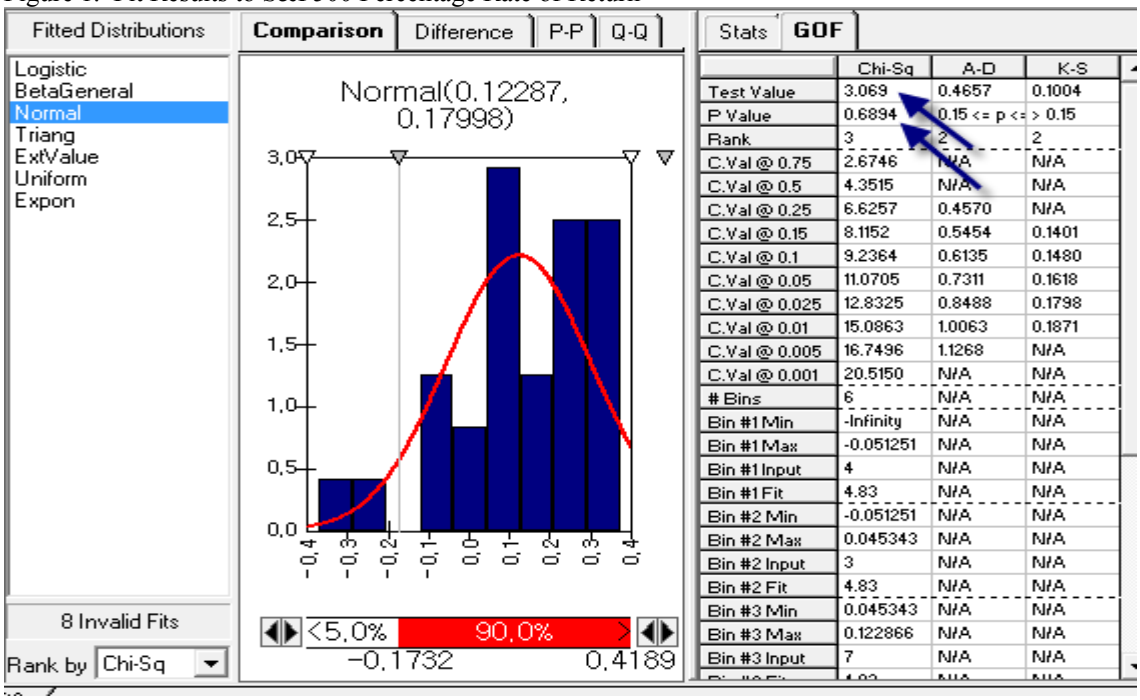
3. @Risk returns the "@Risk-Model [S&P500 Returns.xls]" window. For simplicity[1], select the Normal distribution. We note that there are several different types of distributions that fit the data. We can compare the result of using Excel worksheet functions (Average and St Dev) with that of using @Risk-Model window. Click on tab "GOF" (goodness of fit) in the table, there are *Chi-square* statistics values and the corresponding *p-value* for the hypothesis test [*What is the null hypothesis of this test*?]. [Note: Both of the Anderson-Darling Statistic (A-D) and Kolmogorov-Smirinov statistic (K-S) are described on p. 147-148 in Ch. 6 in the @Risk Manual. They are not used very often to test goodness-of-fit, so we will not introduce and compare these two statistics in this example.]

5. We can see in Fig. 1 that the *Chi-squared* statistic value is 3.069 and the *p-value* of this test is 0.6894. Usually, the lower the *Chi-squared* value is, the better the distributions fitted to the data; while the higher the p-value is, the better the distribution fits the data [*Why? Think about the null hypothesis of this test*]. [Note: The definition of the *Chi-squared* statistic is on p. 147 in Ch. 6 in the @Risk manual]. In this example, we select the Normal distribution.

6. @Risk also allows us to select which distributions to fit to the data. Click on the icon, "**Specifying Distributions To Fit**" in @Risk-Model Fitting Toolbar. From the right-hand side of the window, we can check the boxes for the distributions we are interested in. In the example, We can set the *Exponential* (Expon), *Logistic, Loglogistic, Lognormal* (Lognorm), *Normal, Triangular*, and *Uniform distributions* as the default distributions. We can see from the lower left corner of the Fit Results window in Fig. 1 that there are 8 invalid fits.

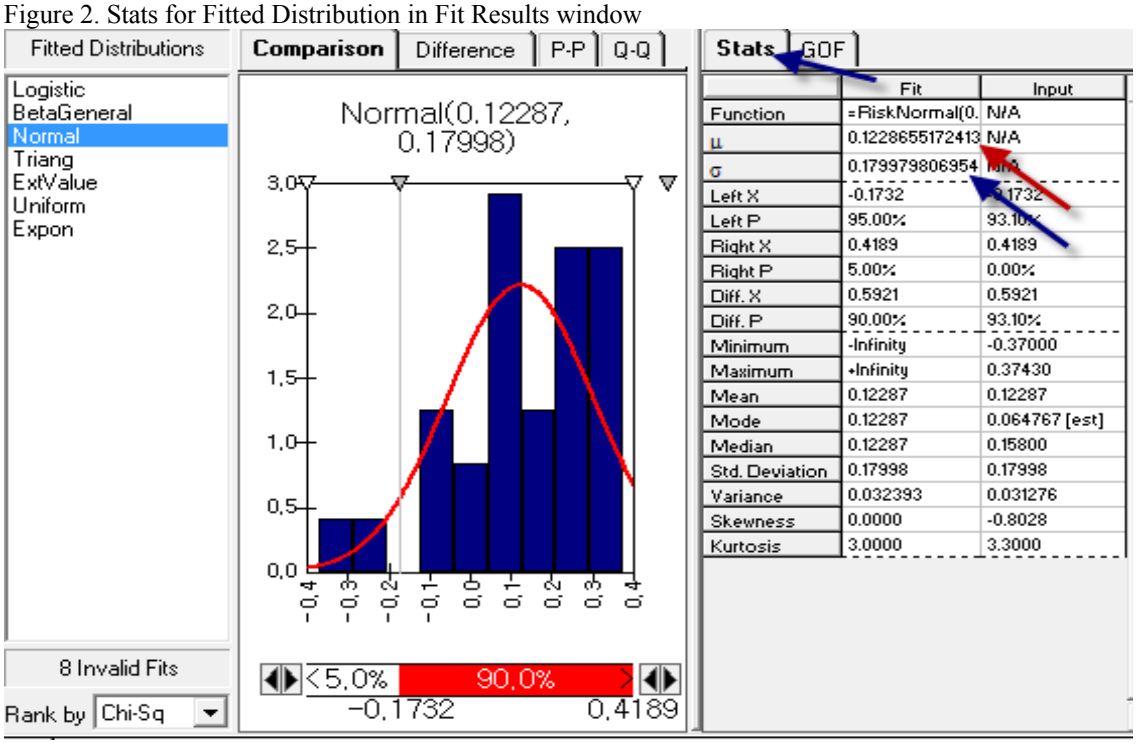
[1] We may select the most highly ranked distribution for simulation, but sometimes it may be a distribution that we are not familiar with, so we will select a distribution here that is easier to interpret. Also, you can select as many distributions to fit as you want if you have sufficient knowledge of these distributions.

Figure 1. Fit Results to S&P500 Percentage Rate of Return



7. Select the **Stats** tab and we can see that the fitted normal distribution is with mean equal to 0.12287 and standard deviation equal to 0.17998 as figure 2 below.

Figure 2. Stats for Fitted Distribution in Fit Results window



8. Click on the icon '**Simulation Settings**' in @Risk Excel Toolbar and select '**Sampling**' tab. Choose '**Monte Carlo'** for both the '**Sampling Type'** and '**Standard Recalc'** and click **OK**.

9. Use the fitted Normal distribution to simulate expected values for the rates of return: To go back to Excel, click on the icon, '**Show Excel Window**' in @Risk Excel Toolbar. Select the first cell (Cell E3) under Simulated Rate of Return. Click on the icon, '**Define Distribution'** in @Risk Excel Toolbar. @Risk returns a window entitled "Define Distribution for E3". Click drop down arrow on '**Source**', select '**Fit Results**' and '**Normal**' and click '**Apply**' at lower right corner of this window. In cell E3 the value of a random variable, which satisfies the fitted Normal distribution, is returned. Copy the E3 cell and paste the range (E4:E12), or just drag and drop to E12. It is meant that we have ten simulated annual rates of return for the S&P500 portfolio. [Note: The values of the 10 simulated rates of return will change automatically since we set up the Monte Carlo simulation at step 5.]

10. Calculate the opening balance, gross return and closing balance for each year. Since only one sample for 10 years is needed from the fitted Normal distribution, we copy this sample (E3:E12) and paste 'value' at the range (I3:I12). Therefore, the value of the opening balance, gross return and closing balance is obtained since each formula for these values are already inserted.

11. Calculate the probability of target rates of return. Since the Normal distribution is a continuous distribution, we have to calculate the probability of receiving a target rate of return. We acquire 0.12287 and 0.17998 as the mean and the standard deviation, respectively, from step 2 and step 7. To calculate the probability of target rates of return, we use the Normdist function in Excel to calculate the cumulative probability. For example, the probability of earning a rate of return larger than 10 % is 55.05%: 1 - Pr $(X \leq x | x=10\%) = 55.05\%$.

FYI: Since the rates of return on S&P500 are randomly drawn from the fitted Normal distribution, the 10 simulated rates of return and the balances and gross returns numbers will be quite different for each student. However, since we use the cumulative Normal distribution to obtain the probability of receiving at least the target rate of return (regardless of the sample that is generated by @Risk) these calculated probabilities for the target rates of return should be the same for all students.

12. Save your completed spreadsheet.

Any Questions?