Homework Problem #4
Option Pricing Models

APEC 4501 Spring 2009

Objective: **To analyze the pricing of options using standard option pricing formulas, to solve for implied volatility using alternative methods, and to simulate the effect of a put option on portfolio returns.**

Excel features used: math and statistics functions, *Data Table*, *Name Cells*, *Goal Seek, @Risk*.

Your references for this homework problem are in Benninga: Ch. 16 (skim), Ch. 17 (p. 443-458), and Ch. 19 (p. 509-517); Sengupta: Ch. 12 (skim) and Ch. 13.

Assignment:

Retrieve **Option Pricing Models.xls** from WebCT or the class web page.

Part I: The Binomial Option Pricing Model

In this problem you will complete Benninga Exercise #6 (p. 479). The solution to the problem will be modeled after examples in Benninga, (p. 449-451 and p. 457-458). Briefly discuss what you conclude when you compare the value of the European put option with the value of the American put option. Explain why this difference exists.

Part II: The Black-Scholes Option Pricing Model

In this problem you will complete Benninga Exercises #1 and #2 (p. 544). The solutions to these problems will be modeled after the examples in Benninga, p. 510-511.

Next to each graph, briefly explain the slope of each graph that you develop in Exercise #2. That is, explain the relationship between each factor (S, σ, T, r, X) in the Black-Scholes option pricing model and price of the option (i.e., the effect on option price) based on your reading and your understanding of basic option pricing theory.

Part III: Finding the Implied Volatility

Based on the Black-Scholes model we know that there is a positive relationship between the price of the option on an asset (stock) and the volatility of the return on the asset. Normally, we assume that the volatility is known (the historical volatility) and then solve for the price of the option. However, we do not actually observe the option volatility, but we do observe the price of the market-traded option.

We can theoretically solve for the volatility (sigma) by rewriting the Black-Scholes model with sigma on the left hand side of the equation. The solution value for that equation is sigma, the implied (or forward-looking) volatility. The problem with this approach is that we cannot solve the equation for sigma directly.

(over)

In both parts of this problem use the following data to set up and solve the problem for the **implied volatility** of the call option: stock price = 45; call price = 3; risk-free interest rate = 8%; exercise price = 50; time to expiration = 1.

Method 1: Use the "trial and error approach." First develop the spreadsheet equivalent of the example shown in Benninga (p. 518) using the B-S model to calculate the call option price instead of the VBA model. Then develop the *Data table* and the associated graph for the call price versus volatility. Then try various values for sigma until you approximate the call price of 3. Report your results.

Method 2: Use either *Goal Seek* or *Solver* to solve for the implied volatility assuming the call price is known. Vary the call price (set cell) using the value of sigma as the change cell (this is suggested in Benninga (p. 516-517) or Sengupta (p. 329). Verify that the values for sigma and the call price that you obtain lie on the curve you plotted on the graph using Method 1. Report your results.

Part IV: Simulating the VaR of a Put Option Strategy

In this problem you will solve a problem like the one shown in the @Risk manual (*Finding VaR of a Portfolio*, Ch. 5, pp. 119-122). You will be comparing: 1) the VaR for a stock portfolio without a protective put option to 2) the VaR for a stock portfolio with a protective put option (i.e., with portfolio insurance). We will be assuming that the underlying stock price is lognormally distributed, and that it is increasing over time at an historical growth rate (g).

Using the parameters below and the steps presented in the solution (pp. 120-121), calculate the stock price at expiration (A), the put value at expiration (B), the percentage gain without the put option (C), and the percentage gain with the put option (D).

Variable	Range name	Parameter value
Current stock price	S	56
Put exercise price	x	50
Put duration (expiration)	d	.5
Risk-free rate	r	.04
Stock price growth rate	g	.20
Volatility	v	.30
Put price (the B-S price)	p	
Stock price at expiration		A (Step 1)
Put value at expiration		B (Step 2)
Pct gain w/o put option		C (Step 3)
Pct gain w/ put option		D (Step 4)

Step 5: Use @Risk to simulate the gains without and with the put option. Save and report your simulation results and discuss what effect the protective put has on the returns to the portfolio. Is there a cost associated with obtaining portfolio insurance? Briefly explain.