Time is money: optimal investment delay in procurement (and concession) contracts*

Chiara D'Alpaos[†], Michele Moretto[‡], Paola Valbonesi[§]

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Abstract

Procurement (and concession) contracts are agreements granting the right to construct public works, operate and provide a service/good. The main advantage of a procurement contract is that it passes full responsibility for investment and operations to the private sector and consequently provides incentives for efficiency.

Although most contracts include penalty/premium clauses to avoid construction risks (i.e. delays), evidence from ongoing procurement contracts shows that there are many delays in making investments. Actually these clauses introduce the flexibility to decide when it is optimal to invest and consequently increase the contract's value for the contractor. Therefore if the contracting authority underestimates penalty/premium fees, these may be totally ineffective in avoiding construction risks.

In this paper we specifically investigate the effects that penalty/premium clauses have on both contract value and reduction of delay. We also focus on the design of optimal penalty/premium rules.

Keywords: procurement/concession contracts, premium/penalty fee, investment timing flexibility

JEL: L33; H57; D81

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[†]DIMEG, University of Padova, E-mail: chiara.dalpaos@unipd.it.

[‡]Department of Economics, University of Padova and Fondazione ENI Enrico Mattei, Italy, E-mail: michele.moretto@unipd.it.

[§]Department of Economics, University of Padova, Italy, E-mail: paola.valbonesi@unipd.it

1 Introduction

Total public procurement in the EU - i.e. the purchase of goods, services and construction works by governments and public utilities[1] - is estimated at about 16% of the European Union's GDP and it was 1500 billion Euros in 2002. Its importance varies significantly between Member States ranging between 11% and 20% of GDP.[2]

Public sector procurement thus represents big business which significantly affects many economic operators and has rapidly increased over the last decade: the total amount of public procurement in the EU increased by 31% from 1995 to 2002.[3] Of the different types of procurement contracts (e.g. public works contracts[4], public works concession contracts[5], public supply contracts,[6] etc.) in the EU experience the most widely implemented procurement contracts are construction works contracts which in 2002 represented 37.3% of the total public procurement value (about 81 billion Euros).[7] Public procurement is therefore an attractive option especially where large investments in infrastructures are needed.[8]

However, evidence from ongoing public procurement contracts shows that delays in the execution time (completion date) are quite common especially for construction works contracts and can be very costly for all the actors involved (e.g. the government, economic operators and consumers). The typical illustra-

[1] The term procurement includes concession contracts also (see http://europa.eu/scadplus/leg/en/lvb/122007.htm; http://europa.eu/scadplus/leg/en/lvb/122011.htm). Under concession contracts, the government retains ownership of the infrastructure but transfers all risk and responsibility for running the utility, including responsibility for financing investments (Marin, 2002).

[2] See http://ec.europa.eu/internal_market/publicprocurement/introduction_en.htm

[3] See http://ec.europa.eu/internal_market/publicprocurement/docs/eprocurement/2004-12-impact-external-vol2_en.pdf _en.htm

[4] Public works contracts are contracts for pecuniary interest concluded in writing between a contractor and a contracting authority for: a) either the execution, or both the execution and design, of works related to one of the activities covered by class 50 of NACE or of a work; b) the execution of a work corresponding to the requirements specified by the contracting authority (Directive 93/37/EEC). According to the Directive 93/37/EEC, a work is "the outcome of building or civil engineering works taken as a whole" - e.g. a hospital or a bridge - "that is sufficient of itself to fulfill an economic and technical function", i.e. it is fully equipped and completed.

[5] A public works concession is the same as a public works contract, except that the consideration is usually in the form of the right to exploit the works - i.e. the profit which the concessionaire will gain depending on his ability to manage the project - but is sometimes pecuniary as well (Directive 93/37/EEC).

[6] Public supply contracts are contracts for pecuniary interest concluded in writing between a supplier and a contracting authority and involving the purchase, lease, rental or hire purchase, with or without option to buy, of products. Delivery of such products may, in addition, include siting and installation operations (Directive 93/37/EEC).

[7] See http://ec.europa.eu/internal_market/publicprocurement/docs/eprocurement/2004-12-impact-external-vol2_en.pdf.

[8] The main advantage of a public procurement contract is that it passes full responsibility for operations and investment to the private sector and so it brings efficiency incentives to bear in all the contract's targets.

tive example for this issue on delay is given by a public procurement contract for roadway resurfacing, rehabilitation and restoration: if these activities are undertaken in heavily urbanized areas, they can cause extreme traffic congestion and severe inconvenience to the travelling public and the business community. Delays in the final completion date thus have a negative impact on users.

In this respect the European Commission promotes the award of procurement contracts by means of open procedures (Directive 2004/18/EC). The reason for incentivating open procedures for the award of contracts is basically that open procedures allow taking into consideration of social costs (Hancher and Rowings, 1981; Zohar et al., 1995) connected with delays on delivery date which are in turn aggravated by the non-verifiability[9] of construction time (Bajari and Tadelis, 2001). The open procedures make the negative trade-off between the price and the delivery date explicit. The private contractor will report a lower bid if he has the ability to postpone the construction time. In other words, the construction time flexibility has a value for the economic operator.

This trade-off does not exist (or is substantially reduced) when procurement contracts are awarded through negotiated procedure. The price is negotiated between the contracting authority and the economic operator on the basis of estimated costs of the project that do not consider the social costs of the non-verifiability of construction time[10]. Most procurement contracts tend to take account of the social costs associated with possible delays in the delivery date by including penalty clauses.

The aim of this paper is to show that the introduction of a penalty clause in a procurement contract where the price is fixed by negotiated procedure may induce an endogenous optimal investment delay which can be very costly for both the government and the taxpayers/users.

We specifically focus on the effects which penalty/premium rules have both on contract value and reduction in investment delay. We carry out the analysis referring to the Real Option Theory that allows for correct evaluation of the increase in contract value induced by penalty/premium rules. As Brennan and Schwartz (1985) and McDonald and Siegel (1985; 1986) highlighted in their seminal works, there is a close analogy between security options and investment timing flexibility (i.e. the possibility of deciding when it is optimal to invest). Within this theoretical and methodological framework we are also able to design the optimal penalty/premium rule, that is, we are able to determine the penalty/premium scheme which the contracting authority should implement in procurement contracts in order to prevent delays in delivery date. In fact when the penalty/premium fee is optimally set, the expected penalty fee equals the value of the option to delay.

The paper is organized as follows. In section 2 the basic model with penalty

[9] Referring to construction time, we have non-verifiability when "no court or other third party will accept to arbitrate a claim based on the value taken by this variable" (Salanié: 1997, p.177).

[10] More specifically, non-verifiability of time-to-completion may lead the parties to renegotiate the terms of the contract which results in ex post transaction costs (Bajari and Tadelis, 2001).

fee is presented and a numerical example is discussed. Section 3 provides a simple penalty/premium scheme and also a numerical example. In section 4 we focus on concession contracts and related optimal penalty design. Section 5 provides a brief summary of the findings and further developments.

2 A simple model of procurement

Let's consider the case where a contracting authority awards a cost-plus contract to an economic operator (i.e. a firm) to construct a public work such as an infrastructure[11] [12] According to the contract the contractor must commit itself to constructing the infastructure immediately (i.e. at the current time t) in return for a fixed payment p negotiated between the contracting body and a contractor. Under these assumptions, the net benefit by a risk-neutral contractor (i.e. the project's NPV) is simply given by:

$$F_t = p - C_t \tag{1}$$

where $C_t \leq p$ is the cost of the infrastructure at time t.

In practice, however, many procurement contracts include a penalty clause according to which the firm pays a penalty if it delays the contract delivery time. The most commonly adopted rule requires that:

A) The contractor pays a constant penalty c for each period (e.g. day, month, year, etc.) it delays the delivery date, and the penalty c is set as a percentage of p.[13]

The reason for the introduction of this rule is quite clear. The contracting authority of recognising that it is not able to induce the economic operator to complete the work within the contractual time. Consequently, by making the firm pay for delays, the authority tries to avoid construction risks and reduce the problem arising from non-verifiability of the delivery date.

In order to correctly analyse the effect of a penalty rule on the delivery date it should be recognized that its introduction alters the original contract provisions since it gives the firm the opportunity to decide when it is optimal

[11] The EU normative setting for public procurement contracts - the present model referred to - is provided by Directive 2004/18/EC, Directive 2004/17/EC and national amending acts (for Italy see: Government Decree n° 163/2006 and D.P.R. n° 554/1999).

[12] *In practice*, the vast majority of contracts are variants of simple fixed-price and cost-plus contracts. In fixed-price contracts, the contracting authority offers the potential contractors a *pre-specified* price for completing the project. A cost-plus contract does not specify a price, but rather reimburses the contractor for costs plus a mark-up.

Fixed-price contracts tend to be awarded through competitive bidding, while cost-plus contracts are frequentely negotiated between a buyer and a contractor (Bajari and Tadelis, 2001).

[13] For the Italian law references on penalty rules in public procurement contracts, see: Government Decree n° 163/2006 and D.P.R. 554/1999.

to invest. This investment timing flexibility has a value that should be added to the project's NPV as described in (16).

Specifically, if the penalty rule does not set any limit on the maximum amount the firm has to pay, and the project's cost evolves according to a geometric Brownian motion, the possibility of deferring the infrastructure's completion date becomes analogous to an American Put option. For any fixed payment p, the value of this option is:

$$\Phi_t = E_t(e^{-r(\tau-t)})F_\tau \tag{2}$$

where $F_\tau = p - C_\tau$ is the net benefit from investing at a general cost of the project $C_\tau < C_t$, r is the risk-free interest rate[14], τ is the exercise time of the option and C_t is driven by $dC_t = \alpha C_t dt + \sigma C_t dz_t$ with $\sigma > 0$ and $\alpha \leq 0$.[15]

Nonetheless in order to benefit from this investment timing flexibility the firm has to pay a penalty whose expected value Λ at time t is given by:

$$\Lambda_t = E_t\left[\int_t^\tau ce^{-r(s-t)}ds\right] = \left[1 - E_t(e^{-r(\tau-t)})\right]\frac{c}{r} \tag{3}$$

Therefore, according to (3), the ex-ante procurement contract value for the firm that decides to defer the investment till time $\tau > t$ turns out to be:[16]

$$P_t = \Phi_t - \Lambda_t \equiv E_t(e^{-r(\tau-t)})\left(F_\tau + \frac{c}{r}\right) - \frac{c}{r} \tag{4}$$

Finally, since by Ito's Lemma F_t is also described by the process $dF_t = \alpha(F_t - p)dt + \sigma(F_t - p)dz_t$, the discount rate $E_t(e^{-r(\tau-t)})$ can be expressed as:[17]

$$E_t(e^{-r(\tau-t)}) = \left(\frac{F_t - p}{F_\tau - p}\right)^\beta \tag{5}$$

where $\beta < 0$ is the negative root of the fundamental quadratic $\frac{1}{2}\sigma^2 x(x-1) + \alpha x - r = 0$. By substituting (5) into (4) we obtain:

$$P_t = \left(\frac{F_t - p}{F_\tau - p}\right)^\beta \left(F_\tau + \frac{c}{r}\right) - \frac{c}{r} \tag{6}$$

[14] Introducing risk aversion does not change the results since the analysis can be developed under a risk-neutral probability measure (Cox and Ross, 1976; Harrison and Kreps, 1979).

[15] dz is the increment of a standard Brownian process with mean zero and variance dt (Dixit, 1993; Dixit and Pindyck, 1994).

[16] Sometimes the contract provisions establish that the contracting body has the possibility of revoking the contract if the total penalty reaches an upper bound , say G, set as a percentage of the project's value p. In this case the option becomes an American Put Option with maturity time T given by:

$$\int_0^T ce^{-rs}ds \equiv \frac{c}{r}(1 - e^{-rT}) = Gp$$

In this case the modelling turns out to be more complicated but the result remains substantially the same.

[17] See Dixit and Pindyck,1994, p. 315-316.

For any fixed p, it will be profitable for the firm not to observe the contract provision on delivery date and consequently defer the delivery date by means of clause A) whenever $P_t > F_t$. The firm will be better off by maximizing (6) with respect to F_τ and determing the optimal delay. Denoting by $F^* \equiv F_\tau$, the net benefit that will trigger the investment is:[18]

$$\begin{aligned} F^* &= \frac{1}{1-\beta}(p + \beta\frac{c}{r}) \\ &= \frac{1}{1-\beta}p - \frac{\beta}{\beta-1}\frac{c}{r} \end{aligned} \tag{7}$$

which provides the following optimal investment rule:

If $F^* > F_t$ it is optimal to wait to invest until the net benefit is equal to F^*.
If $F^* \leq F_t$ it is optimal to build the infrastructure immediately.

In order to illustrate the properties of the above model and get some quantitative idea of the effect of the penalty on the decision to observe the delivery date, in this section we provide some numerical solutions of (6). The choice of parameters was made in the interest of simplicity, following as far as possible some indications found in other studies (Dixit and Pindyck,1994; Herbsman *et al.*, 1995). The parameters take the following values: $p = 1$; $r = 0.05$; $\alpha = 0, -0.05$; $\sigma = 0.3, 0.4$ and finally the penalty c is set in annual terms at $2\%, 10\%$ and 30% of p.[19]

Figure 1 presents both F_t and P_t for $\alpha = -0.05$ and $\sigma = 0.4$. If $c = 0.1$ the optimal trigger $F^*(c = 0.1)$ is approximately equal to 0.3, while if the authority sets $c = 0.02$ the trigger increases dramatically to $F^*(c = 0.02) = 0.659$. With higher penalties, on the other hand, e.g. $c = 0.3$, the trigger becomes negative.[20]

Comparing these triggers with F_t we get the firm's optimal investment rule. For example, if the current investment cost is $C_t = 0.7$, the project's NPV is $F_t = p - C_t = 0.3$ which corresponds to the trigger calculated for $c = 0.1$. Then, the firm finds it optimal to invest immediately. Alternatively if $c = 0.02$ we get $F^*(c = 0.02) = 0.659 > F_t = 0.3$. The firm delays construction, it waits until the contra*c*t value is maximum and then invests. Finally, with $c = 0.3$ or higher, it appears to be optimal for the firm to observe contract provisions and invest immediately.

Figure 1 about here

[18] The first order condition is:

$$\begin{aligned} \frac{\partial P}{\partial F_\tau} &= \beta\left(\frac{F_t - p}{F_\tau - p}\right)^{\beta-1}\left(-\frac{F_t - p}{(F_\tau - p)^2}\right)\left(F_\tau + \frac{c}{r}\right) + \left(\frac{F_t - p}{F_\tau - p}\right)^{\beta} \\ &= \left(\frac{F_t - p}{F_\tau - p}\right)^{\beta}\left[\beta\left(-\frac{1}{F_\tau - p}\right)\left(F_\tau + \frac{c}{r}\right) + 1\right] = 0 \end{aligned}$$

[19] *These* penalties are commonly used in Italian procurement contracts.
[20] Since 0 is an absorbing barrier for F_t, the trigger is set to zero.

Let's now extend the example considering $\alpha = 0, -0.05$ and $\sigma = 0.3, 0.4$. Table 1 shows how the optimal investment triggers vary with σ and with the rate of cost reduction α.

		c=0.02		c=0.1		c=0.3	
		$\alpha = -0.05$	$\alpha = 0$	$\alpha = -0.05$	$\alpha = 0$	$\alpha = -0.05$	$\alpha = 0$
F^*	$\sigma = 0.3$	0.5748	0.4400	0.0885	-0.2*	-1.1259*	-1.8*
F^*	$\sigma = 0.4$	0.6597	0.5753	0.270754	0.0899	-0.7016*	-1.1234*

Table 1: Optimal trigger for different penalties and $\sigma = 0.3, 0.4$ and $\alpha = -0.05, 0$ respectively.
* Since 0 is an absorbing barrier for process F, the trigger is non-negative, therefore the optimal trigger is zero.

The higher the uncertainty, the higher the optimal trigger F^*, that is, the uncertainty incentivates the firm to defer the investment and the lower the cost to be sustained, i.e. the penalty c, the greater the delay. We obtain the same results by analysing the effect of the expected rate of reduction of the construction costs. Whenever the firm expects a future reduction of construction costs, it will always be more profitable for the firm not to invest immediately and consequently defer the investment.

Finally, it is easily shown that if the contracting authority wants to force the firm to respect the completion date, it has to fix a penalty such that $F^* = F_t$. The optimal penalty is then:

$$c^* = \frac{\beta - 1}{\beta} r C_t - rp \tag{8}$$

which, *ceteris paribus*, depends on σ (via β) and C_t. In particular, Table 2 shows that for increasing values of both the current investment costs C_t (i.e. for decreasing mark-up) and the uncertainty about future values of these costs, we get increasing values of the optimal penalty.

	c^*	c^*
C_t	$\sigma = 0.3$	$\sigma = 0.4$
0.7	0.065	0.1
0.8	0.082	0.115
0.9	0.103	0.135

Table 2: Optimal penalty for different C_t and $\sigma = 0.3, 0.4$

3 The penalty/premium model

In the simple case proposed in the previous section, the contract establishes a commitment for the firm to invest and construct the infrastructure at time t.

Many contracts, however, commit the firm to invest and initiate the project at a future, although predefined, date $t' > t$. In this case the current NPV, say N, of the project for the firm awarded with the contract is:

$$\begin{aligned} N(F_t, t') &\equiv N_t = e^{-r(t'-t)}p - e^{-\delta(t'-t)}C_t \\ &= e^{-\delta(t'-t)}F_t + \left[e^{-r(t'-t)} - e^{-\delta(t'-t)}\right]p \end{aligned} \tag{9}$$

where $C_t < p$ is the project's construction cost at time t when the agreement is signed and $\delta = r - \alpha$.[21] [22]

Let's now assume that the agreement includes a clause according to which, if the contractor is able to complete the project ahead of schedule, he will be entitled to a premium (incentive fee), I. If, on the other hand, the contractor delays completion of the project, a penalty (disincentive fee), D, is then assessed by the contracting body. Evidence from ongoing contracts shows that contracting authorities may introduce different premium/penalty (I/D) schemes. We perform our analysis considering the simplest one:

B) The contractor receives a constant premium/penalty fee c for each period (day, month, year, etc.) he anticipates/delays delivery of the project [23].

Since the above I/D rule does not set any limit on the amount corresponding to the premium/penalty, for any fixed price p, the firm's investment decision is still equivalent to exercising a perpetual Put Option whose value is given by (2). However, for the contractor, the value of the premium/penalty scheme at time t becomes (see Appendix):

$$\begin{aligned} \Lambda_t &= \mathcal{E}_t \left[\begin{array}{c} \int_t^{\min(\tau,t')} 0e^{-r(s-t)}ds + \int_{\min(\tau,t')}^{t'} ce^{-r(s-t)}ds + \\ - \int_{t'}^{\max(\tau,t')} ce^{-r(s-t)}ds \end{array} \right] \\ &= \left[E_t(e^{-r(\tau-t)}) - e^{-r(t'-t)}\right]\frac{c}{r} \end{aligned} \tag{11}$$

where the expected value $\mathcal{E}_t$ is calculated with respect to both τ and the probability that τ is lower than t' or vice versa.

[21] $r - \delta$ is the certainty-equivalent rate of return (see Mc Donald and Siegel, 1984; Dixit and Pindyck, 1994).

[22] If t' is set by the authority to allow the contractor to maximize the NPV (9), depending on the parameter values, t' is greater than t only if $r < \delta$. In particular, maximizing (9) we obtain:

$$t' = \max\left[\frac{1}{r-\delta}\log\left(\frac{\delta}{r}\frac{C_t}{p}\right), 0\right] + t \tag{10}$$

which is an increasing function of the current investment cost C_t and *is* always greater than t if $\frac{\delta}{r} < \frac{p}{C_t}$.If $r = \delta$ we get $N_t = e^{-r(t'-t)}F_t$. Since $F_t > 0$ *it is optimal to invest immediately*, i.e. $t' = t$. *If* $r > \delta$ the solution of the first order condition represents a minimum as $\frac{\partial^2 N_t}{\partial(t')^2} > 0$ and then the optimal value is found on one of the boundaries and is given by $\max\left[F_t, \lim_{t'\to\infty} N_t\right]$. However, since $\lim_{t'\to\infty} N_t = 0$ *it is still* optimal to invest immediately.

[23] Generally contracting bodies using the I/D scheme apply the same value for both the incentive and disincentive fee (see Herbsman *et al.*, 1995).

According to (2) and (11) the ex-ante procurement contract's value if the firm decides to defer the investment by means of clause B) is:

$$P_t = \left(\frac{F_t - p}{F_\tau - p}\right)^\beta \left(F_\tau + \frac{c}{r}\right) - \frac{c}{r} e^{-r(t'-t)} \tag{12}$$

which should be maximized with respect to F_τ.

By (12), if the contractual time is very long, i.e. $t' \to \infty$, the second term on the l.h.s. disappears and the firm will get the premium since it invests before t' with probability one. On the contrary, if t' is very short, i.e. $t' \to t$, the second term on the l.h.s. of (12) reduces to $\frac{c}{r}$ as in (6). The firm will incur a penalty since, with probability one, it will invest after the contractual time is over. Finally, as the term $\frac{c}{r}e^{-r(t'-t)}$ is constant, the optimal investment trigger F^* is still given by (20) as well as the firm's investment decision rule. In other words the contractor defers the infrastructure delivery date until F_t reaches for the first time the trigger F^* from below . If F_t reaches the trigger F^* before time t', the firm gains a premium, otherwise it has to pay a penalty.

Let's continue with the numerical example of section 2 by considering an I/D scheme where c is equal to 0.02 and 0.1 respectively.[24] When $c = 0.1$ we obtain $F^* = 0.3$, while when $c = 0.02$, we get approximately $F^* = 0.6$.[25]

Figures 2, 3, 4 and 5 represent N_t and P_t plotted against F_t respectively for $t' - t = 10, 5, 1, 0.1$ years. In all the cases analysed the results are as expected: P_t is greater than N_t. That is, for the contractor it is always preferable to avail itself of an I/D clause.[26] Moreover, by direct inspection of Figures 2, 3, 4 and 5, it emerges that when $c = 0.1$ (i.e. with a high premium/penalty fee) F^* is always lower than the one obtained for $c = 0.02$ (i.e. with a low premium/penalty fee).

In order to appreciate the intuition behind this result we go back to the optimal trigger F^*. When $c = 0.1$, the firm invests earlier due to the combination of two effects. Firstly, when the commitment date t' is far from t (i.e. $t' - t = 10$ or 5 years), the firm will expect $\tau < t'$ and gain premiums by the I/D scheme. Secondly, when t' is close to t (i.e. $t' - t = 1$ or 0.1 years), the firm will expect $\tau > t'$ and pay penalties. In both cases the contractor will maximize its value by investing as soon as possible in order to gain premiums and/or avoid penalties.

The reverse applies when $c = 0.02$. If c is too low the firm does not have any incentive to avoid penalities in the case of $t' - t$ low, and to collect premiums in the case of $t' - t$ high**.** The contractor will exploit the investment timing flexibility by setting the construction time as far ahead as possible.

Figure 2 about here

Figure 3 about here

[24] Generally Italian procurement contracts set premium fees equal to penalty fees. Here we have considered the extremes of the fee interval usually set by the contracting authorities according to the national legal framework.

[25] Both these cases satisfy the condition $\frac{\delta}{r} = \frac{0.1}{0.05} > \frac{p}{C_t} = \frac{1}{0.7}$, consequently $t' > t$ and obviously $N_t > F_t$.

[26] At least as long as the value of the premium/penalty c is not too high (below 0.1).

Figure 4 about here

Figure 5 about here

The difference $P_t - N_t$ represents the loss in terms of contract value that the firm suffers at any time t if it undertakes to observe t' instead of using the I/D clause. When $t \to t'$, the solution obtained coincides with the one in section 2.

To complete the analysis we show how the contracting authority can force the firm to respect the completion date t'. However, since the exercise time τ is stochastic, the contracting authority has to set a policy-rule referring to the probability distribution of τ. For the sake of simplicity, we assume the following simple rule:[27]

$$E(\tau) = t' \tag{13}$$

where the mean time that F_t, with starting point $F_t > F^*$, takes to reach the upper barrier F^* for the first time is (see Appendix):

$$E(\tau) = m^{-1} \log \left(\frac{C_t}{C^*} \right) + t, \tag{14}$$

with $m \equiv (\frac{1}{2}\sigma^2 - (r - \delta))$ and $C^* = p - F^*$.[28]

According to (14) and the rule (13), the optimal penalty is then equal to:

$$c^* = \frac{\beta - 1}{\beta} r C_t e^{-m(t'-t)} - rp \tag{15}$$

which coincides with (8) when $t' = t$.In all other cases it is obviously smaller.

		c^*		
		$C_t = 0.7$	$C_t = 0.8$	$C_t = 0.9$
$\sigma = 0.3$	$t' - t = 0.1$	0,06415043	0,08045763	0,09676483
	$t' - t = 1$	0,05479616	0,06976704	0,08473792
	$t' - t = 5$	0,02166605	0,03190406	0,04214206
	$t' - t = 10$	-0,0054319	0,00093491	0,00730177
$\sigma = 0.4$	$t' - t = 0.1$	0,09212457	0,11242808	0,13273159
	$t' - t = 1$	0,07643191	0,09449361	0,11255532
	$t' - t = 5$	0,02516637	0,03590442	0,04664248
	$t' - t = 10$	-0,0107597*	-0,0051539*	0,00045179

Table 3: Optimal penalty for different C_t, different $t' - t$ and $\sigma = 0.3, 0.4$ respectively

* The contracting authority has to give a premium to the contractor in order to disincentivate him to invest ahead of time

[27] Depending on different assumptions about the authority's risk aversion, the policy-rule can be made more stringent by giving different weights to different moments of the firm's delivery time distribution.

[28] Obviously m should be positive otherwise $E(\tau) = \infty$ (see Cox and Miller, 1965, p. 221-222).

When $t' - t$ is long (e.g. 10 years) we incur in some sort of paradox: the contractor has to be somehow incentivated, by means of a negative penalty (i.e. a premum) to respect the delivery date instead of investing ahead of time.

4 A concession contract

Concessions differ from public contracts in transfer of the responsibilities for operations that they entail. In concessions a public authority entrusts a third party (i.e. the concessionaire) with the total or partial management of an economic activity for which the concessionaire assumes the operating risk [29].

The aim of the section is to show how the base model of section 2 can be used to design optimal penalty for concession contracts.

Let's suppose that a firm has signed a contract for the provision of a service to be started immediately (i.e. at time t) which requires, on the part of the firm, a completely irreversible capital outlay K (e.g. building an infrastructure). Under these assumptions, the value of the concession for the concessionaire is:[30]

$$G_t = V_t - K \tag{16}$$

where V_t is the project's value (i.e. the future discounted cash flow generated by the project and the provision of the service) at time $t = 0$ and $V_0 > K$.

Let's now assume, analogously to section 2, that the contract includes a clause requiring the concessionaire to pay a penalty if the provision of the service is delayed. In particular:

C) The concessionaire pays a constant penalty d for each period (e.g. day, month, year, etc.) it delays starting of the service. The penalty d is set as a percentage of the investment cost K.

This clause gives the concessionaire the option to defer the investment the value of which is analogous to a perpetual American call option:

$$\Psi_t = E_t(e^{-r(\tau - t)})G_\tau \tag{17}$$

where $G_\tau = V_\tau - K$ denotes the net benefit from investing at a general value of the project $V_\tau > V_t$, r is the risk-free interest rate, τ is the exercise time of the option and V_t is driven by $dV_t = \nu V_t dt + \lambda V_t dz_t$ with $\lambda > 0$ and $\nu \geq 0$.

[29] There are two main types of concession contract: works concessions and service concessions. Directive 93/37/EC distinguishes a work concession from a public works contract by the fact that the concessionaire is granted the right to exploit a construction as a consideration for having erected it. Directive 2004/18/EC defines service concessions as contracts of the same type as public service contracts except for the fact that the consideration for the provision of services consists in the right to exploit the service. A service concession exists when the concessionaire bears the risks involved in establishing and exploiting the service and obtains revenues from users by charging fees or tariffs (http://europa.eu/scadplus/leg/en/lvb/122011.htm).

[30] In practice, the franchise is awarded to the bidder reporting the lowest price or to the firm offering the highest up-front payment (concession fee). In both cases, however, at the time the firm has to start operation these bids are already sunk.

By (17), Ψ_t is the expected and discounted profit deriving from exercising the option to invest when the net benefit has increased to $G_\tau > G_t$ instead of doing it now and obtaining G_t. Then, if $\tau = t$, we get $\Psi_t = G_t$.

Nonetheless, the concessionaire has to pay c per unit of time of the delay. The expected value of the penalty is given by:

$$\Delta_t = E_t\left[\int_t^\tau de^{-r(s-t)}ds\right] = \left[1 - E_t(e^{-r(\tau-t)})\right]\frac{d}{r} \tag{18}$$

According to (17) and (18), the ex-ante concession's value for the firm that decides to defer starting the service till time $\tau > t$, is:

$$\begin{aligned} B_t &= \Psi_t - \Delta_t \\ &\equiv E_t(e^{-r(\tau-t)}\left(G_t + \frac{d}{r}\right) - \frac{d}{r} \\ &\equiv \left(\frac{G_t + K}{G_\tau + K}\right)^\gamma \left(G_\tau + \frac{d}{r}\right) - \frac{d}{r} \end{aligned} \tag{19}$$

where $\gamma > 1$ is the positive root of the fundamental quadratic $\frac{1}{2}\lambda^2 x(x-1) + \nu x - r = 0$. By maximizing (19) with respect to G_τ we obtain:

$$\begin{aligned} G^* &= \frac{1}{\gamma - 1}(K - \gamma\frac{d}{r}) \\ &= \frac{1}{\gamma-1}K - \frac{\gamma}{\gamma-1}\frac{d}{r} \end{aligned} \tag{20}$$

and imposing $G^* = G_t$, we get the optimal penalty:

$$d^* = rK - \frac{\gamma-1}{\gamma}rV_t \tag{21}$$

which, *ceteris paribus*, depends on λ (via γ) and V_t.

Defining $r - \nu = \delta$, it is worth noting that if the concessionaire knows for certain the evolution of the investment's future cash flows, the optimal penalty, say d^{**}, is:

$$d^{**} = rK - \delta V_t$$

and comparing d^{**} and d^* we get:

$$d^{**} - d^* = \delta\left(\frac{\gamma-1}{\gamma}\frac{r}{\delta} - 1\right)V_t$$

Since $\frac{\gamma}{\gamma-1} > \frac{\delta}{r} > 1$, we easily obtain $d^{**} < d^*$. In other words according to the NPV rule the "optimal penalty" is lower than the "optimal penalty" calculated considering the investment timing flexibility arising from the introduction of the clause C).

5 Final remarks

In this paper we investigate the effects that the inclusion of penalty/premium clauses in procurement contracts have on both contract value and investment delay reduction. Starting from the analysis of ongoing procurement contracts and given the high number of delays in delivery date recorded in everyday experience, our intuition is that penalty/premium fees are generally ineffective in avoiding construction risks. We can argue that the presence of a penalty/premium scheme allows the contractor to decide when it is optimal to invest and consequently increase the contract's value. In particular, we have shown that, when incorrectly set, the penalty rule induces the contractor to delay the investment and consequently pay the fee instead of committing himself to observing the contract time.

We extended the model to concession contracts and the results still hold.

A Appendix

A.1 Proof of (12)

Let's consider (11), after some calculations and arrangements we obtain:

$$\begin{aligned}
\Lambda_t &= E_t\left[\int_t^{\min(\tau,t')} 0e^{-r(s-t)}ds + \int_{\min(\tau,t')}^{t'} ce^{-r(s-t)}ds - \int_{t'}^{\max(\tau,t')} ce^{-r(s-t)}ds\right] \quad (22)\\
&= E_t\left[c\left(-\frac{1}{r}e^{-r(t'-t)} + \frac{1}{r}e^{-r(\min(\tau,t')-t)}\right) - c\left(-\frac{1}{r}e^{-r(\max(\tau,t')-t)} + \frac{1}{r}e^{-r(t'-t)}\right)\right]\\
&= \frac{c}{r}E_t\left[-e^{-r(t'-t)} + e^{-r(\min(\tau,t')-t)} + e^{-r(\max(\tau,t')-t)} - e^{-r(t'-t)}\right]\\
&= \frac{c}{r}E_t\left[e^{-r(\min(\tau,t')-t)} + e^{-r(\max(\tau,t')-t)}\right] - 2\frac{c}{r}e^{-r(t'-t)}
\end{aligned}$$

where the optimal exercise time τ is defined as

$$\tau = \min\left(t \geq 0 \mid F_\tau = \arg\max P_t\right) \quad (23)$$

According to (23), at time t, the probability of having a bonus is the probability of having an optimal exercise time τ lower than (or equal to) the contractual time t'. In other words, this is the probability of the geometric Brownian motion F_t reaching the critical value F_τ^* within $[t,\ t']$ starting from an initial condition $F_t < F_\tau^*$. This can be expressed as (Harrison, 1985)

$$\Pr\left(\tau \leq t'\right) = N(s_1) + \left(\frac{F_\tau^*}{F_t}\right)^{2(r-\delta)/\sigma^2-1} N(s_2) \quad (24)$$

where:

$$\begin{aligned}
s_1(F_t, F_\tau^*) &= \frac{\ln(F_t/F_\tau^*) + (r-\delta-\sigma^2/2)(t'-t)}{\sigma\sqrt{t'-t}}\\
s_2(F_t, F_\tau^*) &= s_1 - \left(\frac{2(r-\delta)}{\sigma^2} - 1\right)\sigma\sqrt{(t'-t)}.
\end{aligned}$$

By (24), we rewrite (22) as:

$$\begin{aligned}\Lambda_t &= \frac{c}{r}E_t\left[\Pr(\tau\le t')\left(e^{-r(\tau-t)}+e^{-r(t'-t)}\right)+(1-\Pr(\tau\le t'))\left(e^{-r(t'-t)}+e^{-r(\tau-t)}\right)\right]+\\ &\quad -2\frac{c}{r}e^{-r(t'-t)}\\ &= \frac{c}{r}E_t\left[e^{-r(t'-t)}+e^{-r(\tau-t)}\right]-2\frac{c}{r}e^{-r(t'-t)}\\ &= \frac{c}{r}E_t\left[e^{-r(\tau-t)}\right]-\frac{c}{r}e^{-r(t'-t)}\end{aligned}$$

A.2 Proof of (14)

Let's consider the process C_t on an interval $0 < a < C_t < b < \infty$, with left boundary a and right boundary b. Defining $t_{a,b}$ as the stochastic variable that decribes the time it takes C_t to hit for the first time either a or b, we are able to evaluate the first moment (Saphores, 2002):

$$E(t_{a,b}) = \frac{2}{\kappa^2\sigma^2}\left\{\frac{C_t^\kappa-a^\kappa}{b^\kappa-a^\kappa}\left[\left(\frac{b}{C_t}\right)^\kappa-1-\kappa\log\left(\frac{b}{C_t}\right)\right]+\frac{b^\kappa-C_t^\kappa}{b^\kappa-a^\kappa}\left[\kappa\log\left(\frac{C_t}{a}\right)+\left(\frac{a}{C_t}\right)^\kappa-1\right]\right\}$$

where $\kappa = 1-\frac{2(r-\delta)}{\sigma^2}$.Since $\kappa > 0$, letting $b\to\infty$ and $a \to C^* < C_t$ we obtain the expected time the construction cost takes to reach the lower boundary C^* starting from C_t.

$$\lim_{a\to C^*,b\to\infty} E(t_{a,b}) = E(t_{C^*}) =$$

$$\begin{aligned}&= \frac{2}{\kappa^2\sigma^2}\left\{\frac{C_t^\kappa-a^\kappa}{b^\kappa-a^\kappa}\left[\left(\frac{b}{C_t}\right)^\kappa-1-\kappa\log\left(\frac{b}{C_t}\right)\right]+\frac{b^\kappa-C_t^\kappa}{b^\kappa-a^\kappa}\left[\kappa\log\left(\frac{C_t}{a}\right)+\left(\frac{a}{C_t}\right)^\kappa-1\right]\right\}\\ &= \frac{2}{\kappa\sigma^2}\log\left(\frac{C_t}{C^*}\right)\end{aligned}$$

To prove this limit let's consider the first and the second term separately:

$$\lim_{b\to\infty}\frac{b^\kappa-C_t^\kappa}{b^\kappa-a^\kappa}\left[\kappa\log\left(\frac{C_t}{a}\right)+\left(\frac{a}{C_t}\right)^\kappa-1\right] = \kappa\log\left(\frac{C_t}{a}\right)+\left(\frac{a}{C_t}\right)^\kappa-1$$

$$\begin{aligned}&\lim_{b\to\infty}\frac{C_t^\kappa-a^\kappa}{b^\kappa-a^\kappa}\left[\left(\frac{b}{C_t}\right)^\kappa-1-\kappa\log\left(\frac{b}{C_t}\right)\right]\\ &= \lim_{b\to\infty}\frac{C_t^\kappa-a^\kappa}{b^\kappa-a^\kappa}\left(\frac{b}{C_t}\right)^\kappa-\lim_{b\to\infty}\frac{C_t^\kappa-a^\kappa}{b^\kappa-a^\kappa}-\lim_{b\to\infty}\frac{C_t^\kappa-a^\kappa}{b^\kappa-a^\kappa}\log\left(\frac{b}{C_t}\right)^\kappa\\ &= \lim_{b\to\infty}\frac{b^\kappa}{b^\kappa-a^\kappa}\frac{C_t^\kappa-a^\kappa}{C_t^\kappa}-0-0=\frac{C_t^\kappa-a^\kappa}{C_t^\kappa}=1-\left(\frac{a}{C_t}\right)^\kappa\end{aligned}$$

Putting together the two limits, we get:

$$\frac{2}{\kappa^2\sigma^2}\left\{1-\left(\frac{a}{C_t}\right)^{\kappa}+\kappa\log\left(\frac{C_t}{a}\right)+\left(\frac{a}{C_t}\right)^{\kappa}-1\right\}=\frac{2}{\kappa^2\sigma^2}\left\{\kappa\log\left(\frac{C_t}{a}\right)\right\}$$

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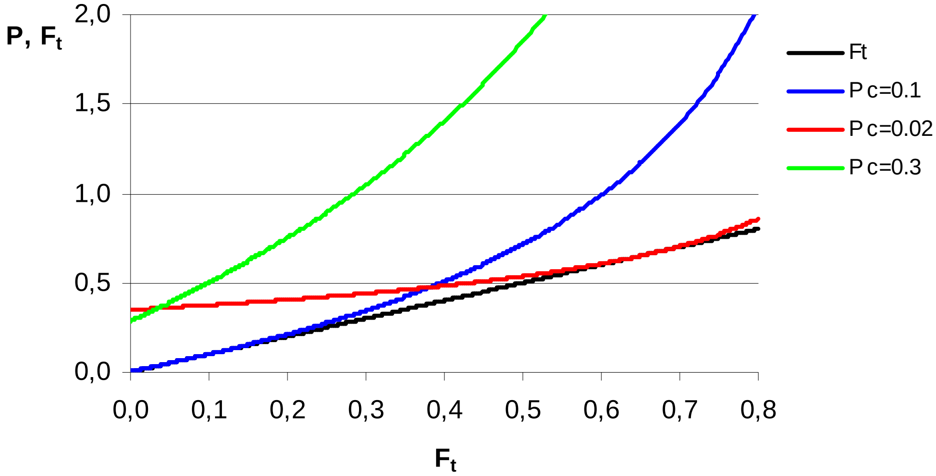

Figure 1 - F_t and P for $c = 0.02, 0.1, 0.3$ and $\sigma = 0.4$

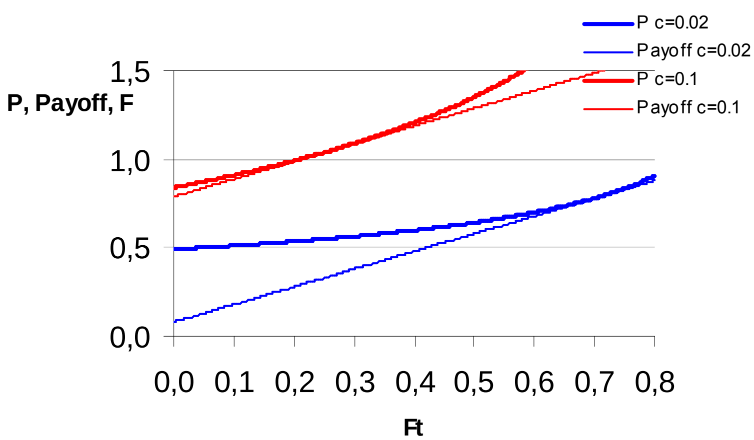

Figure 2 - F and P for different c and $t' - t = 10$ years

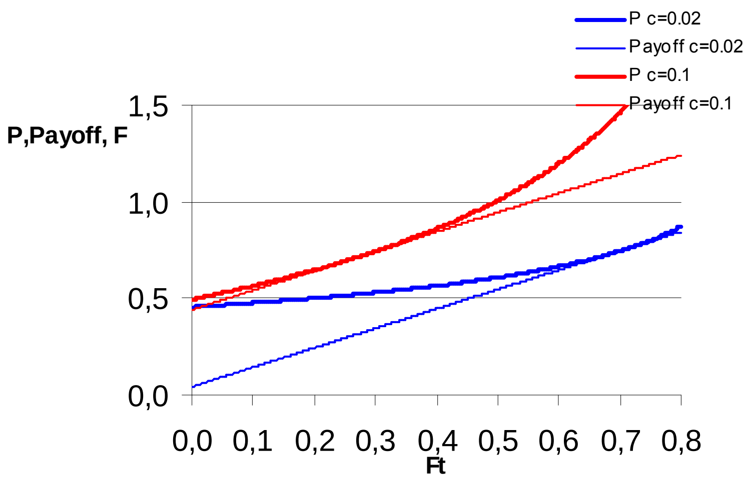

Figure 3 - F and P for different c and $t' - t = 5$ years

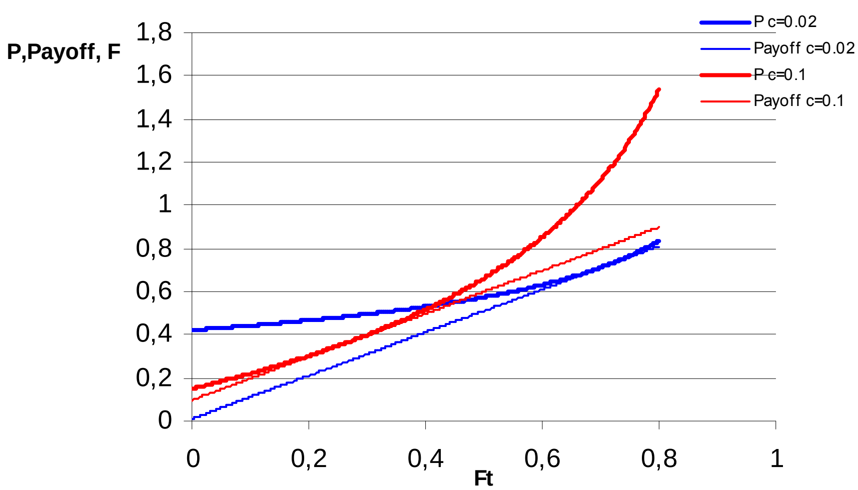

Figure 4 - F and P for different c and $t' - t = 1$ year

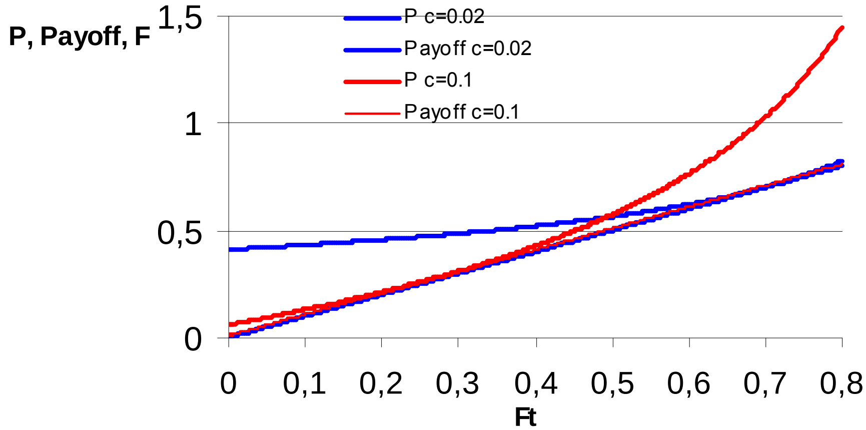

Figure 5 - F and P for different c and $t' - t = 0.1$ year